ESSENTIALS OF
Applied Physics

A FOUNDATION COURSE FOR
TECHNICAL, INDUSTRIAL, AND
ENGINEERING STUDENTS

By

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Preface

Physics is a prerequisite for courses in the curriculum of junior colleges, evening engineering schools, technical institutes, and advanced trade schools, owing to the fundamental position of the subject in all branches of engineering work. This book is one of a series of applied science textbooks designed to meet the needs of schools where a more concise course is given than is found in the average college physics textbook, and where numerous topics not found in a preparatory course in physics are essential.

The orthodox arrangement of, first, mechanics, then sound, heat, electricity, and light is followed. Numerous illustrative problems are completely worked out. A summary of the irreducible minimum of algebra, geometry, and trigonometry necessary for a clear understanding of physics is included in the appendices.

Modern viewpoints on light have been employed, while at the same time the full advantage of the wave theory of light has been retained. The electron current is used exclusively, rather than the conventional positive current. The practical electrical units are used instead of the two c.g.s. electrical systems of units. As preparation for this, the kilogram-meter-second system, as well as the English system of units, is used in mechanics. Likewise the kilogram-calorie is used instead of the gram-calorie.

This work is the outgrowth of the author's experience in teaching engineering physics to many groups of students in evening engineering schools. The material was developed and tested in the class room over a period of many years. It has proven effective for students whose needs for practical and applied knowledge of mechanics, heat, light, and electricity were paramount.

Acknowledgments

The pen sketches at the heads of the chapters and some of those in the body of the text are the contributions of Louise A. Frye. The diagrams, in addition to many of the pen sketches, were done by Ralph E. Wellings. A great many of the illustrative problems, as

well as the index, were prepared by Virginia M. Brigham, who also typed the manuscript. The author is indebted to Robert E. Hodgdon for numerous suggestions made during the course of many years' association in the teaching of the physics of engineering.

It is impossible for the author to make adequate acknowledgement to a long line of predecessors in the field of physics to whom he is indebted.

ROYAL M. FRYE

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Contents

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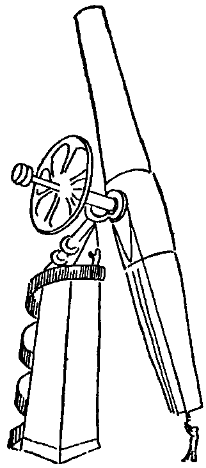
CHAPTER I



Introduction

1-1. Why Study Physics? By far the larger group of subjects in the curriculum of the average school is that containing history, psychology, biology, sociology, languages, and philosophy, which depend for their importance on their direct relations to living, intelligent beings. The smaller group contains, for example, physics, chemistry, astronomy, and geology, all of which deal with inanimate nature; we study these either out of a sheer desire for knowledge for its own sake, or because of possible applications of this information in our daily lives. Mathematics occupies something of a middle position; it consists of a set of rules in accordance with which a series of operations are performed, but in this case it is we who devise the rules. All we ask of these rules is consistency. Most of us hope that the mathematical rules will also be useful (and it is true that they usually are); yet there is gossip to the effect that certain mathematicians have been guilty of praying that no practical use would ever be found for their particular creations.

But mathematics is a subject that requires rigorous concentration for its mastery, and therefore is not overpopular. Much of physics is hidden from the nonmathematician. The demand for physicists considerably exceeds the supply. This book contains a minimum of mathematics. It is written for those who quite frankly intend to use physics as a prerequisite for engineering.

1-2. What Is the Territory of Physics? Pure physics concerns itself with things that our senses reveal to us: heat, electricity, natural forces, forms of energy, properties of matter, sound, and light; we also find it convenient to add to this list all sorts of devices made by man which depend on a knowledge of natural phenomena. By means of the telescope and spectroscope, the sense of sight is extended to such enormous distances that we are enabled to tell the sizes, chemical constitutions, temperatures, physical states, amount, and direction of motion of objects completely invisible to the naked eye. We also have knowledge of particles so small that they are beyond the power of being made visible by the best optical or electron microscope that man has yet invented. And the science of physics is still growing. We continue to observe facts about nature. We are still inventing theories to fit these facts. The theories often lead us to suspect the existence of new facts as yet undiscovered. Then we carry out experiments in search of these supposed new facts. Sometimes we discover that the "facts" do not exist, and as a result we have to throw away the theory which involved them. If on the other hand the facts are there, our respect for the theory increases. *Physics is a study of the facts of the nonliving part of nature together with those interconnecting theories that so far have stood the test of experiment.*

1-3. Why Is Physics the Basis of All Engineering Training? Engineering schools train students to be civil engineers, mechanical engineers, metallurgical engineers, electrical engineers, illuminating engineers, biological engineers, chemical engineers, sanitary engineers, marine engineers, torpedo engineers, public health engineers, naval engineers, and aeronautical engineers. Almost anyone reading this list will take pleasure in adding to it. But all of these branches of engineering grow directly from the subdivisions of physics itself or from the closely associated sciences of chemistry and biology. Physics itself includes at present the subjects of mechanics, sound, heat, magnetism, electricity, and light. Formerly all the natural sciences combined, including physics, chemistry, astronomy, and biology, were considered to be within the capabilities of single individuals to master. But as these sciences grew in scope, it became increasingly difficult for any one man to master them all, or even any one of them. Today it is the business of chemistry to study several hundred thousand compounds; of astronomy, to catalogue nearly 100 billion stars in our own galaxy along with a billion other galaxies; and of biology, to classify hundreds of thousands of zoological and botanical species. Yet in these three sciences the relationships between entities are far more important than the large numbers of entities involved. And even as chemistry and astronomy are already considered as separate sciences, it may well be that other portions will in the future be detached from physics, but physics will still remain basic, not only to the other "physical sciences," but to all branches of engineering.

1-4. Physical Facts. The two important things in our universe as we know it are *energy* and *intelligence*. The latter we leave to psychologists, biologists, and philosophers, and confine our attention to the former. At a suitable point, we shall define energy, and later we shall see that one of the manifestations of energy is *matter*. For the present, however, we shall find it convenient to take over a few terms from everyday life such as *time* and *space*, and by means of these, define more terms for technical use. Once we have defined a technical term, we shall be careful not to use that word in any other way, and physical facts of a general type (often called *laws* or *principles*) will be stated using these technical terms. Although *matter* is sometimes defined as that which occupies space, we must remember that a vacuum (absence of matter) also occupies space, and furthermore that a vacuum has pronounced physical properties. Consequently it will be better at present to think of matter as the substance of which physical bodies are made, and reserve until later a discussion of the method of measuring quantity of matter, or *mass*. We may temporarily think of *energy* as a storehouse out of which comes the ability to change either the shape or the state of motion of matter. A *physical fact* may be described as something that actually can be demonstrated in the laboratory to a high degree of precision (although never to a precision of one hundred per cent, for both practical and theoretical reasons). We shall not be surprised at the necessity of discarding a theory occasionally for a better one, but we do expect our physical facts, once established, to remain physical facts.

1-5. Physical Theories. A large collection of isolated physical facts without any interconnecting theory would be hard to keep in mind, and for this reason would lose much of its usefulness to the engineer. The mathematical network, as self-consistent as geometry, which has been developed slowly over the years, and which weaves together the vast accumulation of physical data into one integrated whole, is referred to as *physical theory.* Thus we talk of the theory of elasticity, electrical theory, theory of light; or even in connection with mechanical devices we are apt to ask, "What is the theory back of that machine?" The importance of theory, however, increases as the student becomes more advanced. In this elementary treatment of physics, we shall be much more concerned with facts than with theory.

1-6. Units. In concluding this chapter, it is proper to say a few words about *units.* Outside of the field of electricity, the engineer finds that he can get along very well with just three fundamental units: a unit of time, say the *second;* a unit of distance, such as the *foot* or the *meter*; and a unit of force, for example the *pound* or the *newton.* In defining the *second,* it is customary to divide the length of the average "solar day" into 86,400 equal parts ($24 \times 60 \times 60$). The second is common to both the English and metric systems. As a basis for the units of the metric system, there are carefully preserved two pieces of metal at as nearly as possible constant conditions. The distance between two fine scratches on one of them is taken by the scientific world as the definition of the *meter,* and the mass of the other piece of metal defines the *kilogram.* A *newton* is somewhat smaller than the kilogram; a kilogram weighs about 9.8 newtons. In London there exist similarly the *standard yard* and the *standard pound.* Such units as the *foot per second,* the *foot-pound,* and so on are obvious combinations of these fundamental units. There are 3.2808 feet in a meter, and 2.2046 pounds in a kilogram. In the United States, we are legally on the metric system; our *foot* is *defined* as $\frac{1200}{3937}$ of a meter, and our *pound* as $\frac{1}{2.204622}$ of a kilogram.

SUMMARY OF CHAPTER 1

Technical Terms Defined

Physics. Physics is a study of the facts of inanimate nature together with the theories that thus far have stood the test of experiment.

Fact. Facts, in physics, are the direct result of physical experimentation and observation.

Theory. An assumption or system of assumptions not only mutually consistent, but also consistent with all known facts.

Physical Unit. An arbitrary portion of a physical quantity, of a convenient size, and established by general agreement.

Second. $\frac{1}{86400}$ of a mean solar day.

Meter. Distance at the temperature of melting ice between two scratches on a platinum-iridium bar preserved at the International Bureau of Weights and Measures, Paris, France.

Yard. In England, the distance between two scratches on a standard bar preserved at London. In the United States $\frac{3600}{3937}$ of a meter. This makes one meter equal to 3.2808 feet.

Kilogram. The amount of matter in a certain platinum cylinder also preserved at Paris, France.

Newton. A unit of force or weight which will be found to tie in with both the metric system and the practical system of electrical units. One kilogram weighs about 9.8 newtons.

Pound. The United States pound is defined by law as $\frac{1}{2.204622}$ of a kilogram.

EXERCISES AND PROBLEMS

1-1. Name ten practical illustrations of physical principles, so distributed that at least one application will be drawn from each of the five branches of physics.

1-2. As an illustration of the terms *fact* and *theory*, state a nonphysical fact; also a nonphysical theory.

1-3. Mention several important industries of today which owe their existence entirely to theories developed during the previous century.

1-4. From the data in section 1-6, find the number of inches in a meter; also the number of kilograms in an ounce.

1-5. How many newtons are there in a pound?

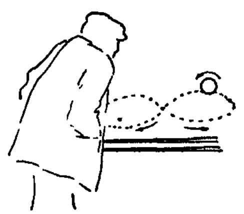
1-6. If there are 62.4 pounds of water in a cubic foot, find the number of kilograms of water in a cubic meter.

CHAPTER 2

Newton's Laws

2-1. Historical. One of the earliest books on physics was written by Aristotle (385–322 B.C.). He was a remarkable man, and is credited with having possessed the most encyclopedic mind in all history. However, Aristotle lived before the experimental era, and for this reason he made many statements that could have been disproved easily by simple trial. One of these statements, concerning falling weights, was not shown to be false until the time of Galileo (1564–1642). Galileo made numerous scientific discoveries, but due to ecclesiastical and civil opposition, he never reached the point of generalizing his findings; on the contrary, he was forced to renounce some of them as false! Sir Isaac Newton (1642–1727) was born in England the year Galileo died in Italy. He too had a most unusual mind, and an almost uncanny sense regarding physical phenomena. Moreover, he had the advantage of living at a time when it had become customary to perform scientific experiments before drawing physical conclusions. Newton published a book in 1687 (written in Latin, which was then a universal scientific language), in which he summarized Galileo's work in the form of three laws that are known to this day as Newton's first, second, and third laws respectively. These laws are the basis of what is known as *Newtonian mechanics*. They hold for distances somewhat greater than those between atoms up to astronomical distances. (Advanced students will learn that, for atomic dimensions, we have to use what is known as *quantum mechanics*, a form of mechanics which automatically becomes Newtonian mechanics with increased distances). Therefore, for the purposes of the engineer, there is no need of questioning the exactness of Newton's laws.

2-2. Newton's First Law. If we should pass by a store window in which a croquet ball was busily engaged in rolling about in such a way as to describe figure eights, our intuition would tell us,



"Something is wrong; there is more here than meets the eye!" We all have in mind a notion of what an object ought to do when left to itself, and it is *not* to describe figure eights. If we start an object sliding along a smooth surface, then leave it to itself, the object will move more and more slowly in a straight line and finally come to rest. If we repeat the experiment on a still smoother surface, say some glare ice, the object will take much longer to come to rest, and still continue to travel along a straight line. But it is not correct in either of these two cases to say that the object is left to itself. In both cases, forces of friction were slowing down the moving object. If there were actually zero friction, the object would never come to rest when "left to itself." This statement constitutes a part of Newton's first law. A more complete statement is as follows: *A body left to itself will remain at rest if it is already at rest, and if it is already in motion, it will continue in motion with uniform velocity in a straight line.*

Newton's first law represents such an idealization that we never encounter a pure case of it in practice. No object that we have ever met can be said to be "left to itself." Gravitation is always present to pull objects toward the earth; friction or air resistance is always acting to slow down the motion of bodies. In fact, it would even be difficult to say just what we mean by "at rest." Any table in front of us which appears to be at rest is moving about 700 miles per hour due to the rotation of the earth, about 66,000 miles per hour due to the earth's orbital motion about the sun, and faster yet on account of galactic rotation. In general we consider it a sufficiently good illustration of Newton's first law if we find ourselves nearly plunging over the seat in front of us on a trolley when the motorman suddenly applies the brakes. We were in motion and physical law does its best to keep us in motion! Another illustration is the possibility of removing a book from under a pile of books by means of a quick jerk. The books on top were at rest and they therefore tend to remain so.

The property of matter by virtue of which it is necessary to apply a force in order to change its condition of rest or motion is called the *inertia* of matter. The inertia of a body at a particular point on the earth's surface is proportional to (but not equal to) its weight.

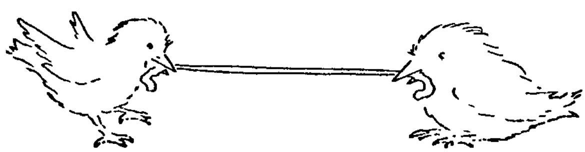
2-3. Technical Terms. By the time we have completed this course in physics, we shall find ourselves using in a very particular way a list of something under a hundred words, one of which (inertia) was mentioned in the previous section. Many of these words are already familiar to us; we shall merely restrict their rather long list of everyday meanings to some one scientific meaning. *Time* and *distance* may well appear at an early point on our list of technical terms. *Time* in physics means *measured* or *measurable duration*, and nothing else. It does not mean the pleasant or unpleasant evening we have just spent or the jail sentence we did or did not serve. Similarly, *distance* in physics means *measured* or *measurable space*, and nothing else. It does not mean the separation in relationship between a couple of third cousins, or the lack of cordiality in manner affected by one's former friend. Another word that we must define to get well started on our subject is *force.* A *force* is defined as *that which will tend to produce a change in the size or shape of an object.* A force will also produce other effects such as changes in the motions of objects, but this relationship will be reserved to enable us to define *mass* when the time comes. Everyday terms which are practically equivalent to *force* or at least special cases of *force* are: push, pull, resistance, tension, effort, attraction, repulsion, friction, thrust, compression, and so on. With combinations of the three words, *time*, *distance*, and *force*, we shall find it possible eventually to produce a fairly comprehensive list of technical terms.

2-4. Newton's Second Law. The next question to be asked concerns the behavior of an object when it is *not* left to itself, that is, when a push or a pull is applied. Under these conditions the object deviates from its uniform straight-line motion in accordance with the size and direction of the force that is being applied. This is Newton's second law. If the force is applied to the object in the direction of the motion and not balanced by an opposing force, such as friction, the object will move faster and faster. In practice it becomes difficult after a time to continue to apply this unbalanced force, otherwise there would be no limit to the velocities which could be acquired.

2-5. Newton's Third Law. A force is always exerted *by* some object *on* some other object. The only one of these bodies that interests the engineer is the one *on* which the forces act; these forces determine the subsequent motion of the object involved, or help to hold it in equilibrium, or tend to change its size or shape. A force that is exerted *by* an object will have no direct influence on that object, but will affect some other object *on* which the same force is being exerted. Newton's third law, however, states that forces always exist in pairs, and that a force exerted *on* an object is to be paired with an equal and opposite force exerted *by* the object, the latter being of no interest unless we decide to include in our investigation the other body upon which that happens to be acting. A more useful statement of Newton's third law will include for each force the object exerting the force as well as the object upon which the force is exerted. Thus, Newton's third law may be restated as follows: *If body A exerts a force on body B, then under all conditions and with no exceptions, body B will simultaneously exert an equal and opposite force on body A.* From what has just been said, it will be clear that only one of these two forces will affect body A and the other will affect body B. A common way of stating this law is to say that "action and reaction are equal."

Isaac Newton would be somewhat surprised if he should return to earth and hear some of the erroneous statements occasionally made about this law. For example, one such boner makes the law apply only at uniform speeds, and another leaves the impression that it applies only in such cases as tugs of war with the teams evenly balanced. But as a matter of fact, if there were any exceptions at all to this law, one of the most important generalizations of all physics would cease to hold, namely the law of conservation of energy. THERE ARE NO EXCEPTIONS TO NEWTON'S THIRD LAW.

Another point in connection with this law sometimes disturbs the student. If the two forces involved in the law are always equal and opposite to each other, why do they not balance each other, and since there are no cases where the law does not hold, then how can



anything ever happen in the physical universe? A careful reading of the first paragraph of this section will help to answer this question. Two forces will never balance each other unless they act upon the same body. For example, if I exert an upward force of 25 pounds upon a suitcase and you exert a downward force of 25 pounds upon some spot on the floor, the forces will be equal and opposite, but they will not balance each other because they act upon different bodies. Body A and body B are two different bodies, and since one force is exerted on each, there is no chance of the forces balancing each other.

2-6. Examples of Forces Which Do and Do Not Illustrate Newton's Third Law. The two sparrows and the worm furnish several illustrations of Newton's third law as well as several combinations of forces that do not illustrate the law. The ground pushes up on the left-hand sparrow and this sparrow pushes down on the ground with an equal and opposite force. This illustrates the law. But these two forces do not balance each other because one acts on the sparrow and the other on the ground. The left-hand sparrow exerts a force toward the left on the worm and the worm exerts an equal force toward the right on the sparrow. These forces also illustrate the law, and again they do not balance each other because one force acts on the worm and the other on the sparrow. Now consider some forces that do balance each other and therefore do not illustrate Newton's third law. Gravity pulls down on the left-hand sparrow and the ground pushes up on this sparrow. These forces are equal and opposite to each other and they both act upon the same object, the sparrow. They balance each other but they do not illustrate Newton's third law. Similarly both sparrows exert opposing forces on the worm. If these forces are numerically equal, they balance, but they do not illustrate Newton's third law. When two forces balance each other, they do not illustrate Newton's third law; and when two forces illustrate Newton's third law, they do not balance each other.

2-7. Newton's Law of Gravitation. It has long been understood that bodies free to do so "fall," but it was not until the time of Sir Isaac Newton that the relations between the forces and the masses of the objects involved were clearly stated. Since the forces are small except when objects of astronomical size are concerned, it will probably be best to get the astronomer's point of view in this discussion, although once more the law under consideration is perfectly general and holds *between* two small objects just as well as for two large objects like the sun and earth. The sun and the earth each exert an attracting force on the other. By the third law, stated in the previous section, the forces exerted by the sun and earth on each other are equal and opposite; by the law now about to be stated, these forces each depend on the distance between, as well as on the masses of, the sun and the earth. As the mathematician would put it, either of these forces is directly proportional to the product of the two masses and inversely proportional to the square of the distance between them. But, if we are not yet expert mathematicians, it may be well to put it somewhat differently. Any object in the universe exerts a gravitational attraction upon every other object. When we compare these attracting forces we find two things to be true: (1) *if* we double the mass of either body the force will also double, or if we increase either mass by any number of times the force will increase the same number of times, and (2) *if* we double the distance between these objects the force will be reduced to one quarter of the original value, or if we multiply the distance between the bodies by any factor, the new force will be found by dividing by the square of this factor.*

2-8. Illustrations. As an illustration of how small the force of gravitation is for small objects, imagine two spheres made of about the heaviest material on our planet. Gold is 19.3 times, and osmium 22.5 times as heavy as water. Let one of these spheres weigh one ton and the other two tons, and let their centers be two feet apart. The gravitational force of attraction that either of these spheres would exert on the other would figure out to be just under half a grain (0.466 grain). Since there are 7,000 grains to the pound avoirdupois, this is rather a feeble force. Yet if one of the two objects to be considered is the earth itself (about 6,570,000,000,000,000,000,000 tons) and the other is, say a one-pound body on the earth's surface, with the centers of the two objects now four thousand miles apart (approximately the earth's radius), then either will exert on the other an attracting force of one pound. The gravitational force exerted by the earth on some object upon its surface is known technically as the *weight* of this object. If our one-pound body is now removed from the surface of the earth to a distance of 240,000 miles, which is about sixty times as far from the earth's center, its weight will then be reduced to $\frac{1}{3600}$ of a pound. However, its mass, that is, the amount of matter in it, will still remain the same.

2-9. How the Law Was Discovered. The story of the discovery of the law of gravitation involves principally three men: Tycho Brahe (1546–1601), a Danish astronomer; Johann Kepler (1571–1630), a German astronomer and mathematician; and Isaac Newton. Brahe made a series of painstaking observations on the positions of the planets of our solar system over a considerable period of time, making no particular effort to deduce anything therefrom. With Brahe's mass of data before him, Kepler drew the conclusions (1) that the paths of the planets about the sun are ellipses, and not circles as had been previously supposed, and that the sun was at a focus and not at the center, (2) that a line, joining the center of the sun with the center of the planet, sweeps over equal areas in

* The mathematical statement of this law is as follows. If m_1 and m_2 represent two masses expressed in kilograms, d is the distance between them in meters, and F the gravitational force in newtons pulling each mass toward the other, then

$$F = \frac{m_1 m_2}{d^2} k_g,$$

where k_g is the physical quantity 6.66×10^{-11} newton-meter² per kilogram².

equal times, and (3) that the time that it takes for each planet to go around the sun once is proportional to the square root of the cube of its average distance from the sun. But Kepler made no effort to answer the question as to why the planets behave in this way. This was Newton's problem; he discovered that the law of gravitation described in section 2-7 would just account for Kepler's conclusions. As a final test, Newton then tried his law of gravitation on the moon, using data then available. To his dismay, the law failed to account for the data; so Newton tucked his work away in a drawer and busied himself with other things. Years afterward, his attention was called to new data bearing on the problem; he dug out his nearly forgotten work, and lo! it now checked beautifully. And so he published his results without further delay.

2-10. Illustrative Problem. If a man can leap five feet in a standing high jump contest on the earth, how high could he leap in a similar contest on the moon, assuming (1) that he had the same strength on the moon as on the earth, (2) that the mass of the moon is one eightieth of that of the earth, and (3) that the radius of the moon is one fourth of the earth's radius?

Solution: If we first focus our attention on the fact that the two objects involved in the problem are now the man and the moon instead of the man and the earth, remembering that the force of gravitation increases when either mass increases and vice versa, our first conclusion is that the man now appears to weigh only one eightieth as much on the moon as on the earth. This would mean that he could jump eighty times five feet or 400 feet! But the gravitational force also depends on the distance between the gravitating bodies. On the moon the centers of the two objects (the moon and the man) are only one fourth as far apart as on the earth, and since four squared is sixteen, the force of gravitation will be increased sixteen times on this account. This will decrease the height to which he can jump by a factor of sixteen. Four hundred divided by sixteen is twenty-five; our answer is therefore that the man can leap to a height of twenty-five feet on the moon. He could thus jump over a small house without difficulty.

SUMMARY OF CHAPTER 2

Technical Terms Defined

Time. *Time* is measured or measurable duration.

Distance. *Distance* is measured or measurable space of one dimension.

Force. A *force* is that which will tend to produce a change in the size or shape of an object. Familiar synonyms are *push, pull.*

Inertia. *Inertia* is that property of matter which causes it, in the absence of all forces, to remain at rest when at rest, and when in motion, to continue in motion at uniform speed in a straight line. An equivalent definition is: *inertia* is that property of matter by virtue of which it is necessary to apply a force in order to change its condition of rest or motion.

Gravitation. The phenomenon that any two particles in the universe exert attracting forces on each other.

Weight. The gravitational attraction in the special case when one of the objects concerned is the earth and the other object is a body on the surface of the earth. Common synomyms are "the pull of gravity" or "the force of gravity."

Laws

Newton's First Law. Inertia is a universal property of matter.

Newton's Second Law. When a force is applied to a body, the body will deviate from its condition of rest or uniform straight-line motion in accordance with the size and direction of the force that is being applied.

Newton's Third Law. If one body exerts a force upon a second body, the second body will simultaneously exert an equal and opposite force upon the first body.

Newton's Law of Universal Gravitation. Every particle in the universe exerts on every other particle a force of attraction that is directly proportional to the masses of the two particles, and inversely proportional to the square of their distance apart.

EXERCISES AND PROBLEMS

2-1. A man standing on a stepladder pushes downward on the stepladder. According to Newton's third law, by what is the reacting force exerted, and on what is it exerted?

2-2. If every force is accompanied by an equal and opposite reacting force, why do the acting and reacting forces never balance each other? How would one proceed to accomplish something in a physical world that was put together on the basis that whenever a force acted on a body, another force, equal and opposite to the first, acted on the same body?

2-3. If one of the sparrows in section 2-5 pulled harder than the other, would there still be an illustration of Newton's third law in the sketch? Give a reason for your answer.

2-4. In order for an automobile to start forward from rest, something must push forward on the car. Can the car itself exert this forward push on itself? What outside agency is capable of exerting a forward push on the car, and why?

2-5. Give an illustration of Newton's first law.

2-6. Give an illustration of Newton's second law.

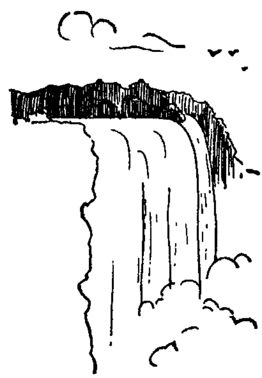
2-7. If the sun weighs 300,000 times as much as the earth and has a diameter 100 times that of the earth, how much would an object that weighs 150 pounds upon the surface of the earth weigh upon the surface of the sun?

2-8. If a 2,000-pound projectile can be made to rise 100 miles above the surface of the earth, what does it weigh at this altitude?

2-9. The earth is flattened at its poles. Where would a certain gold brick weigh more, in Alaska or in New York?

2-10. Isaac Newton is said to have constructed a horseless carriage which went by jet propulsion. Which one of his laws was involved most in the operation of the carriage?

CHAPTER 3



Force; Work; Energy; Power

3-1. Force. Consider a boy pushing a sled along a straight route. The sled is traveling in a more or less irregular fashion subject to two varying forces, one of which opposes the motion and is due to friction, while the other is the force exerted by the boy pushing the sled. If the two forces were uniform as well as equal and opposite, then, in accordance with Newton's first law, the sled would continue moving along its straight path with constant speed if it were already moving, or remain at rest if it were already at rest. If the route remains straight, the greatest complication that we can have in the matter of forces will be as to whether they are positive or negative, that is, whether the forces act along the line in one direction or the other.

3-2. Work. In physics, the word *work* has a very limited and important use. Technically, no work is done unless something moves. An artist's model does no work in the physical sense while he is posing, although the model earns a living thereby and is tired at the end of a long session. In order to compute the numerical value of the work done on a body, it is necessary to multiply the force exerted on the body by the distance that the body moves. Also, in computing the work done, the force that is multiplied by the distance must be parallel to that distance. If we multiply a force, expressed in pounds, by a distance, expressed in feet, the product is said to be expressed in foot-pounds. We shall, then, define *work* as *the product of a force acting on a body multiplied by the distance through which the body moves, in a direction parallel to the force.*

If the boy mentioned in the preceding section exerts a constant forward force of 20 pounds on a sled which during the process moves forward 50 feet, we may compute the work he has done by multiplying 20 pounds by 50 feet. The product has a two-fold aspect: one part is numerical, 1,000 in this case, and the other feature is the unit involved, foot-pounds, obtained by combining the feet with the pounds through the use of a hyphen. Thus the work done by the boy is 1,000 foot-pounds. Other possible units of work are foot-tons, newton-meters (which are called *joules*), and so on; any unit of force may be hyphenated with a unit of distance to obtain a unit of work.

Work also may be negative. Suppose a man to walk along a track behind a slowly moving freight car for 10 feet, exerting a backward force of 40 pounds on the car. Since the force and the distance are in opposite directions, it is customary to call one of them positive and the other negative; which is which is immaterial so long as we make a definite choice. The product of a positive 10 feet and a negative 40 pounds is a negative 400 foot-pounds of work done by the man on the freight car.

Another illustration of negative work can be obtained from the boy-sled problem. Suppose a force of friction of 20 pounds to oppose the motion of the sled throughout the 50-foot distance. Then the work would be a negative 1,000 foot-pounds of work done by friction on the sled.

Another way of looking at negative energy may seem more reasonable. According to Newton's third law, when a man exerts a backward force of 40 pounds on a freight car, the freight car exerts a forward force of 40 pounds on the man. If we multiply this forward 40-pound force by the distance the man moves, 10 feet, we obtain a positive 400 foot-pounds of work. That is, while the man does a negative 400 foot-pounds of work on the car, the car does a positive 400 foot-pounds of work on the man. Since this happens every time work is done, it begins to look as if work is something that can not be created out of nothing; whenever there is positive work done, negative work is also done to the same extent. Of the two objects or bodies involved, one is the giver and the other is the recipient; something is passed along.

3-3. Energy. Conservation of Energy. The "something" that is involved in the work situation mentioned in the previous section is called *energy*. *Energy* is *the ability to do work;* it is measured in terms of the same units that we use for work, foot-pounds, joules, and so on. The idea at the end of the previous section may thus be expressed in terms of energy: whenever a body gains energy it is always at the expense of some other body. It therefore seems reasonable to assume that the total energy in the universe is a constant. This statement is known as the "law of conservation of energy," which is a direct consequence of Newton's third law, and to the present, no exceptions to it have been found either in nature or in the laboratory. The law of conservation of energy denies the possibility of such things as perpetual motion machines, which are efforts to create energy out of nothing.

In the boy-sled illustration, there were two transfers of energy: 1,000 foot-pounds of energy passed from the boy to the sled and 1,000 foot-pounds of energy were transferred to the surfaces responsible for the frictional force, this time into the form of heat-energy. Frictional forces may or may not be accompanied by motion. When there is motion against friction, heat energy is always developed, which may be computed by multiplying the frictional force by the distance. If there is no motion, no energy relations are involved.

3-4. Illustrations of Energy. A partial list of the many ways in which it is possible to do work and therefore to store energy is as follows: (1) by compressing a gas, which can expand and give the work back, (2) by coiling a spring, which can uncoil and drive a watch, (3) by raising a weight, (4) by setting a body in motion, (5) by raising the temperature of a body, (6) by changing the state of a body from solid to liquid or from liquid to gas, (7) by charging a storage battery or condenser, (8) by creating a magnetic field, (9) by creating waves in liquids or solids, or sound waves in air, and (10) by producing radiation, which may travel for thousands of years from one star to another. There are other forms of energy which are quite interesting. If we should divide a piece of paper into sufficiently small pieces, there would come a time at length when any further subdivision would result in substances that were no longer paper, but more simple chemicals, namely, carbon, hydrogen, and oxygen. The smallest portion that could still be called paper is termed a molecule; one molecule of paper consists of a group of carbon, hydrogen, and oxygen atoms. Random molecular motion, that is, haphazard motions of individual molecules relative to each other, constitutes heat energy. Rearrangements of atoms to form different kinds of molecules involve changes in chemical energy; the burning of paper, the rusting of iron, and the explosion of gunpowder are examples. The atoms themselves are also complicated structures which we shall describe later. It is difficult to disintegrate atoms; for many years scientists considered it completely impossible. But when certain atoms are split up (for example, by exposure to slowly moving subatomic entities called neutrons), tremendous energy changes take place such as those involved in radioactivity, both natural and artificial, and in "atomic bombs"; when the atom is split, we notice that some matter disappears and energy appears in its place. That is, matter itself is one form of energy.

All sorts of energy can be converted into all other kinds of energy, but to varying extents. For example, any form of energy may be converted one hundred per cent into heat energy, but it is possible to convert only a relatively small per cent of heat energy into mechanical, electrical, chemical, or other forms of energy.

3-5. Potential Energy. The type of energy that results from raising a weight ((3) in section 3-4) is called *energy of position*, or *potential energy*. When we wind a cuckoo clock we store energy of this kind. To find its value we have merely to compute the work done in raising the weight. The necessary force is upward and is equal numerically to the weight W lifted; the distance h is also upward, therefore the potential energy is the product of W and h, and may be measured in foot-pounds, joules, or any other convenient unit of work.

$$\text{Potential energy} = Wh$$

Putting two quantities next to each other, as Wh, implies multiplying one by the other. Division is represented in algebra by the fraction notation; for example six divided by two equals three would be written $\frac{6}{2} = 3$.

3-6. Kinetic Energy. The type of energy that results from setting a body in motion ((4) in section 3-4) is called *kinetic energy*, or *energy of motion*. It will be discussed again in connection with the question of change of velocity, but its formula will be stated here for reference purposes.

$$\text{Kinetic energy} = \frac{Wv^2}{2g}$$

In this expression, v is the velocity of a moving body the weight of which is W, and g represents a constant the numerical value of which is 32.2 feet per second per second or 9.80 meters per second per second. We have met this constant in its metric form as the ratio between a kilogram and a newton. Both forms will appear on numerous occasions again. Note that when W is in pounds, v in feet per second (which may be written ft/sec.), and g in feet per second per second (which may be written ft./sec.^{2}), the kinetic energy will come out in foot-pounds. That is

$$\text{foot-pounds} = \frac{(\text{pounds})\left(\dfrac{\text{feet}}{\text{second}}\right)^2}{\dfrac{\text{feet}}{\text{second}^2}}$$

If W is in newtons, v in meters per second, and g in meters per second squared, the kinetic energy will come out in newton-meters (or joules).

3-7. Illustrative Problem. One boy is drawing another on a cart. The tension in the tongue (which is horizontal) is 30 pounds. Find the work done in drawing the cart a horizontal distance of 200 feet. Express the answer both in foot-pounds and in joules.

The product of the horizontal forward force (30 pounds) and the horizontal distance (200 feet) is 6,000 foot-pounds, which is one answer. Since there are 4.45 newtons in a pound (see problem 1-5) and 0.305 meter in a foot, the forward force of 30 pounds may be expressed as 133.5 newtons and the 200 feet as 71.0 meters. Therefore the work may also be expressed as the product of 133.5 newtons by 71.0 meters or 9,480 newton-meters, or 9,480 joules, which is the second answer required.

3-8. Second Illustrative Problem. A ball weighing one pound is thrown vertically upward with a speed of 100 feet per second. What is the kinetic energy given it by the thrower? If the ball continues to rise until its kinetic energy is zero and its potential energy is equal to the original value of the kinetic energy, how high will it rise?

Since the kinetic energy of the ball is $Wv^2/2g$, where $W = 1.00$ pound, $v = 100$ feet per second (this is written 100 feet/second because distance must be divided by time to get the speed), and $g = 32.2$ feet/second^2, the kinetic energy is $(1.00)\ (100)^2/(2)\ (32.2)$ or 155.3 foot-pounds.

If the ball rises until the kinetic energy is zero and the potential energy is 155.3 foot-pounds, we must set $Wh = 155.3$ foot-pounds. Since $W = 1\ 00$ pound, $h = 155.3$ feet, which is the distance that the ball will rise.

3-9. Power. Rate of doing work is called *power*. Whenever the expression "rate of" is used in physics, division by time is implied. Therefore another way of defining power is to say that it is the amount of work done divided by the time required to do this work. Speed, or rate of motion, is similarly distance divided by time, so that another definition of power is "the product obtained by multiplying the force that caused the motion by the speed."

It will be seen that these two definitions are equivalent since

$$\text{power} = \frac{(\text{work})}{(\text{time})} = \frac{(\text{force})\,(\text{distance})}{(\text{time})} = (\text{force})\,(\text{speed})$$

By either method of computing the power the unit will come out foot-pounds/second. This unit, however, is not large enough to be very useful, so another unit, equal (very closely in this country and exactly in England) to 550 foot-pounds per second and called the *horsepower* is used in practice. A joule per second is called a *watt;* 1,000 watts is called a *kilowatt.* There are exactly 746 watts in a horsepower in the United States.

3-10. Illustrative Problem. If a 200-pound man runs up a flight of stairs 20 feet high in seven seconds, at what rate is he working? Express the result in foot-pounds per second, horsepower, and watts.

First, find the work done in climbing from one floor to the next, which is equal to the potential energy gained.

$$\text{Work} = Wh = (200 \text{ pounds})\,(20 \text{ feet}) = 4{,}000 \text{ foot-pounds}$$

Since the rate of doing this work, that is, the power, is the work divided by the time, we find the quotient obtained from (4,000 foot-pounds)/(7 seconds) or 571 foot-pounds per second. This is equivalent to 571/550 or 1.037 horsepower or in terms of watts, (1.037) (746) = 774 watts. It is perfectly possible for a man to work at the rate of a horsepower for a short time; it is also possible for a horse to work at the rate of many horsepower for a short time; but it takes a first-class horse to work at the rate of one horsepower for an entire day.

3-11. Another Illustrative Problem. A horse pulls a plow at the rate of two feet per second and exerts a forward force on the plow of 250 pounds. At what rate does the horse work? Express the result in foot-pounds per second, in horsepower, and in watts. How much work does the horse do in five hours?

The rate of doing work is the power, one formula for which is the product of the force and the speed. Since the force is 250 pounds and the speed is 2.00 feet/second, the required power is (250 pounds) (2.00 feet/second) or 500 foot-pounds/second. Since there are 550 foot-pounds/second per horsepower, this power is 500/550 or 0.909 horsepower. It will be noticed that the numerator together with its units is 500 foot-pounds per second, and the denominator together with its units is 550 foot-pounds per second per horsepower, or 550 foot-pounds/horsepower-second. When we divide the numerator by the denominator, all the units cancel except horsepower, which being in the denominator of the denominator can be transferred to the numerator and survives in the result. By multiplying 0.909 horsepower by the conversion factor 746 watts/horsepower, the horsepower cancels giving 678 watts. Since power is the ratio of work to time, it follows that work is the product of power and time. We may therefore multiply any one of our three answers by an amount of time corresponding to five hours and get the work done by the horse. Five hours is 18,000 seconds. When we multiply 500 foot-pounds/second by 18,000 seconds, the seconds cancel and we have 9,000,000 foot-pounds. If we multiply 0.909 horsepower by 5.00 hours, we have 4.54 horsepower-hours. If we multiply 678 watts by 5.00 hours, we have 3,390 watt-hours. Since 1,000 watts equals one kilowatt, this is equivalent to 3.39 kilowatt-hours.

3-12. Units of Energy. If we multiply both sides of the equation

$$1 \text{ horsepower} = 550 \text{ foot-pounds/second}$$

by one second, we obtain

$$1 \text{ horsepower-second} = 550 \text{ foot-pounds}$$

The horsepower-second is thus a unit of energy. Similarly the horsepower-minute (33,000 foot-pounds) and the horsepower-hour (1,980,000 foot-pounds), also the watt-hour and the kilowatt-hour are units of energy. We often use the kilowatt-hour to measure electrical energy. We have now accumulated so many units of both energy and power that it will perhaps be well to relate them in tabular form. So the table will go as follows:

1 joule	=	1 newton-meter
1 joule	=	1 watt-second
3,600 joules	=	1 watt-hour
1,000 watt-hours	=	1 kilowatt-hour (kw.-hr.)
0.746 kilowatt-hour	=	1 horsepower-hour (hp.-hr.)
550 foot-pounds	=	1 horsepower-second
3,600 hp.-seconds	=	1 hp.-hr.
0.738 foot-pound	=	1 joule

Some other units of energy which we have not yet met may also be tabulated here for reference.

10,000,000 ergs	=	1 joule
1 erg	=	1 dyne-centimeter
4,190 joules	=	1 Calorie* (used to measure heat energy)
778 foot-pounds	=	1 British thermal unit
3.97 British thermal units	=	1 Calorie

It will be noticed that since

$$\text{energy} = (\text{power})\ (\text{time}) \text{ and}$$
$$\text{energy} = (\text{force})\ (\text{distance})$$

*The unit Calorie is sometimes called a ***kilogram-calorie***.

there are two types of energy units, the watt-hour being an example of the first, and the foot-pound an example of the second.

3-13. Power Units. A similar table may be constructed with power units:

1 joule/second	=	1 watt
1,000 watts	=	1 kilowatt
746 watts	=	1 horsepower
550 ft.-lb./sec.	=	1 horsepower
33,000 ft.-lb./min.	=	1 horsepower

SUMMARY OF CHAPTER 3

Technical Terms Defined

Work. Product of a force by a distance in the same direction as the force.

Energy. Ability to do work.

Power. Rate of doing work, that is, work divided by the time consumed in doing the work.

Laws

Conservation of Energy. Energy can be neither created nor destroyed. But it can be passed along from one body to another, or changed from one form to another with efficiencies ranging from very small values up to 100 per cent.

PROBLEMS

3-1. How much work is done in winding a church clock if the weight weighs 50 pounds and has a vertical motion of 30 feet? Express the answer in foot-pounds, foot-tons, horsepower-seconds, and joules.

3-2. One boy is drawing another on a sled. The tension in each of the two sled ropes is 10 pounds, the ropes are horizontal, and the distance covered by the sled is 200 feet. Compute the work done, and express the result in foot-pounds and in joules. (3.28 feet equal one meter and 4.45 newtons equal a pound.)

3-3. A 200-pound man climbs a flight of stairs which is 50 feet along the slant and which rises vertically 20 feet. How much work does he do?

3-4. If Niagara Falls is 160 feet high, how much potential energy is changed into kinetic energy when two pounds of water drop from the top to within an infinitesimal distance from the bottom? What is the velocity of the water just before it strikes the bottom? How much heat in British thermal units is produced when the two pounds of water strike the bottom?

3-5. What is the kinetic energy of a 3,000-pound automobile moving at the rate of 90 feet per second? What will be the speedometer reading corresponding to 90 feet per second?

3-6. A mule, walking along the tow-path on the bank of a canal, exerts a force on a canal boat of 100 pounds. How much work does the mule do in a mile?

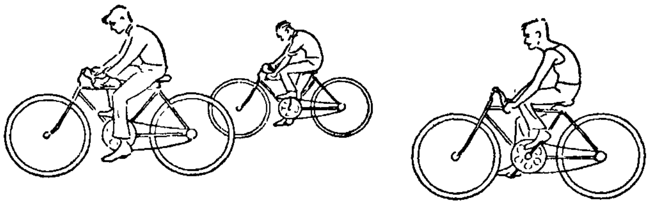
3-7. If the mule of the previous problem walks at the rate of three miles per hour, at what rate is work being done? Express the answer in horsepower and in kilowatts.

3-8. A 150-pound boy runs up a flight of stairs in six seconds. The vertical distance between floors is 22 feet and the slant length of the stairs is 46 feet. At what rate is the boy working?

3-9. A man exerts a force of 200 newtons on the chain of a differential pulley while pulling 10 meters of rope through his hand. How much work does he do? If it takes him a minute to do this, find the power in watts, also in horsepower.

3-10. Assume that the force necessary to move an airplane through the air is 1,000 pounds when the plane moves at 100 feet per second and doubles every time the speed doubles. Find the power necessary to drive the plane at 200 feet per second; at 400 feet per second.

CHAPTER 4



Efficiency; Mechanical Advantage; Coefficient of Friction; Simple Machines

4-1. Efficiency. A machine is a contrivance that transfers energy from one body to another. Very few machines are capable of transferring all the energy received, however, and so it is customary to refer to that fraction of the energy received by a machine which is handed on as the efficiency of the machine. Stated mathematically, the *efficiency* of a machine is the ratio between the output of a machine and its input. The output and input may both be considered as energy handled in a given time or, better, they may both be expressed as power. The efficiency is a pure number, that is, a number without units. No machine can be expected to deliver in a given time more energy than it receives; it does well if it delivers as much energy as it receives. The only occasion when we have one hundred per cent efficiency is when the output is in the form of heat energy. If we could have frictionless processes, they would also result in one hundred per cent efficiencies. The latter are talked about in physics courses (for the sake of simplicity) but never realized in practice. The only case approaching frictionless motion that we know of is that of heavenly bodies through empty space; the planets apparently move in their orbits around the sun with practically no friction. Occasionally one hears of the invention of a perpetual motion machine. It has already been pointed out (section 3-3) that such machines are impossible; in the present connection it may be stated that a perpetual motion machine would be a machine the efficiency of which is greater than one hundred per cent! To summarize then

$$\text{efficiency} = \frac{\text{power delivered } by \text{ the machine}}{\text{power delivered } to \text{ the machine}}$$

or more simply

$$\text{efficiency} = \frac{\text{output}}{\text{input}} \tag{a}$$

4-2. Mechanical Advantage. It is also convenient to speak of the *force* exerted on or by a machine. For example, in the case of a bicycle, we exert a large force on the pedals, and in turn the bicycle exerts a comparatively small force on the road. In the case of a pulley, we exert a small force on the machine and the machine exerts a large force. But we must remember that these statements represent only half of the story. The large force on the pedals of the bicycle is exerted through a small distance, and the small force on the road is exerted through a large distance; with the pulley it is just the other way around. The ratio between the force exerted by a machine, considered frictionless, and the force exerted on the machine is called the *ideal mechanical advantage* of the machine. In this case there is no limit to the value of the ratio; it may be less than one or it may be greater than one. But in any case it is again a pure number. In order to save words, engineers agree to call the force exerted *on* a machine the *effort*, and the force exerted *by* the machine the *resistance*. Similarly we call the distance (or displacement) through which the effort is exerted the *effort displacement*, and the corresponding distance for the resistance the *resistance displacement*. Therefore the output of a machine in a given time is the product of the resistance by the resistance displacement. Likewise the input in the same time interval is the product of the effort by the effort displacement. Therefore, using equation (a) of the previous section, we have

$$\text{efficiency} = \frac{(\text{resistance})\ (\text{resistance displacement})}{(\text{effort})\ (\text{effort displacement})}$$

Multiplying both sides of the equation by (effort displacement) and dividing both sides by (efficiency) (resistance displacement), this equation becomes

$$\frac{\text{effort displacement}}{\text{resistance displacement}} = \frac{\text{resistance}}{(\text{effort})\ (\text{efficiency})}$$

Each of these fractions may be called the ideal mechanical advantage of the machine. Therefore

$$\text{ideal mechanical advantage} = \frac{\text{effort displacement}}{\text{resistance displacement}} \tag{b}$$

$$\text{ideal mechanical advantage} = \frac{\text{resistance}}{(\text{effort})\ (\text{efficiency})} \tag{c}$$

If the efficiency = 1.00 (100 per cent), the last equation reduces to

$$\text{mechanical advantage} = \frac{\text{resistance}}{\text{effort}} \tag{d}$$

This ratio of resistance to effort is often called *actual* mechanical advantage, regardless of the efficiency, just as equation (b) gives the ideal mechanical advantage regardless of the efficiency. However it is the ideal mechanical advantage which is commonly used because this is the one that depends solely on the dimensions of the machine. The actual mechanical advantage fluctuates with the condition of the machine. From here on, if it is not specified which mechanical advantage is being used, it will be assumed that the ideal is intended. Most machines have a mechanical advantage greater than one. A mechanical advantage of less than one is sometimes desired for purposes of convenience, as in the case of tongs, or in situations where we wish to gain speed at the expense of force, an example of which is the bicycle.

4-3. Coefficient of Friction. A third ratio (or pure number) used in mechanics is called *coefficient of friction.* Friction is due to the roughness of two surfaces that are in contact. This roughness, magnified, becomes miniature hills and valleys in the surface. The more two surfaces are pressed together, the harder it is to move one surface over another, because the hills and valleys of one surface sink farther into the valleys and hills of the other surface. However, when one surface is moving against the other, the friction is not so great, because in that case the hilltops of one surface merely ride over the hilltops of the other surface, and do not have time to sink into the valleys. The *coefficient of friction* is the ratio of the force necessary to pull one surface against friction along the other to the perpendicular force pressing the two surfaces together. The latter force is often called the *normal force* because in mathematics *normal* means *perpendicular.* Therefore

$$\text{coefficient of friction} = \frac{\text{force of friction}}{\text{normal force}}$$

The coefficient of friction may be either greater or less than one. We also distinguish between the value of the coefficient of friction when the surfaces are at rest relative to each other, and when they are in motion. The former is called *static coefficient of friction* and the latter *kinetic coefficient of friction.* As has been indicated, the kinetic is less than the static coefficient. When a surface is moved against friction, heat is produced, and the amount of heat in foot-pounds may be found by multiplying the force of friction by the distance moved. The heat may be expressed in either British thermal units or Calories, the familiar heat units used in dietetics, by using the facts that

$$\begin{aligned} 778 \text{ foot-pounds} &= 1 \text{ B.t.u.} \\ 4180 \text{ joules} &= 1 \text{ Calorie} \\ 3.97 \text{ B.t.u.} &= 1 \text{ Calorie} \end{aligned}$$

When the fraction is computed by using the equation

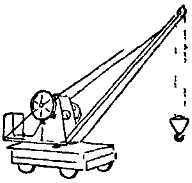
$$\text{force of friction} = (\text{coefficient of friction})\ (\text{normal force})$$

which is another form of the equation occurring earlier in this section, it is necessary to remember that friction is not an active force; friction can only oppose an active force. An active force, for instance, could cause a book lying on a table to start moving. In the absence of any other forces, friction will not start the book moving, but is to be subtracted from any force that does tend to move the book. If there is no active force from which to subtract the friction, we assume that there is also no frictional force. Or, if the application of the coefficient of friction formula gives a force greater than the active force, we use only that part of the friction necessary to neutralize the active force. Friction always tends to oppose the relative motion of two surfaces.

4-4. Illustrative Problem. A 100-pound weight is to be dragged four feet along a floor; the coefficient of kinetic friction is 0.2. What horizontal force is required and how much energy is converted into heat in the process?

In this case the normal force exerted by the floor on the 100-pound weight is exactly equal and opposite to the 100-pound pull of gravity on this weight. Therefore the force of friction is (0.2) (100) or 20 pounds, which necessitates a 20-pound horizontal force to keep the weight moving along the floor once it is started. The heat that is produced is the product of this 20 pounds by the four feet, that is, 80 foot-pounds. Converted into British thermal units, we have 80/778 or 0.1028 B.t.u. or 0.0259 Calorie.

4-5. Simple Machines; Compound Machines. In a few of the sections following, certain simple machines are discussed, such as the lever, pulley, inclined plane, screw, wheel and axle, and hydraulic press. There are others, such as the wedge, which we are not yet ready to discuss. Most actual machines represent combinations of simple machines; such a combination is called a *compound machine.* Bicycles and derricks are examples of compound machines. The mechanical advantage of a compound machine is the product of the mechanical advantages of its constituent parts. The mechanical advantage of the compound machine also may be found directly from its resistance and effort or their displacements.



4-6. The Lever. A lever is a rigid bar upon which, in the simplest case, only three forces act, each one perpendicular to the bar. Two of these forces are the effort, usually small, and the resistance, usually large. The third force acts at the axis, which is called the fulcrum, and which may be either at the end of the bar or somewhere between the ends, but usually nearer to the point of application of the resistance. One type of lever is diagrammed in figure 4-1, where the third force is not shown, but if it were, the force would be applied upward at the axis and would be equal in magnitude to the sum of both effort and resistance.

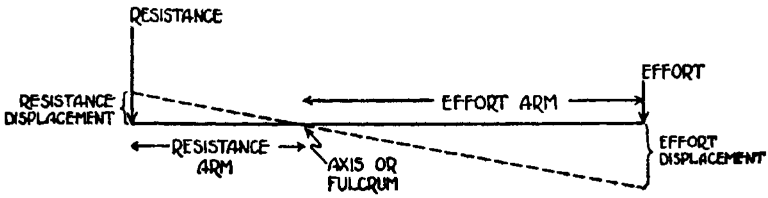


Figure 4-1.

By applying the geometrical theorem concerning the proportionality of corresponding sides of similar triangles, we see from figure 4-1 that

$$\frac{\text{effort displacement}}{\text{resistance displacement}} = \frac{\text{effort arm}}{\text{resistance arm}}$$

It therefore follows from equation (b) of section 4-2 that

$$\text{ideal mechanical advantage of lever} = \frac{\text{effort arm}}{\text{resistance arm}}$$

4-7. The Pulley. In figure 4-2, in order to raise the lower block one foot, it is necessary to exert the force E (the effort) through a distance of four feet. By section 4-2, equation (b), the mechanical advantage is the ratio of the effort displacement to the resistance displacement, which in this case is 4/1 or 4. It will be observed that in order to find the mechanical advantage of a pulley of this simple type, it is only necessary to count the number of ropes against which the resistance pulls. In more complicated pulley arrangements, where some of the ropes have twice or three times the tensions of others, it is necessary to fall back on the more general relation, equation (b).

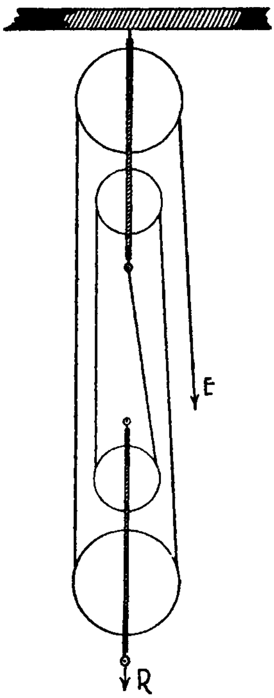


Figure 4-2.

4-8. The Inclined Plane. In such operations as putting a box into a truck, an inclined plane furnishes a convenient means for raising a given weight by exerting a force considerably less than the weight. Let an inclined plane of length s make an angle with the horizontal, and let us assume that the problem is to raise the weight W (figure 4-3) a vertical distance h by exerting a force E which is less than W (which will now be called R, the resistance). R is a force acting vertically downward.

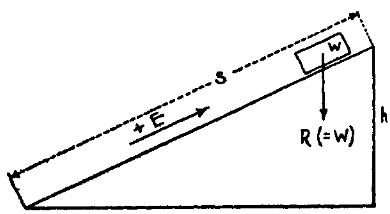


Figure 4-3.

We shall find the mechanical advantage of the inclined plane from a consideration of the law of conservation of energy, assuming no friction. The potential energy gained by raising the level of W pounds h feet is Wh foot-pounds. If this energy is obtained by doing the work represented by exerting the force E pounds through the distance of s feet, then Es foot-pounds should equal Wh or Rh foot-pounds. Therefore the ideal mechanical advantage is $R/E = s/h$; that is, the

$$\text{ideal mechanical advantage of inclined plane} = \frac{s}{h}$$

or, stated in words, the ideal mechanical advantage equals the ratio of the length of the plane to the vertical height of one end of the plane with respect to the other end.

4-9. Problems Illustrating Inclined Plane. A 1,000-pound block is pulled up a rough inclined plane by a force of 925 pounds. The plane rises 50 feet in a slant height of 100 feet. The block starts from rest at the bottom and has acquired a speed of 17.94 feet per second by the time it arrives at the top. Find (1) the mechanical advantage of the plane, (2) the work done by the 925-pound force, (3) the gain in potential energy from the bottom of the plane to the top, (4) the gain in kinetic energy, (5) the part of the work that goes into heat, and (6) the force of friction. (7) If the normal force is 866 pounds, find the coefficient of friction.

(1) The mechanical advantage of the plane is 100 ft./50 ft., or 2.

(2) The work done by the 925-pound force is the product of 925 pounds by 100 feet, since the two are parallel. The product is 92,500 foot-pounds.

(3) The gain in potential energy is Wh, the product of the weight of the block and the vertical height through which it rises. Notice that the weight (a vertical force) and the height are parallel with each other. Numerically, therefore, the gain in potential energy is (1,000 pounds) (50 feet) or 50,000 foot-pounds.

(4) At the bottom of the incline, the kinetic energy was zero since the block was at rest there. At the top, the kinetic energy is by section 3-6 equal to $Wv^2/2g$; we may write the equation

$$\text{kinetic energy} = \frac{(1{,}000)\ (17.94)^2}{2(32.2)} = 5{,}000 \text{ foot-pounds}$$

(5) The total work, 92,500 foot-pounds, accounts for three things, the gain in potential energy of 50,000 foot-pounds, the gain in kinetic energy of 5,000 foot-pounds, and the heat developed. By subtraction, therefore, we find that 37,500 foot-pounds of heat are developed.

(6) Since the heat is the product of the force of friction and the slant height of the plane, the force of friction is the quotient of 37,500 foot-pounds and 100 feet, or 375 pounds.

(7) The coefficient of friction is the quotient of the force of friction, 375 pounds, and the normal force, 866 pounds, or 0.433.

4-10. The Jackscrew. Figure 4-4 represents a jackscrew. Let the effort be applied through one whole circumference of the dotted circle of radius r. The effort displacement is therefore $2\pi r$. The corresponding resistance displacement is called the *pitch* of the screw and may be represented by p. It is the vertical distance that the weight rises when the screw is turned through an angle of 360 degrees. The mechanical advantage is, then, by section 4-2, equation (b), given by the expression

$$\text{ideal mechanical advantage of jackscrew} = \frac{2\pi r}{p}$$

The efficiency of a jackscrew is fairly low; more than half of the work is done against friction. As a matter of fact, it would be inconvenient to have the efficiency of a jackscrew greater than fifty per cent; this would mean that when the effort dropped to zero, the weight of the load would turn the screw back down again. One may think of the jackscrew as a modification of the inclined plane in which the "plane" has been twisted into a helix.

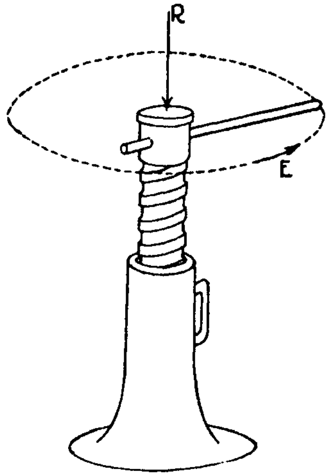


Figure 4-4.

4-11. Illustrative Problem. (1) A jackscrew has four threads per inch, and an efficiency of 25 per cent. How great a weight can the jack lift when a force of 100 pounds is exerted at the end of a two-foot bar in order to turn the screw? (2) What force must be applied at the end of the two-foot bar in order to let the same weight back down again with the same jack?

(a) Use the equation, efficiency equals output divided by input. The efficiency is 0.25. The input is the effort, 100 pounds, multiplied by the effort displacement, (2) (π) (24 inches). The output is the product of the resistance, W pounds, by the resistance displacement, 0.25 inch. 0.25 inch is the pitch of the screw, since there are four threads to the inch. With these values substituted, the equation becomes

$$0.25 = \frac{0.25\,W}{(2\,\pi)\,(24)\,(100)}$$

The efficiency is a pure number; both output and input are in inch-pounds. Divide both sides of the equation by 0.25 and multiply both sides by $4{,}800\,\pi$, and the equation becomes $4{,}800\,\pi = W$, or $W = 15{,}070$ pounds. It will be noticed that during this one revolution the input is 15,070 inch-pounds, the output is 3,700 inch-pounds, and 11,300 inch-pounds of heat are developed. The difference between input and output is practically always heat.

(b) When we let the weight back down again, there is no output except heat, and when heat is the only output, a process is always 100 per cent efficient. The output is now 11,300 inch-pounds of heat, and the input is the sum of 3,770 inch-pounds recovered from the load together with the work done by exerting an unknown force, F, through a distance of $(2\,\pi)$ (24) inches. The efficiency equation therefore becomes

$$1.00 = \frac{11{,}300}{3{,}770 + (2\,\pi)\,(24)\,(F)}$$

Clear of fractions by multiplying both sides of the equation by the denominator of the right-hand side

$$3{,}770 + 150.7\ F = 11{,}300$$

Subtracting 3,770 from both sides of this equation, which is called "transposing the 3,770," we have

$$150.7\ F = 7{,}530$$

Dividing both sides of the equation by 150.7 gives

$$F = 50.0 \text{ pounds, answer}$$

The method employed in solving this problem is typical of a large number of physics problems. In general the successive steps are (1) recognition of the physical principle involved and the selection of an equation embodying this principle, (2) substitution of the numerical values of the problem into the equation, leaving the unknown values represented by letters, and (3) algebraic manipulation in order to solve for the unknown quantities.

It is very important to make sure that the physical quantities are expressed in the proper units. When the units are correct, they may be introduced into the equation along with the numerical values, and it will then be found that the units completely cancel.

4-12. Problem Illustrating "Wheel And Axle." The wheel of a "wheel and axle," figure 4-5, has a radius of two feet and the axle, a diameter of six inches. Assuming no friction, compute the force which must be applied to the rim of the wheel in order to lift a 400-pound weight by means of a rope wrapped around the axle. What is the mechanical advantage?

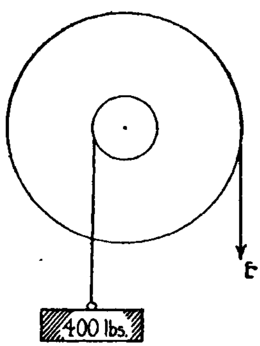


Figure 4-5.

If the wheel is turned through one complete revolution, the unknown force on the rim (the effort E) acts through a distance of (2π) (2) feet, the circumference of the wheel: this is the effort displacement. At the same time the resistance, 400 pounds, is lifted a distance of (2π) (0.25) feet, the circumference of the axle: this is the resistance displacement. The ideal mechanical advantage may be found immediately by dividing the effort displacement by the resistance displacement and obtaining 8. Since in this case we are assuming no friction, the efficiency is 100 per cent and the ideal mechanical advantage is also equal to the resistance divided by the effort. Therefore

$$8 = \frac{400}{E}$$

and solving for E, we obtain 50 pounds for the effort.

4-13. The Hydraulic Press. The hydraulic press consists of two hollow cylinders of different diameters and therefore different cross-sectional areas, a and A, connected by means of a tube (see figure 4-6). Each cylinder is equipped with a tightly fitting piston, and the whole is filled with some liquid. A small force E, exerted through the distance n, results in the exertion of a large force R, exerted through the small distance m. Levers are also used in practice to increase further the mechanical advantage but will not be included in this discussion.

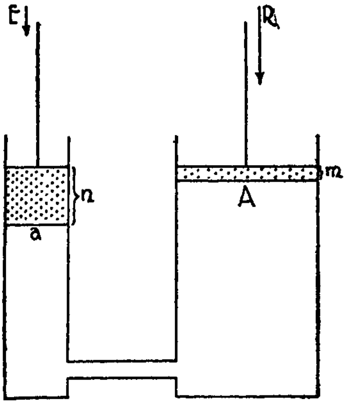


Figure 4-6.

By section 4-2, equation (b), the mechanical advantage is equal to n/m. The volume of the liquid na, which leaves the small cylinder is necessarily equal to the volume of liquid mA, which enters the large cylinder. Therefore $n/m = A/a$, another expression for the mechanical advantage.

4-14. Pressure. In section 3-4, the first illustration of energy (1) could be termed pressure energy. *Pressure* may be defined as the ratio of a force exerted at right angles to the surface of a fluid to the area of the fluid upon which the force is acting.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

that is, pressure is *force per unit area.* The unit of pressure is of the form, pounds per square foot, pounds per square inch, or newtons per square meter. It should be emphasized that the force is perpendicular to the area and directed toward the area. We shall see eventually that pressure is a special case of a "stress."

4-15. Pressure Energy. When a fluid is compressed, we may state with sufficient accuracy for our purpose that the pressure energy stored by the process is equal to the product of the pressure and the decrease in volume. If the pressure is expressed in pounds per square foot and the volume in cubic feet, the energy will come out in foot-pounds. Consider the following example: a tightly fitting piston moves a distance of d feet into a cylinder of cross section A square feet against a constant pressure of p pounds per square foot. The force necessary to do this is therefore pA pounds, and since this force is parallel to the distance d, work will be done by the force equal to pAd foot-pounds. But the volume V moved through by the piston is the cross section A times the distance d. Therefore the energy put into the fluid is pV foot-pounds.

$$\text{Pressure energy} = pV$$

The hydraulic press affords another illustration of pressure and pressure energy. The area of one of the pistons, multiplied by the pressure in the liquid, will equal the force exerted by the liquid on that piston, and also, by Newton's third law, will equal the force exerted by the piston on the liquid. That is, force equals pressure times area. The pressure throughout the liquid has practically a constant value; therefore, disregarding friction, the forces on the two pistons are proportional to the cross-sectional area of the pistons. The smaller force may be taken as the effort (E, figure 4-6) and the larger as the resistance R, and the

$$\text{mechanical advantage} = \frac{R}{E} = \frac{A}{a}$$

where A and a are the areas of the two pistons.

This relation may also be obtained from the energy point of view, still considering the machine frictionless. If n and m are the effort displacement and the resistance displacement respectively, then Rm is the output and En the input energy; these energies could be written in terms of pressures and volumes by putting areas in both numerators and denominators, that is

$$\left(\frac{R}{A}\right)(Am) \quad , \quad \left(\frac{E}{a}\right)(an)$$

Therefore $Rm = En$, since the efficiency is one hundred per cent. If we divide $Rm = En$ by $mA = na$, we obtain $R/A = E/a$ which may be rewritten $R/E = A/a$.

SUMMARY OF CHAPTER 4

Technical Terms Defined

Simple Machine. A device for transferring energy.

Compound Machine. A combination of two or more simple machines.

Input. Rate of doing work on a machine.

Output. Rate at which a machine does work.

Efficiency. Ratio of output to input, usually expressed in percentage.

Effort. Force exerted on a machine.

Resistance. Force exerted by a machine.

Effort Displacement. An arbitrary distance through which the effort is exerted.

Resistance Displacement. The corresponding distance through which the resistance is exerted.

Ideal Mechanical Advantage. Ratio of effort displacement to resistance displacement.

Actual Mechanical Advantage. Ratio of resistance to effort. The mechanical advantage of a compound machine is the product of the mechanical advantages of its parts.

Normal Force. A force perpendicular to the surface of contact exerted by one object upon another object which rests or slides upon the first.

Coefficient of Friction. Ratio of force of friction to normal force; called "static coefficient of friction" when surfaces are at rest and "kinetic coefficient of friction" when in relative motion.

Pressure. Ratio of force to area; force must be normal to area and directed toward it.

Pressure Energy. Product of pressure by change of volume produced by the pressure.

Laws

Heat may be changed to other forms of energy at low efficiencies.

Other forms of energy may be converted into heat with one hundred per cent efficiency.

The difference between input and output is rate of production of heat, except when the output itself is rate of production of heat.

Displacement multiplied by force of friction is heat energy.

PROBLEMS

4-1. Find the force of friction between sled and snow if the sled and load weigh 100 pounds and the coefficient of friction is 0.1. How much work will be done in pulling the sled 50 feet with the rope horizontal? What becomes of this work?

4-2. Show that in the case of a windlass, the mechanical advantage is R/r, where R is the length of the crank and r is the radius of the axle on which the rope is wound.

4-3. A man weighing 180 pounds is lowered into a well by means of a windlass, the arm of which is 30 inches long and the axle of which is 6 inches in diameter. Assuming no friction, find the force required to let him down with uniform speed.

4-4. What horsepower is necessary to run a 700-watt generator, the efficiency of which is 90 per cent?

4-5. What wattage is necessary to drive a one-horsepower motor, the efficiency of which is 80 per cent?

4-6. State the data necessary to determine the mechanical advantage of a bicycle (1) if one is to be restricted to an examination of the bicycle itself; (2) if one is allowed to experiment with the bicycle on the road.

4-7. A jackscrew with four threads to the inch, lifting a weight of 10,000 pounds, is turned by a capstan rod, and the force required to turn the screw is 100 pounds, the lever arm being 18 inches. Find the efficiency.

4-8. Prove that a jackscrew, the efficiency of which is 50 per cent, requires no force to let the load back down after being raised.

4-9. An automobile is stuck in the mud. Given: a horse, a long rope, a pair of triple pulleys, and a tree growing at a convenient spot. Show how to rig the pulleys so as to obtain the maximum mechanical advantage. If the efficiency of the pulleys is 50 per cent and the horse can exert a pull of 700 pounds, what pull can be exerted on the car?

4-10. The wheel of a "wheel and axle" has a radius of two feet and the axle a diameter of six inches. Assuming that 25 per cent of the applied force is necessary to overcome friction, compute the force which must be applied to the rim of the wheel in order to lift a 400-pound weight by means of a cord wrapped around the axle. What is (1) the mechanical advantage; (2) the efficiency?

4-11. In figure 4-7, which illustrates a differential pulley (sometimes called a chain hoist or chain fall), the two upper pulleys (radii r_1 and r_2 respectively) turn together. Show that the ideal mechanical advantage is $\dfrac{2r_2}{r_2 - r_1}$. Does the radius of the lower wheel affect the mechanical advantage?

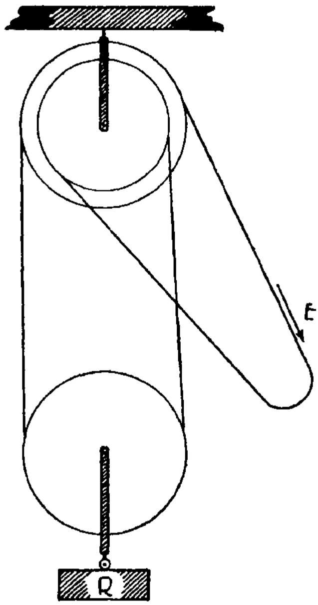
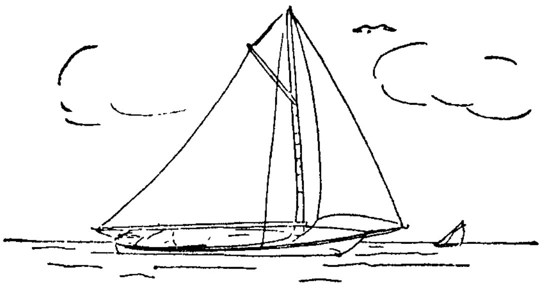


Figure 4-7.

4-12. A differential chain hoist has one wheel nine inches in diameter and the other ten inches. If the efficiency is 40 per cent, how large a force on the chain is necessary to lift a one-ton load? What is the largest efficiency that the chain hoist may have without dropping back when the effort is removed?

CHAPTER 5



Fluids

5-1. Boyle's Law. Liquids and gases are both fluids; neither of them has a fixed shape, but both take the shape of the container. The distinction between a liquid and a gas is that a liquid has a definite volume, and therefore a free surface, and stays in the bottom of the receptacle, whereas a gas occupies the whole volume of the container. For this reason the expression "volume of a gas" means no more than the volume of the container. The pressure exerted by the walls of the container on a gas may be computed if we know the weight and temperature of the gas, and the volume of the container. If the mass and temperature of a gas remain constant, decreasing the volume of a gas increases its pressure in accordance with the equation

$$\frac{V_1}{V_2} = \frac{P_2}{P_1}$$

where P_1 and V_1 represent the original pressure and volume and P_2 and V_2, the new values. It will be noticed that $P_1V_1 = P_2V_2$ expresses the same mathematical fact more concisely; still another way is to say that the product of the pressure and the volume is constant. This relation goes under the name of *Boyle's law.* The pressure here is the total pressure, not the excess over and above atmospheric pressure, which generally goes under the term "gage pressure."

5-2. Density and Specific Gravity. The conceptions of density and specific gravity are particularly useful in dealing with fluids.

The *density* (sometimes called *weight density* to distinguish it from *mass density*, a term often used in theoretical physics) of a substance is determined by dividing its weight by its volume. That is

$$\text{density} = \frac{\text{weight}}{\text{volume}}$$

A typical unit of density will therefore be lb./ft.[3] Since 62.4 pounds of water has a volume of one cubic foot, and one gram (one thousandth of a kilogram) of water has a volume of one cubic centimeter, it follows that the density of water in two common systems of units is 62.4 pounds per cubic foot, and one gram per cubic centimeter. (The mass density in the standard kilogram-meter system of units is 1,000 kilograms per cubic meter.) *Specific gravity* is the ratio of the density of the substance under consideration to the density of water. Both densities must be expressed in the same units before dividing. This means that specific gravity itself is a pure number; it has no units. For this reason the specific gravity of a substance will be the same in one system as it is in any other. Specific gravity may also be defined as the ratio of the weight of a given volume of that substance to the weight of an equal volume of water. Since specific gravity is the ratio of two weights, it will still come out a pure number.

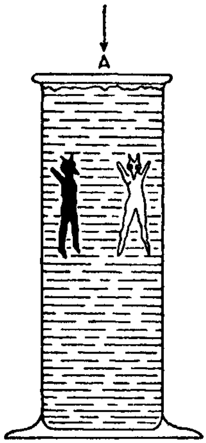


Figure 5-1.

5-3. Pascal's Principle. In section 4-15, it was assumed that increasing the pressure under the smaller piston of a hydraulic press had the effect of increasing the pressure by the same amount under the larger piston. This is, in fact, the case. We may make this statement more general as follows: when a fluid (gas or liquid) is confined within a given volume, an increase in the pressure of any part of the fluid will result in the same increase of pressure everywhere else in the fluid. This is *Pascal's principle.* It is often illustrated on the lecture table by the so-called Cartesian divers. They are weighted to have about the density of water. If the pressure is increased in any part of the apparatus, as at A (figure 5-1), the air inside the divers is compressed, water enters, and both divers become heavier and sink simultaneously.

5-4. Hydrostatic Pressure. The concept of density becomes useful when we are given the volume of a substance and wish to know its weight; the weight will be the product of the volume and the density. As an illustration of this use of density, let us compute the pressure at the bottom of a rectangular tank filled with some liquid. Let the density of the liquid be D, the horizontal cross section of the tank A, and the height of the liquid above the point in question (the bottom of the tank) h. The volume of the liquid in the tank is therefore Ah, and the weight of the liquid DAh. The weight is the force exerted on the bottom of the tank. Since the pressure on the bottom is the ratio of the force to the area, the pressure in this case is DAh/A or hD. That is, the area cancels out, leaving the pressure a function of the density and depth of liquid only.

$$P = hD$$

In other words, at a given depth in a liquid there will be a given pressure, everywhere the same at this level. If there is already a pressure at the upper surface of the liquid, this pressure must be added to hD to get the total pressure. When a force is the product of a pressure by the area over which the pressure is distributed, the force is always at right angles to the area and pushing toward it. This means that if we had a vertical surface in the liquid we could still use the formula we have just derived ($P = hD$) to compute the pressure; after that, to compute the horizontal force on a given vertical area, the necessary formula would be $F = PA$, which we have met before. The pressure is computed for the center of the area in question.

5-5. First Illustrative Problem. If the volume of an air bubble is 10 cubic centimeters when 34 feet below the surface of a pond, what will the volume be just below the surface?

The pressure on the air bubble just below the surface is one atmosphere or 14.7 pounds/square inch. The pressure 34 feet below the surface will be more than one atmosphere by hD, where h is 34 feet and D is the density of water, 62.4 pounds/cubic foot. The additional pressure is therefore (34) (62.4) or 2,120 pounds per square foot. In accordance with slide-rule precision, this number has been rounded off to three significant figures. Since there are 144 square inches in a square foot, this additional pressure is the same as 2,120/144 or 14.7 pounds per square inch, making a total pressure at a depth of 34 feet of 29.4 pounds per square inch.

Assuming that the temperature is the same in both places, Boyle's law holds, or, $P_1V_1 = P_2V_2$. In this case P_1 is 29.4 pounds per square inch, V_1 is 10 cubic centimeters, P_2 is 14.7 pounds per square inch, and V_2 is unknown. Therefore

$$(29.4)\ (10) = (14.7)\ (V_2)$$

Dividing both sides by 14.7, we obtain $V_2 = 20$ cubic centimeters. That is, when the pressure is halved, the volume is doubled.

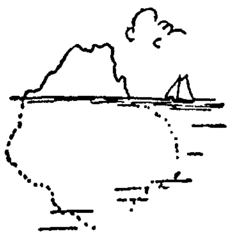
5-6. Second Illustrative Problem. A tank is three feet wide, four feet deep, and six feet long. If it is filled with water, find the average pressure on one side and on one end in pounds per square foot, also find the force on one side and on one end, in pounds.

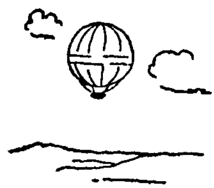
The center of one side, also the center of one end, is two feet from the top. Two feet is therefore the value of h. Since the fluid is water, the density is 62.4 lb./ft.^{3} Therefore the average pressure for the side is the same as the average pressure for the end, and both are equal to (2 ft.) (62.4 lb./ft.^{3}) which is 124.8 lb./ft.^{2} Notice the cancellation of the feet. The force on one end is PA, in this case (124.8 lb./ft.^{2}) (12 ft.^{2}) or 1,498 pounds. Similarly the force on a side is (124.8 lb./ft.^{2}) (24 ft.^{2}) or 3,000 pounds.

5-7. Third Illustrative Problem. As a third illustration of this type of problem, imagine a brick suspended by a wire below the surface of a liquid the density of which is 1.5 gm./cm.^{3} Let the dimensions of the brick be 5 by 10 by 20 cm., and let the 10 by 20 side be on top and immersed 30 cm. below the surface of the liquid. Find the forces exerted by the fluid on all six surfaces of the brick.

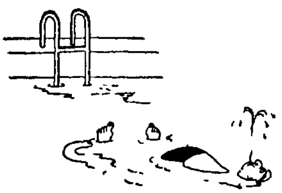
We can name these surfaces top, bottom, sides, and ends. The values of h for these surfaces are 30 cm. for the top, 35 cm. for the bottom, and 32.5 cm. for the sides and ends. Therefore the pressures will be 45 gm./cm.^{2} for the top, 52.5 gm./cm.^{2} for the bottom, and 48.8 gm./cm.^{2} for the sides and ends, since the density of the fluid is 1.5 gm./cm.^{3} The forces will be 9,000 grams down on the top, 10,500 grams up on the bottom, two horizontal forces of 2,440 grams each (in opposite directions), one on each end, and two forces of 4,880 grams each, one on each side, these numbers being found by multiplying each pressure by the appropriate area. This solves the problem, but let us continue and find the total force on the whole brick. The forces on the ends cancel each other, likewise the forces on the sides, but the top and bottom forces add (algebraically) to 1,500 grams, upward. This means that the liquid actually pushes up on the brick with a force of 1,500 grams, which is, incidentally, exactly the weight of the liquid that could be contained in the volume of the brick.

5-8. Buoyant Force; Archimedes' Principle. Let us now consider whether the agreement mentioned in the previous sentence is a coincidence or not. Imagine the vessel that contains the liquid, discussed in the previous section, before the brick has been suspended in the fluid. The space later to be occupied by the brick is at that time filled with 1,500 grams of liquid. This 1,500 grams of liquid is at rest; it has a tendency neither to rise nor fall. The only way in which to account for this is to assume that the surrounding liquid is supporting it by exerting upon it an upward force of just 1,500 grams. This is known as a *buoyant* force. And it will be exerted by the surrounding fluid on whatever material occupies that particular 1,000 cubic centimeters of space. Therefore it was no coincidence when we found at the end of the previous section that the upward force of the liquid on the brick was 1,500 grams. This general fact is known as *Archimedes' principle* and may be stated as follows: a body immersed in a fluid experiences an upward force exerted by the surrounding fluid and equal to the weight of the fluid displaced by the body. If the buoyant force is greater than the weight of the object, the object will rise, as in the case of a balloon, or a stick of wood placed under water. In the latter instance, a portion of the wood will finally rise above the surface of the water, after which the wood will displace less water, until the weight of the water displaced is equal to the weight of the piece of wood. The floating piece of wood will then be in equilibrium. Archimedes' principle holds just as rigorously for a floating as for a submerged object.





5-9. Determination of Specific Gravity. A little consideration will show that Archimedes' principle furnishes a direct method of determining the specific gravity of a body. For this purpose, we need to know both the weight of the body and the weight of an equal volume of water. But this "weight of an equal volume of water" is simply the buoyant force when the object is immersed in water. It may readily be found by subtracting the weight of the object when it is supported under water from its weight in air, because (figure 5-2) the sum of the two upward forces B and F must equal the downward force W in order to maintain equilibrium. W is the weight in air, B is the buoyant force, and F, the additional force needed for equilibrium, is called the "weight in water." If the density of the object is less than that of water, a modification of this method is necessary. A sinker, attached to the object, is kept under water while the object is weighed both in air and in water. Thus the difference between the two weights still gives the buoyant force on the given object.



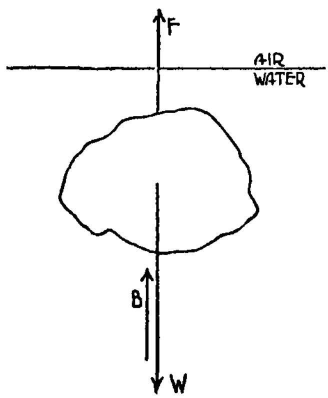


Figure 5-2.

5-10. Illustrative Problem. A block of wood weighs 200 grams. A sinker is fastened to it, and when the sinker is below the surface of water with the wood above the surface, the two together weigh 500 grams. If both are below the surface, the combination weighs 150 grams. Find the volume and the specific gravity of the block of wood.

When only the sinker is submerged, the upward force necessary for equilibrium is 500 grams. When both block and sinker are submerged, the necessary upward force is only 150 grams, 350 grams less. This means that in the second case the buoyant force is 350 grams more. In the first position the wood is out of water and in the second position the wood is under water, therefore the 350 grams represents the buoyant force of the water on the wood, and by Archimedes' principle, is the weight of the water displaced by the wood. Since one gram of water occupies one cubic centimeter, the volume occupied by the wood is 350 cubic centimeters.

The specific gravity of the wood is the ratio of the weight of the wood to the weight of an equal volume of water. The weight of the wood is 200 grams and the weight of the same volume of water is 350 grams, therefore the specific gravity is (200 grams)/(350 grams) or 0.571, a number without units.

If the density had been required, it would have been necessary to find the quotient, (200 grams)/(350 cubic centimeters), and the result would have been 0.571 gram/cubic centimeter.

5-11. Bernoulli's Principle. The discussion has so far concerned fluids at rest; if we let the fluids move, there will be deviations from the laws already stated. For instance, in a fluid at rest, the force on a unit area at a certain depth has the same numerical value no matter whether the unit area be vertical, horizontal, or slanted. But if the fluid is in motion, this force will depend on the direction of the motion. Moreover, let us consider the relations involved when a frictionless, incompressible liquid flows without any eddying motion through the pipe shown in figure 5-3. The potential energies of small quantities of water at *B*, *C*, and *D* are all equal and less than

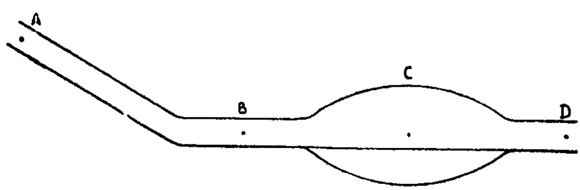


Figure 5-3.

the potential energy at *A*, because *B*, *C*, and *D* are all on the same level and *A* is at a higher level. The velocities at *A*, *B*, and *D* are equal and greater than at *C* because the pipe has the same cross section at *A*, *B*, and *D*, and a larger one at *C*; therefore the kinetic energies are greater at *A*, *B*, and *D* than at *C*. The pressure energies at *B* and *D* are equal and greater than at *A*, because the lower down a fluid, other things being equal, the greater the pressure. But other things are not equal at C. Since the kinetic energy at C is small, some other energy must be greater there to compensate. Since this cannot be the potential energy, it must be the pressure energy. A statement of this fact is known as *Bernoulli's principle,* and is as follows: when the velocity of an enclosed moving fluid changes, the pressure changes in the other direction in such a way as to keep the total energy per unit volume constant.

5-12. Illustrations of Bernoulli's Principle. Some peculiar experiments illustrating Bernoulli's principle can be performed. As one example, push a common pin through a card, and hold the card against the hole in a spool so that the pin enters the hole. The card can naturally be held against the spool by drawing air through the hole in the spool from the opposite end because of the low pressure of the air as it moves rapidly between the card and spool. But for the same reason, the card can also be held in place nearly as well by blowing air through the hole toward the card. Another illustration is that of a light ball riding on an air jet. (See figure 5-4). Three forces act on the ball: A, B, and W the weight. The forces A and B are due to air pressures. B is greater than A because the speed of the air on that side is less than on the side where A is. Therefore, in whatever way the ball starts to fall out of the jet, the action of the forces will be such as to push the ball back up into the jet. The suction between two ships moving parallel to each other, the curving of baseballs, the Venturi meter, and the hydraulic suction pump (or aspirator) so commonly used in laboratories are all illustrations of Bernoulli's principle.

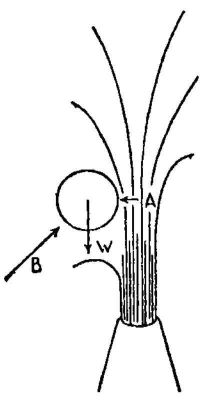


Figure 5-4.

SUMMARY OF CHAPTER 5

Technical Terms Defined

Fluid. A state of matter such that it conforms to the shape of the container; includes liquids and gases.

Liquid. A fluid with a definite volume.

Gas. A fluid which tends to expand indefinitely.

Weight Density. Ratio of weight to volume.

Specific Gravity. Ratio of the density of substance under consideration to the density of water, or ratio of the weight of a given volume of a substance to the weight of an equal volume of water.

Derived Relations. Increase of hydrostatic pressure with increase in depth is equal to the product of the average density of the fluid by the increase in depth.

Laws

Boyle's Law. Given a definite quantity of gas by weight, at a definite temperature, the product of its total pressure and the volume of its container will be a constant.

Pascal's Principle. A local increase in pressure is transmitted to all parts of an enclosed fluid.

Archimedes' Principle. A body immersed in a fluid experiences an upward force (buoyant force) exerted by the surrounding fluid and equal to the weight of fluid displaced by the body.

Bernoulli's Principle. At a given level of a moving fluid, changes of velocity are accompanied by compensating changes of pressure in such a way as to keep the total energy constant in a given volume.

PROBLEMS

5-1. If 2 cubic feet of sulphur weigh 250 pounds, compute (1) the density in English units, (2) the specific gravity, and (3) the density in grams per cubic centimeter.

5-2. Find the excess above atmospheric pressure 50 feet below the surface of a pond in pounds per square foot; in pounds per square inch.

5-3. A cubical tank, six feet on each edge, is half full of water. The upper half contains oil of specific gravity 0.8. Find the excess of the force exerted by the water on the lower half of one side of the tank, over what it would be if the tank were filled with air.

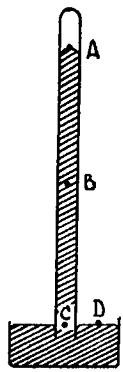


Figure 5-5.

5-4. Figure 5-5 represents a vertical tube open at the bottom and closed at the top. It was originally filled with mercury (density = 13.59 gm./cm.^{3}), the end closed with the thumb and inverted into another dish of mercury. Some of the mercury ran out when the thumb was removed, leaving a vacuum above A. The pressure at A is therefore zero. (1) If B is 38.0 centimeters below A, find the pressure at B. (2) Find the pressure at C, 76.0 centimeters below A. (3) If C and D are on the same level, how do their pressures compare? (4) Express the pressure at D in pounds per square inch. (5) If a small hole were made in the tube at B, would mercury run out or air go in? This apparatus (without the hole at B) is called a *mercury barometer* and is used to measure atmospheric pressure.

5-5. Figure 5-6 represents a vessel containing water into which a tube, also full of water, dips. The tube is closed at the lower end. Compare the pressures at points A, B, and D, all on the same level. Compare qualitatively the pressures at points C, D, and E. If the lower end of the tube is opened, will air enter the tube or will water run out? A tube used in this way is called a *siphon*. Why will it not work when the vertical distance from B to C is such that the difference in pressure between these two points is greater than one atmosphere?

5-6. A rock weighs 250 pounds in air and 150 pounds in water. Find its specific gravity and its density.

5-7. What force must be exerted on a force-pump piston that is three inches in diameter to raise water 100 feet?

5-8. The specific gravity of iron is 7.6. A hollow piece of iron weighs six grams in air and four grams in water. What is the volume of the cavity in the iron?

5-9. A sinker weighing 38 grams is fastened to a cork weighing 10 grams, and the two together are in equilibrium when immersed in water. Find the specific gravity of the sinker if that of the cork is 0.25.

5-10. A dam is 10 feet in height and 100 feet long. If the water level is even with the top of the dam, find the thrust of the water against the dam.

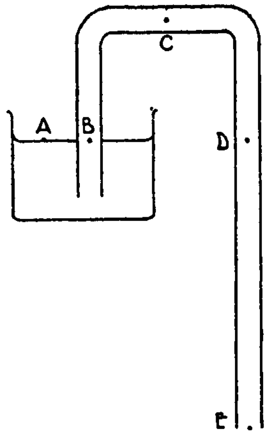
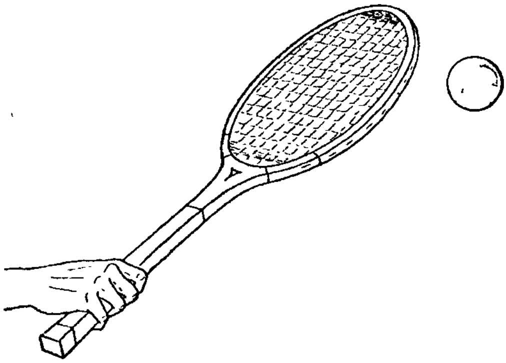


Figure 5-6.

5-11. A horizontal water pipe two square inches in cross section widens out to four square inches in cross section. If the speed of the water is six feet per second in the narrower part, what is the speed in the wider part? If the gage pressure in the narrower part of the pipe is five pounds per square inch, what is the gage pressure in the wider part?

CHAPTER 6



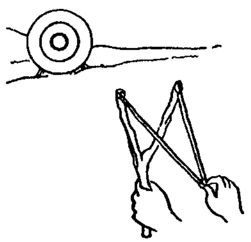
Elasticity

6-1. Elasticity. In everyday life the concept of elasticity is probably more definitely associated with rubber than with any other substance. But when we use the word in its technical sense, we are obliged to admit that rubber is elastic not because it can be stretched so far, but because after being stretched, it has a tendency to return to its original dimensions. *Elasticity* is the tendency of a body after being deformed to return to its original dimensions. In order to discuss elasticity intelligibly, we must become familiar with the use of two more technical terms: stress and strain.

6-2. Stress. *Stress* is the ratio between a force and an area over which the force is applied. We shall discuss three types of stress. The force may be perpendicular to the area or in the plane of the area. If the force is perpendicular to the area, it may push on the area or pull on the area. We have already met the case where the force pushes perpendicularly on the area, and have called this type of stress *pressure* (see section 4-15). When the force pulls on the area, we have a *tensile stress*, and its tendency is always to lengthen the wire or whatever object the force is acting upon. If the force is exerted in the plane of the given area, it gives rise to what is known as a *shearing stress*. In all three cases, the unit of stress is the pound per square inch, the newton per square inch, the newton per square meter, or some similar unit.

$$\text{stress} = \frac{\text{force}}{\text{area}}$$

6-3. Strain. A strain has no units; it is always a pure number. Each type of stress produces its own type of strain, or deformation.



A pressure tends to decrease the volume of the object on which the pressure is applied; and the numerical measure of the volume strain is the ratio of the decrease in volume to the original volume.

$$\text{volume strain} = \frac{\text{decrease in volume}}{\text{original volume}} = \frac{v}{V}$$

Since a tensile stress tends to increase the length, the accompanying strain is the ratio of the increase in length to the original length.

$$\text{tensile strain} = \frac{\text{increase in length}}{\text{original length}} = \frac{e}{L}$$

A shearing stress tends to distort a cube into a solid figure having two rhomboid, two oblong, and two square faces. A reference to figures 6-1 and 6-2 shows the nature of the change in shape. For example, imagine a force exerted on the cover of a thick book, parallel to the direction of a line of reading matter on the cover.

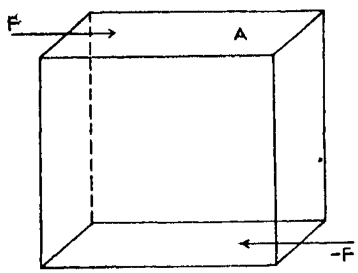


Figure 6-1.

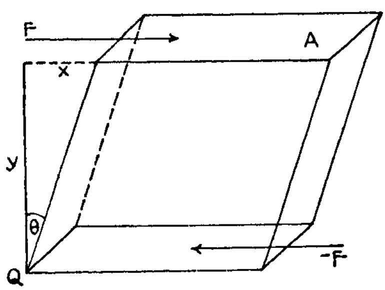


Figure 6-2.

The result is to change the shape but not the volume of the book. The shearing strain is the ratio of x to y in figure 6-2.

It is thus roughly proportional to the angle θ with vertex at Q. In all three of these cases, if the substance is elastic, the strain disappears when the stress is removed, and the object resumes its original shape and size.

6-4. Modulus of Elasticity. It is possible to produce a unit strain in a rubber band; that is, the stretch may be made equal to the original length. The stress necessary to produce this unit strain is the numerical measure of the elasticity of the rubber band; stress is also the force per unit cross section with which the rubber band resists the stretching and tries to return to its original dimensions. Half of this stress would be enough to produce a strain of 0.5. In general the stress and the strain are proportional, so that the elasticity may also be obtained by dividing the existing stress by the accompanying strain. This is fortunate because rubber is nearly the only substance the elasticity of which could be determined by measuring the stress necessary to produce unit strain, although it is possible in the case of any substance to measure a given stress and the corresponding strain and obtain the ratio. The ratio of the stress to the strain is the *modulus of elasticity.*

$$\text{modulus of elasticity} = \frac{\text{stress}}{\text{strain}}$$

The numerical value of a modulus of elasticity is always *very large.* Any fraction may be increased by increasing its numerator or by decreasing its denominator. Therefore, since stresses (in the numerator of the modulus) are large, and corresponding strains (in the denominator of the modulus) are small, we are not surprised to find, for example, that the stretch modulus (called *Young's modulus*) for steel is in the neighborhood of 30,000,000 pounds per square inch.

6-5. Hooke's Law. In the preceding section the statement was made that in general, stress and strain are proportional. This fact is known as *Hooke's law,* and is true providing the stress does not become too great. If the stress does become too great for Hooke's law to hold, we say that the elastic limit has been exceeded. Mathematically, the two statements, "Stress is proportional to strain," and "Stress is equal to strain times a constant" are equivalent. The proportionality constant is the modulus of elasticity.

Since for a given cross section, stress is proportional to force, and for a given original length, strain is proportional to elongation, it is also possible to state Hooke's law: for a given specimen, elongation is proportional to stretching force. This may be expressed by the equation

$$F = ke$$

where k is a constant for a given specimen. We may also set up a similar expression for the restoring force which, by Newton's third law, is equal and opposite to the stretching force, F. This equation is

$$F = -ke$$

6-6. Illustration of the Use of Hooke's Law. A force of 10 pounds will produce in a certain helical spring a strain of 10 per cent. What additional force will produce an additional strain of 10 per cent of the new length, assuming that the elastic limit is not reached?

Using Hooke's law in the form, elongation is proportional to stretching force, we have

$$\frac{0.1\ L}{10} = \frac{0.1\ (L + 0.1\ L)}{F}$$

where L is the original length and F is additional force which we seek. The right-hand side may be rewritten

$$\frac{0.1\ (1.1L)}{F} \text{ or } \frac{0.11L}{F}$$

so that the original proportionality becomes

$$\frac{0.1L}{10} = \frac{0.11L}{F}$$

Canceling the L's and solving for F, we find that F is 11 pounds.

6-7. Illustrations of the Use of Young's Modulus. (1) Let us consider the following problem. What force is necessary to produce a stretch of an eighth of an inch in a steel wire 10 feet long (120 inches long) and one hundredth of an inch in diameter?

The strain in this case is 0.125/120 or 0.001042. Since the material is steel, we shall take the modulus to be 30,000,000 lb./in.^{2} By multiplying both sides of the equation in section 6-4 by (strain), we obtain the relation that stress = (strain) (modulus). The necessary stress in the present case is therefore (0.001042) (30,000,000) lb./in.^{2}, or 31,300 pounds per square inch. The cross section of this wire is $[\pi(0.01)^2/4]$ in.^{2}, or 0.0000785 in.^{2} Since stress is the ratio of force to area, the force is the product of area and stress, or in this case (0.0000785 in.^{2}) (31,300 lb./in.^{2}) = 2.46 pounds.

(2) As a second illustration, assume a literal problem. A certain wire has a cross section of A square inches, and a length of L inches. If a force of F pounds produces an elongation of e inches, find Young's modulus, Y.

The stress is, then, F/A, and the strain e/L. Therefore the modulus, Y, is (F/A)/(e/L) or

$$Y = FL/Ae$$

Since we have used letters rather than numbers, the result is general, that is, it may be taken as a formula useful in solving any problem involving Young's modulus.

6-8. Bulk Modulus. The modulus obtained by dividing pressure by volume strain is called the *bulk modulus*. The reciprocal of the bulk modulus is called the *compressibility*. If B represents the bulk modulus, V the original volume, and v the decrease in volume, then v/V is the volume strain, and the formula connecting the four quantities is

$$B = \frac{PV}{v}$$

where P represents the pressure (the stress). Solids, liquids, and gases all have volume elasticity, but solids alone have stretch moduli and shear moduli.

6-9. Illustration of Bulk Modulus. (1) If the specific gravity of sea water is 1.03, find the increase in pressure that one would encounter five miles below the surface of the ocean. (2) If the bulk modulus of sea water is 31,900,000 lb./in^2, how much would a cubic foot of water decrease in volume if removed from the surface and placed at a point five miles below the surface?

(1) We find the increase in pressure from the formula $P = hD$ (section 5-4). In this case h = (5) (5,280) feet and D = (1.03) (62.4 lb./ft.^{3}), therefore P = (5) (5,280) (1.03) (62.4) pounds per square foot, or 1,697,000 lb./ft.^{2} By dividing this by 144 we can change the result to 11,790 pounds per square inch.

(2) Since $B = PV/v$, it follows that $v = PV/B$. Filling in the numerical values, we have v = (11,790 lb./in.^{2}) (1 ft.^{3})/(31,900,000 lb./in.^{2}) = 0.000370 ft.^{3} In other words, water is so nearly incompressible that its density at the bottom of an ocean five miles deep is practically the same as the density at sea level.

6-10. Shear Modulus.

In figure 6-2, if the force F is applied to the area A (to which it is parallel) the shear modulus is

$$S = \frac{Fy}{Ax}$$

which is the ratio of the stress, F/A, to the strain, x/y.

6-11. Illustration of Shear Modulus. The shear modulus of glass is 2×10^{10} newtons per square meter. A glass brick, 3 by 4 by 5 centimeters, rests on the 4 by 5 centimeter face. If a shearing stress of 2,000,000 newtons per square meter is applied to the upper surface, find the shearing strain, the relative displacement of the upper and lower surfaces, and the shearing force.

Modulus = (stress)/(strain); modulus = 2×10^{10} newtons/m^2 and stress 2×10^6 newtons/m^2. Substituting the numerical values into the equation

$$2 \times 10^{10} = \frac{2 \times 10^6}{\text{strain}}$$

Multiplying both sides by strain and dividing both sides by 2×10^{10}, we obtain strain = 10^{-4}, or strain = 0.0001. This strain is x/y in figure 6-2, and y = 3 centimeters. We therefore have

$$0.0001 = \frac{x}{3}$$

Solving for x, we obtain $x = 0.0003$ centimeter, the relative displacement of the upper and lower surfaces. The shearing force may be obtained from the equation

$$\text{shearing stress} = \frac{\text{shearing force}}{\text{area of surface on which force is applied}}$$

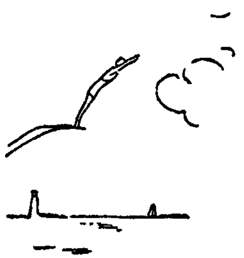
Substituting into this equation the numerical values, shearing stress = 2×10^6 newtons/m^2 and area = (0.05) (0.04) or 0.0020 square meter, we have

$$2 \times 10^6 = \frac{F}{0.0020}$$

Solving for the shearing force, we obtain $F = 4{,}000$ newtons. The result may be changed to kilograms by dividing by 9.80. This will give, to three significant figures, 408 kilograms for the shearing force.

To give an idea of the magnitude of this force, we can multiply it by 2.2 (there are 2.2 pounds in a kilogram) and obtain its equivalent in English units, which would be 898 pounds.

6-12. Bending of Beams; Twisting of Rods. In more extended treatises, other applications of elasticity would be discussed.



One of these could well be the sag to be expected in a beam of given dimensions and given material, supported at the ends and carrying a given load with a given distribution. This expression would involve Young's modulus. The behavior of a spring board can also be computed by the help of Young's modulus. Another common problem concerns the amount of twist that could be expected in a rod under given conditions. This expression would involve the shear modulus, which may also be called the *rigidity modulus*. It will be noticed that bending and twisting do not involve any new moduli of elasticity. In fact it may be shown that the three moduli that we have discussed are all related, so that if we knew any one of them for a given material, we could compute the other two.

6-13. Ultimate Strength. In testing materials, it is important to know not only Young's modulus, the compressibility and rigidity modulus, and the elastic limits, but also the stresses that will cause the specimen to fail. For example, it is possible to carry the tensile stress beyond the elastic limit to a point such that the specimen will break in two. This stress is known as the *ultimate strength* of the material. Data of this type may be found in engineering handbooks.

SUMMARY OF CHAPTER 6

Technical Terms Defined

Stress. Ratio of a force to the area on which it is applied.

Strain. Ratio of deformation to original dimension.

Elasticity. Ratio of stress to strain for a given material.

Young's Modulus. Coefficient of stretch elasticity. It is the ratio of the tensile stress to the resulting tensile strain.

Bulk Modulus. Coefficient of volume elasticity. Ratio of the pressure to the resulting volume strain.

Compressibility. Reciprocal of the bulk modulus. Ratio of the volume strain to the pressure producing it.

Shear Modulus. Coefficient of rigidity. Ratio of the shearing stress to the resulting shear.

Bending. Involves Young's modulus.

Twisting. Involves the shear modulus.

Elastic Limit. Stress beyond which specimen will not return to its original dimensions when released.

Ultimate Strength. Stress necessary to break the specimen.

Laws

Hooke's Law. Within the elastic limit, stress is proportional to strain.

PROBLEMS

6-1. A steel bar one quarter inch square is ten feet long. When the bar, in a vertical position, is made to support a one-ton weight, the bar is stretched one hundredth of a foot. Compute the stress, strain, and Young's modulus for this sample of steel.

6-2. If a bar, one half inch square and 20 feet long, made from a sample of steel the modulus of which is 32,000,000 pounds per square inch, be substituted in place of the bar in the preceding problem, what will be the stress, the strain, and the elongation?

6-3. What should be the diameter of a circular steel rod 10 feet long, if the permissible tensile stress is 10,000 pounds per square inch, in order to support a load of 50 tons? If Young's modulus is 32,000,000 pounds per square inch, how much will the rod stretch?

6-4. If Young's modulus for a sample of steel is 30,000,000 pounds per square inch, what is the value of this modulus in newtons per square meter?

6-5. If a pressure of 2,000 pounds per square inch decreases the volume of a copper sphere, one foot in diameter, by 0.258 cubic inch, compute the bulk modulus of copper.

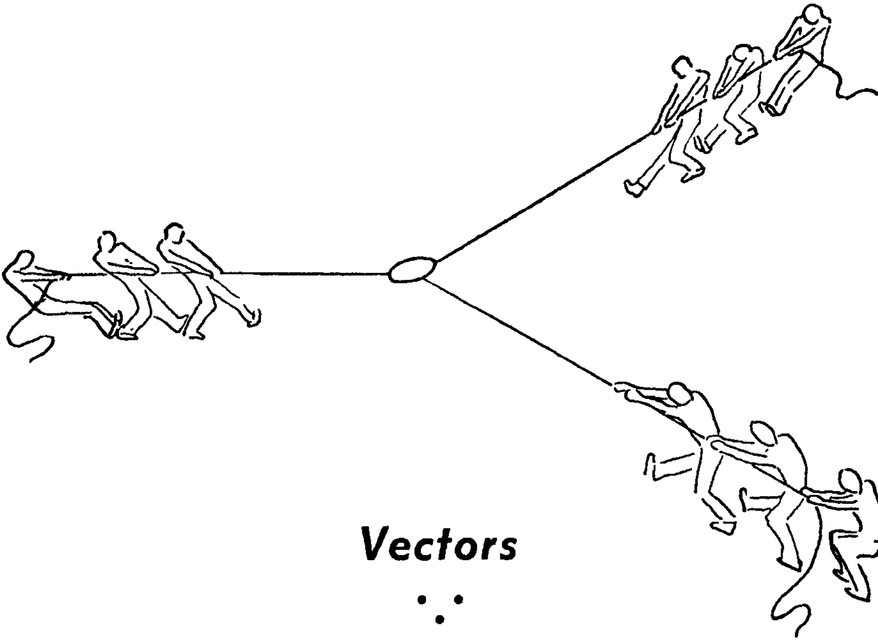
6-6. Two opposite forces of 5 newtons each are applied, as in figure 6-1, to opposite faces of a cubical block of jelly 10 centimeters on an edge, and produce a relative displacement of one centimeter. Find the stress, the strain, and the shear modulus.

6-7. A rectangular block of brass (shear modulus = 5,500,000 pounds per square inch) 10 inches high rests on a horizontal table. A force which is parallel to the surface of the table is applied to the upper surface of the block and produces a displacement of an eighth of an inch. What is the shear? Find the shearing stress.

6-8. If a tensile stress of 20,000 lb./in.^{2} produces in a wire a strain of 0.000625, applying Hooke's law, what stress will produce a strain of 0.1 per cent?

6-9. Neglecting the friction of the plunger, how much work would be done in pulling an airtight piston far enough out of a cylinder against atmospheric pressure (14.7 pounds per square inch) to leave a vacuum under it the volume of which is 10 cubic inches?

CHAPTER 7



Vectors

7-1. Scalars and Vectors. A distinction to which we must become accustomed in physics is that between *scalars* and *vectors*. If a physical quantity is *not* associated with the idea of direction, it is known as a *scalar*. Such values may be read from *scales*, such as dials of watches, steel tapes, speedometers, barometers, and the like. A *scalar* quantity has magnitude but not direction. Examples are: ten seconds, two cubic feet, or five pounds of sugar. It would be meaningless to speak of ten seconds "up," two cubic feet "west," or five pounds of sugar "south!" On the other hand, physical quantities that *are* associated with direction are called *vectors*. A *vector* is a quantity having both magnitude and direction; a vector is not completely described until both are given. A vector is a combination of a quantity read from a scale and the associated orientation in space of that quantity. Examples of vectors are: a velocity of 50 miles per hour due north, a 25-pound pull vertically down, or a displacement of 20 feet to the east. If at any time we wish to discuss merely the magnitude of a vector without reference to its direction, we thereby reduce the vector to a scalar for the time being. For example, the speedometer of an automobile gives the speed but not the direction of motion; speed is therefore a scalar. In physics we reserve the word *speed* to describe the scalar rate and use the word *velocity* for the vector concept. A vector is conveniently represented by an arrow. The length of the arrow is made proportional to the magnitude (the numerical part) of the vector, and the direction of the arrow corresponds to the direction of the vector quantity.

7-2. The Triangle Method of Adding Vectors. There is a branch of mathematics called *vector analysis* which deals with the addition, multiplication, differentiation, and so on, of vectors, but for our purpose it will be sufficient if we learn to *add* vectors. The simplest illustration of a vector is a displacement; a displacement is the change in position that would be necessary to transfer an object in a straight line from a reference point (usually called the *origin*) to the point that it now happens to occupy. For example, after a man has walked due north four miles, his displacement is four miles due north; the displacement would still be the same if he had reached this point by a circuitous route. If after that, he should walk three miles due east, his displacement would then be five miles in a direction about 37 degrees east of north (see figure 7-1). We may consider that we have added a displacement of 4 miles due north to a displacement of 3 miles due east and that the vector sum (or *resultant* as it is often called) is 5 miles in a direction north, 37 degrees east. The student may verify this result either by drawing the figure to scale and measuring the length of the hypotenuse and using a protractor to obtain the angle, or he may use the Pythagorean theorem and observe that $4^2 + 3^2 = 5^2$, and then use trigonometry to get the angle.

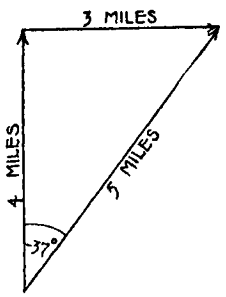
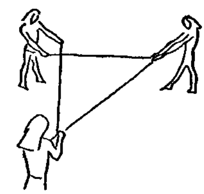


Figure 7-1.

The ancient Egyptian "rope-stretchers," the equivalent of our modern surveyors, used ropes with knots at convenient places to enable them to form right angles quickly through the use of 3-4-5 triangles.



The general rule for the triangle method of adding vectors is to put the vectors together, head to tail, and the vector sum or resultant will be obtained by drawing an arrow straight from the beginning of the first vector to the end of the second. The method may be extended to add several vectors at a time. In that case we talk of the polygon method, and place all the vectors together head to tail in any order, and connect the beginning of the first to the end of the last.

7-3. The Parallelogram Method of Adding Vectors. When the vectors under consideration are forces, it is usually more convenient to use another method of combining them, because the vectors representing the forces are all acting at the same point. Consider for example the three-cornered tug-of-war depicted at the head of this chapter. If each team exerts a force of 500 pounds and the angles are all 120 degrees, which team is winning? One could put up a superficial argument to the effect that any one of the three teams is losing, for is it not opposing a mere 500 pounds to 1,000 pounds? But this can not be true; all three teams can not be losing! Actually we have to add two of the 500 pounds vectorially and compare this resultant with the third force. And in this case we can see from symmetry that the tug-of-war is a tie.

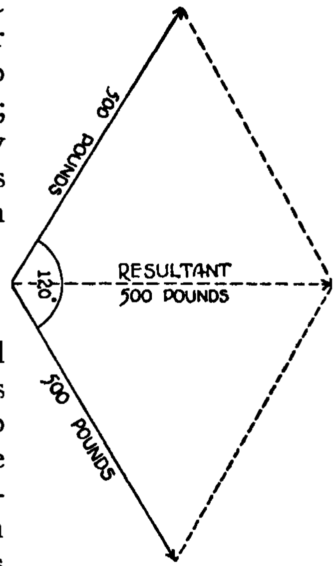


Figure 7-2.

The parallelogram rule which we find convenient in this case may be stated as follows: assuming that the two vectors to be added are both drawn from the same point, complete the parallelogram by drawing two more lines parallel to the given vectors. The parallelogram now consists of the two original arrows and the two additional lines. The vector sum or resultant will be the arrow drawn from the two coinciding tails to the opposite corner of the parallelogram. In the case of the three-cornered tug-of-war, the diagram will be as shown in figure 7-2. In this case the resultant cuts the parallelogram into two equilateral triangles and we have the rather unusual result that the vector sum of two 500-pound forces 120 degrees apart is itself another force of 500 pounds. This is sufficient to offset the third team.

Only two forces at a time can be handled by the parallelogram method. If we had three or more vectors to add, we should add two of them, then add the third to the resultant of the first two, and so on.

7-4. Another Illustration. On the physics lecture table it is convenient to illustrate the parallelogram law by the apparatus shown in figure 7-3. Three strings are tied together at one point and weights of 3, 4, and 5 pounds respectively are attached to the strings. By letting the 5-pound weight hang directly down and using two pulleys, it is possible to exert three forces on A of 3, 4, and 5 pounds respectively. If the strings are displaced from their position of equilibrium in any direction, they will come back to a position such that the strings with the tensions of 4 and 3 pounds will make an angle of 90 degrees with each other and angles of 37 degrees and 53 degrees

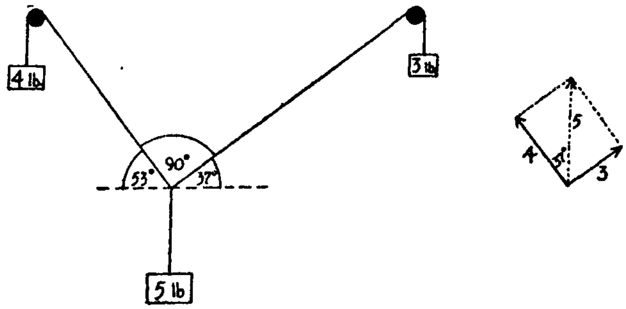


Figure 7-3.

respectively with the vertical. If a parallelogram be constructed using arrows with lengths proportional to 4 and 3 pounds respectively, and in the directions indicated in figure 7-3, it will be found that their resultant, the diagonal of the parallelogram, will be vertical, and equal to 5 pounds. Thus the 5-pound weight exerts enough force downward to balance the vector sum of the other two forces.

7-5. Resolution of Forces Into Components. The procedure of combining vectors may be reversed: a single vector may be replaced by two vectors. In this process, which is called *resolving a vector into components*, a parallelogram (nearly always a rectangle) is drawn with the given vector as its diagonal. Suppose the two forces C and D, figure 7-4, to be at right angles to each other, and let the resultant be called R. In this case, C and D are rectangular components of R. But the same diagram may be constructed in the reverse order: given R and the direction of either C or D, it is possible to construct a rectangle such that R shall be its diagonal and such that the sides of the rectangle shall be either parallel or perpendicular to the given di-

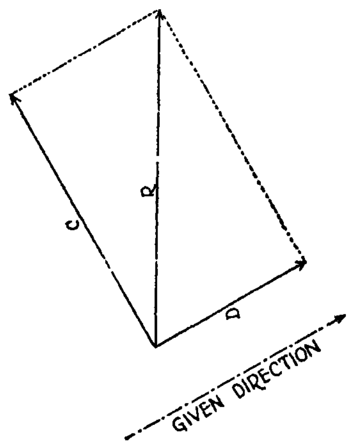


Figure 7-4.

rection. Given any force, it is always possible to resolve it into two components, one parallel, and the other perpendicular, to a given direction.

7-6. Properties of Certain Triangles. In this chapter we shall confine ourselves to four kinds of right triangles (figures 7-5, 7-6, 7-7, and 7-8). The student will recall from his geometry two facts, one of which we have already mentioned: (1) in a right triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse (Pythagorean theorem) and (2) if the three angles of one triangle are equal to the three angles of another triangle, the triangles are *similar*, and the sides of one triangle are proportional to the cor-

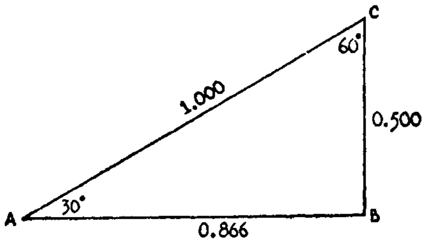


Figure 7-5.

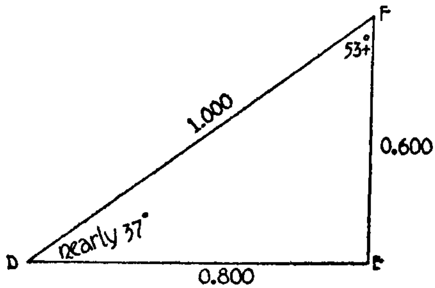


Figure 7-6.

responding sides of the other triangle, and conversely. Applying the first of these two propositions, we see that in the figures $(0.500)^2 + (0.866)^2 = (1.000)^2$; $(0.600)^2 + (0.800)^2 = (1.000)^2$ (which gives the

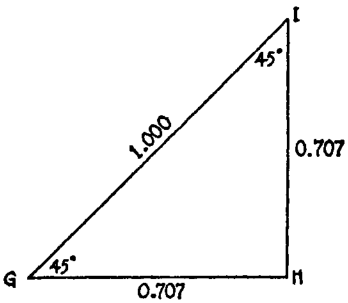


Figure 7-7.

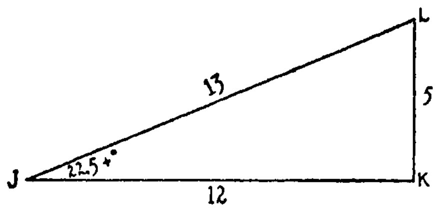


Figure 7-8.

same set of ratios as $3^2 + 4^2 = 5^2$); $(0.707)^2 + (0.707)^2 = (1.000)^2$; and that $5^2 + 12^2 = 13^2$. Applying the second proposition we could say that if we had a 30-, 60-, and 90-degree triangle with the hypotenuse equal to 20 feet, the shorter leg would be 10 feet and the longer leg 17.32 feet.

7-7. Examples of Addition of Forces. (1) Suppose that two horizontal forces act on a post, one due north equal to 30 pounds, and one due east equal to 40 pounds. Find a single force which would have the same effect as the forces combined.

In order to solve the problem, we draw the arrow PR (figure 7-9) in a direction such as to suggest north and of such a length as to suggest 30 pounds. For example, we could let each quarter inch represent 10 pounds, in which case 0.75 inch would represent 30 pounds. Similarly, from the same point draw another arrow PS pointing east and of such a length as to represent 40 pounds (one inch on the scale suggested). Complete the parallelogram $RPSQ$ and draw the diagonal PQ. It will measure 1.25 inch and represent 50 pounds. Without measuring it we could compute its value by comparing figures 7-6 and 7-9. The triangles DEF and PSQ are similar because $DE/PS = EF/SQ$, or $0.800/40 = 0.600/30$. Therefore $DE/PS = DF/PQ$, or $0.800/40 = 1.000/PQ$, which gives us 50 pounds for PQ.

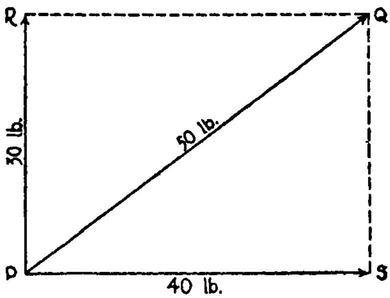


Figure 7-9.

(2) Given a force of 100 pounds due northeast (that is, the angle with both the north direction and the east direction is exactly 45 degrees), find its northerly and its easterly components. In other words, find two forces, one due north and one due east that together will be equivalent to the single 100-pound force.

Draw a diagram (figure 7-10) in which an arrow the length of which is proportional to 100 pounds (say an eighth of an inch represents 10 pounds) points toward the northeast; call it TU. Now draw a rectangle in such a way that the sides will run north-south and east-west, and so that the 100-pound force TU shall be the diagonal. By comparison with figure 7-7, it will be seen that each component is 70.7 pounds. This fact may also be determined by measuring the arrows TV and TW.

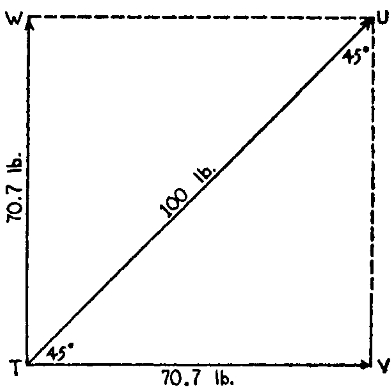


Figure 7-10.

7-8. Problem Illustrating Addition of Several Forces. The following horizontal forces act on a point: (1) a force of 750 pounds directed due southwest, (2) a force of 750 pounds due south, (3) a force of 200 pounds directed 60 degrees north of west, and (4) a force of 1,600 pounds directed 30 degrees east of north. Find the northward and eastward components of the resultant.

The first step in solving this problem is to draw the diagram as indicated in figure 7-11 with a scale of, say, one centimeter representing 200 pounds. To lay off the 750 pounds due southwest, a distance representing 750 pounds is measured on a line making a 45-degree angle with both the south

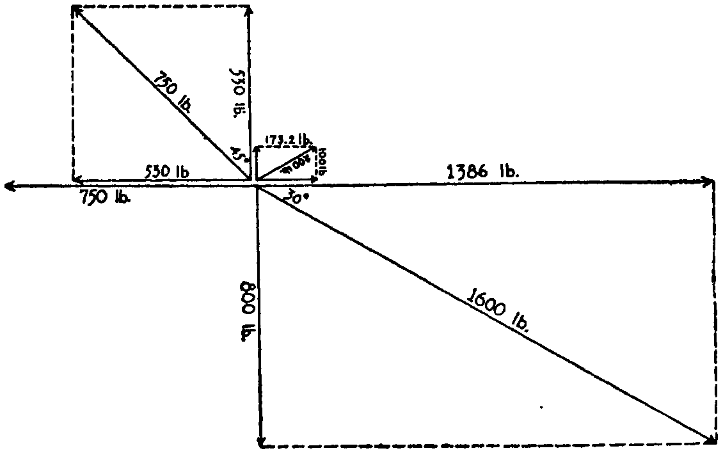


Figure 7-11.

and the west lines. The 200 pounds 60 degrees north of west is measured on a line between the west and north directions making an angle of 60 degrees with the west, and similarly with the other forces.

Next each of these forces must be broken up into components along the north-south and east-west axes. This is done by drawing the sides of a rectangle of which the original force is the diagonal. By reference to the sample triangles of section 7-6, the values of these components may be found. The components of the 1,600-pound force are (1,600) (0.866) or 1,386 pounds north and (1,600) (0.500) or 800 pounds east. The 200-pound force has components (200) (0.866) or 173 pounds north and (200) (0.500) or 100 pounds west. The 750 pounds south is already along the north-south line and therefore does not need to be resolved. The 750-pound force southwest has components of (750) (0.707) or 530 pounds south and also 530 pounds west.

Now, let forces to the north and to the east be considered positive while those toward the south and west are considered negative. Combining the components just found, we have

$$+ 1{,}386 + 173 - 750 - 530 = 279 \text{ pounds north}$$
$$+ 800 - 100 - 530 = 170 \text{ pounds east.}$$

Although this completes the problem as stated, the student may be interested in checking the fact that the vector sum of these two components is a force of 327 pounds in a direction 31.3 degrees east of north. This may be done by constructing to scale the rectangle having for its sides 279 and 170, measuring the length of its diagonal and measuring the angle with a protractor.

7-9. Illustrative Problem Involving a Simple Truss. For the purpose of simplification, assume that the parts of the truss in figure 7-12 are weightless. Which are tension members and which are compression members? Must point A be supported from beneath or held down? Find the tension or compression in each member.

To find which are the compression members and which the tension members, consider the effect of breaking each member in turn while every other part remains as it is in the diagram. For example, if AC were broken, C would not fall toward A but would move farther from it. Therefore AC is a tension member. Applying the same test to CD, we see that it is also a tension member. If BD were broken, D would promptly move toward B, therefore BD is a compression member. If BC were broken, ACD would try to form a straight line between A and D, bringing C toward B, therefore BC is a compression member. Similarly AB is a compression member.

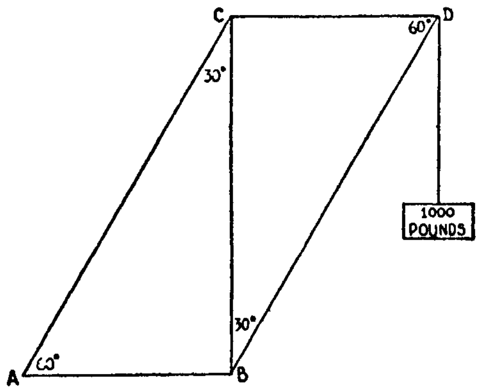


Figure 7-12.

The whole structure somewhat resembles a seesaw with fulcrum at B, that is, point B must be supported from below while point A must be held down.

Since the entire structure is in equilibrium, each point is separately in equilibrium (at rest in this case); therefore the simplest method of finding the numerical values of the tensions and compressions is to consider the forces acting at each point in turn. Since we already know one of the forces acting at D, let us consider that point first. Three forces act on point D (see figure 7-13), 1,000 pounds straight down, the tension in CD horizontally to the left, and the compression in BD in a direction slanting upward 30 degrees to the right of the vertical. This slant force must be resolved into a vertical and a horizontal component, which from now on are to be thought of as replacing the slant force. Since D is in equilibrium, the upward force must equal the downward force which is 1,000 pounds. Reference to figure 7-5 shows that the longer leg of a 30-degree right triangle is 0.866 of the hypotenuse; therefore the hypotenuse of our triangle is 1,000/0.866 or 1,155 pounds, which is the compression in the member BD. On account of the equilibrium at point D, the tension in the member CD is equal to the horizontal component of the slant force of 1,155 pounds. Further reference to figure 7-5 shows that the shorter leg is equal to half the hypotenuse, or in this case, 577 pounds, which is the tension in member *CD*.

The tension in member *CD* exerts a pull to the left of 577 pounds on *D* and a pull to the right of the same amount on *C*. Similarly, since we are ignoring the weight of the truss members, the compression in member *BD* exerts a push upward to the right on *D* of 1,155 pounds and a push downward to the left on *B* of the same amount. We are therefore in a position to solve for all the forces at point *C* just as we did at point *D*. The result is that we discover the tension in *AC* to be 1,155 pounds and the compression in *BC* to be 1,000 pounds.

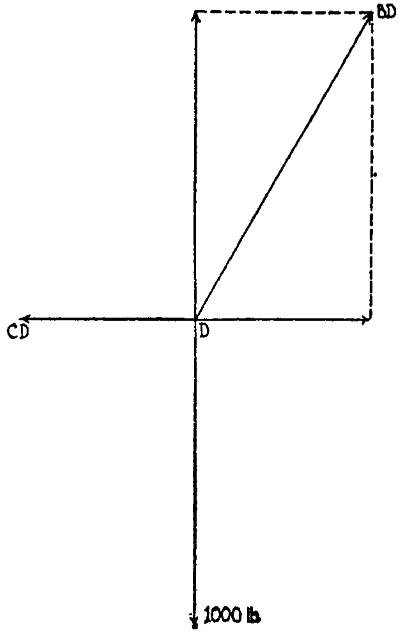


Figure 7-13.

When we solve for the forces acting at point *B*, we discover that in order to get equilibrium, we must have an upward force of 2,000 pounds exerted by whatever the truss is resting on at point *B*. In solving for the forces at point *B*, we also discover that the compression in member *AB* is 577 pounds.

When we solve for the forces at point *A*, we find that in order to obtain equilibrium, there must be a downward force of 1,000 pounds in addition to the tension in *AC* and the compression in *AB*. It will be left to the student to carry out the actual work of solving for the forces at points *C*, *B*, and *A*.

SUMMARY OF CHAPTER 7

Technical Terms Defined

Scalar. A physical quantity which may be read off of a single scale, e.g., time, length, speed.

Vector. A physical quantity which has both magnitude and direction, e.g., displacement, force, velocity. Vectors are conveniently represented by arrows.

Resultant or Vector Sum. A single vector which is equivalent to two or more vectors combined.

Triangle or Polygon Addition of Vectors. If several arrows representing vectors are placed together head to tail without altering any of their lengths or directions, an arrow drawn straight from the tail of the first to the head of the last is the resultant of the group.

Parallelogram Method of Adding Vectors. If two arrows representing vectors are both drawn from the same point, the resultant will be a third arrow also drawn from the same point to the opposite vertex of a parallelogram formed with the two given vectors as two of its sides.

Rectangular Components. Any two mutually perpendicular vectors which will add vectorially to a given vector are said to be rectangular components of that vector. The process of finding the two components is called "resolving a vector into rectangular components."

PROBLEMS

7-1. Find the vector sum of a three-pound force due north and a four-pound force due west. Would there ever be any practical use for the *arithmetic* sum of these two quantities?

7-2. Find the vector sum of an eastward force of 50 pounds, a westward force of 90 pounds, and a southward force of 30 pounds.

7-3. Given a force of 50 pounds acting in a direction 30 degrees south of east, resolve it into (1) two components, one due east, and the other due south; (2) resolve it into two components, one 15 degrees north of east, and the other 15 degrees east of south.

7-4. Velocities are vector quantities and are to be added in the same way as forces. A stream flows southward at a speed of five miles per hour. A motor boat driven westward relative to the water at a speed of twelve miles per hour actually travels in a direction about 22.5 degrees south of west. Find its actual speed.

7-5. A ten-pound weight is held in position by two strings, one horizontal and the other making an angle of 30 degrees with the vertical. Compute the tension in each string.

7-6. A motor boat can travel at the rate of 12 feet per second in still water. Disregarding the time lost in starting and stopping, how long will it take to cross a river, 1,200 feet wide, the drift speed of which is 5 feet per second, if the boat heads directly across? How far down on the opposite bank will the boat land?

7-7. Find the resultant (direction and magnitude) of the following seven forces, all horizontal: 100 pounds due north, 100 pounds due northeast, 100 pounds east, 80 pounds 30 degrees south of east, 60 pounds due south, 50 pounds 30 degrees south of west, and 40 pounds due west. The method of section 7-8 is best for a problem of this type.

7-8. A triangular frame in a vertical plane has its ten-foot member horizontal and its six-foot and eight-foot members above the ten-foot member. Neglect the weight of the members. A 100-pound weight hangs from the junction of the six-foot and eight-foot members. Compute (1) the compression in the six-foot member, (2) the compression in the eight-foot member, and (3) the tension in the ten-foot member.

7-9. Draw the diagrams similar to figure 7-13 for the points C, B, and A, in the truss-work of figure 7-12.

7-10. Figure 7-14 represents a bridge truss with a span of 48 feet. Assume that the support at A exerts a force vertically upward on the truss of 500 pounds and compute the various compressions and tensions in the members, assumed weightless, which are hinged at the joints. The load at G is 2,000 pounds and all the triangles are of the type shown in figure 7-6.

7-11. A swimmer's speed is 50 yards per minute in still water. If a river flows at the rate of 30 yards per minute, find (1) the time required to swim to a point 100 yards upstream and back, also (2) the time needed to swim 100 yards across-stream and back. The considerations of this problem led indirectly to Einstein's famous theory of relativity.

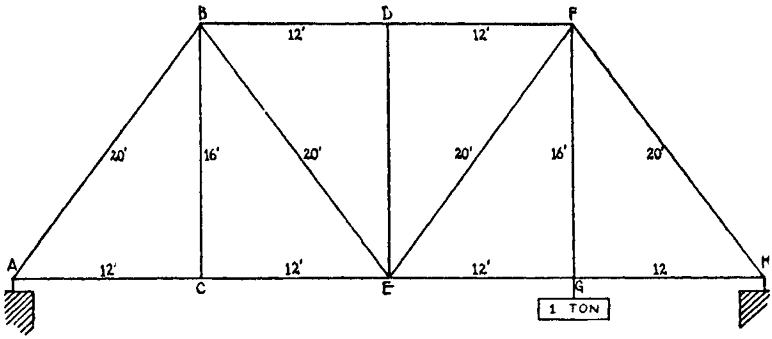


Figure 7-14.

7-12. How great a force (and in what direction) does the atmosphere exert on one square foot of a vertical surface? Why is the total force exerted by the atmosphere on the whole earth equal to zero? What is the total force on any stationary object?

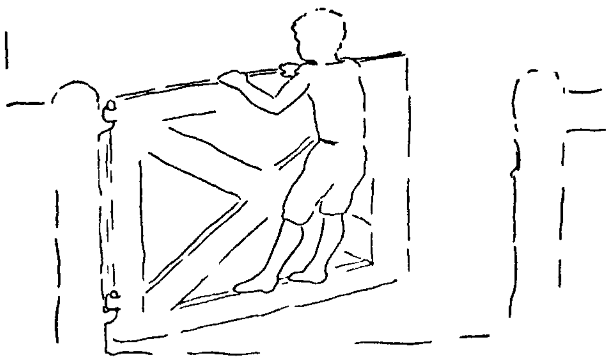
7-13. How much work will be done in pulling a sled and load, which together weigh 100 pounds, horizontally for a distance of 50 feet, if the coefficient of friction is 0.1 and the rope makes an angle of 45 degrees with the horizontal?

7-14. How great a force must be exerted on a 45-degree wedge to enable the wedge to exert two normal forces of 130 pounds each. What is the mechanical advantage of this wedge?

7-15. How much force parallel to the plane is required to support a 200-pound weight on a smooth inclined plane 10 feet long and 6 feet high? Also find the normal force exerted by the plane on the weight.

7-16. If the coefficient of friction in the preceding problem is 0.2, how much more force must be exerted in drawing the weight up the plane rather than in lowering it?

CHAPTER 8



Moment of Force; Center of Gravity

8-1. Translatory Versus Rotatory Motion. All motions, no matter how complicated, can be thought of as being combinations of two simple kinds of motion; one of these is called *translatory* motion and the other *rotatory*. We seldom have either of these in its pure state, though they are both readily visualized. Pure translatory motion is motion of such a sort that any line drawn on the body under consideration remains parallel to its original position during the motion. An example of pure translatory motion is the behavior of a compass needle as the compass is moved about. Even though the compass itself is carried round and round on a merry-go-round, the needle will continue to point in the same direction and thus remain parallel to its original position. That is, the needle is moving with pure translatory motion.

But the merry-go-round itself is a good example of pure rotatory motion. This may be defined as that type of motion where the center of gravity remains at rest, but a line drawn at random in the body moves so that it makes continually changing angles with its original position. The phrase "at random" was used in the preceding sentence, because one line (of the infinitude of lines that could be drawn) is in the axis of rotation; this line does not rotate nor does any line in the body parallel to it. Another example of pure rotation is the motion of a flywheel on a stationary engine. On the other hand, in the case of a closing door, the motion is a combination of translation and rotation, because the center of gravity of the door moves, likewise lines drawn at random on the door are making continuously changing angles with their previous positions.

8-2. Causes of Motion. If a single force is applied to a body in line with its center of gravity, this force will produce pure translation; if on the other hand, the force is not in line with the center of gravity, the result will be a combination of translation and rotation. If the center of gravity is held at rest by one force and an equal and opposite force is made to act somewhere else on the body, the result will be pure rotation. As an example of the last case, consider a small emery wheel at rest. It can be rotated by exerting a force tangent to the circumference. Suppose this force to be two ounces and suppose the radius of the wheel to be three inches, it is then customary to multiply the two ounces by the three inches and announce that a torque of six inch-ounces acts on the wheel. The product of a force by a distance from the axis perpendicular to the force is called a *torque*. Forces produce translation and torques produce rotation. But forces and torques may be balanced, in which case we have equilibrium.

8-3. Moment of Force. If a boy weighing 50 pounds were to balance another boy, who weighs 100 pounds, on a seesaw, it is common knowledge that the lighter boy would have to sit farther from the pivot than the heavier; in this case just twice as far. If in our illustration we take these distances to be six feet and three feet respectively, then we could form a proportion as follows: 50 pounds is to 100 pounds as three feet is to six feet. The simplest way of writing a proportion is to put it in the form of an equation of two fractions. For instance, A is to B as C is to D is usually written $A/B = C/D$. This is also equivalent algebraically to $(A)\ (D) = (B)\ (C)$. That is, the product of the extremes equals the product of the means. So in our numerical illustration we can say

$$50 \text{ lb.}/100 \text{ lb.} = 3 \text{ ft.}/6 \text{ ft.}$$

or

$$(50 \text{ lb.})\ (6 \text{ ft.}) = (100 \text{ lb.})\ (3 \text{ ft.})$$

Carrying out the multiplication, we find that 300 pound-feet equals 300 pound-feet. We have again come upon a torque. This physical quantity expressed here in pound-feet is also called *moment of force*. But since the expression is a bit lengthy it has become customary for engineers to shorten it to *torque;* we shall use both expressions. A *moment of force* is the product of a force and a distance measured perpendicularly from some given pivot called an *axis*. It must be emphasized that the distance is perpendicular to the force. A torque is either *clockwise* or *counterclockwise*. In the illustration given in figure 8-1, (50 lb.) (6 ft.) is counterclockwise because if it were the only torque in the diagram, it would cause a counterclockwise rotation about the point Q.

8-4. Equilibrium. A body is in *equilibrium* when it is at rest or when it is moving with uniform speed in a constant direction. In this book most of our cases of equilibrium will also be cases of rest. If a body is in equilibrium, (1) the sum of the components of the forces acting on the body in any given direction will just balance the sum of the components in the opposite direction, and (2) the clockwise torques will just balance the counterclockwise torques. For instance, in figure 8-1, we shall not have equilibrium unless the down-

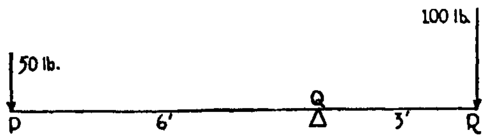


Figure 8-1.

ward forces at P and R are balanced by an upward force at Q, and this upward force must be 150 pounds. (See figure 8-2). It is important in an equilibrium problem to make sure that all the forces act *on* the body under consideration. A mixture of forces exerted *by* the body and forces exerted *on* the body will lead to incorrect results.

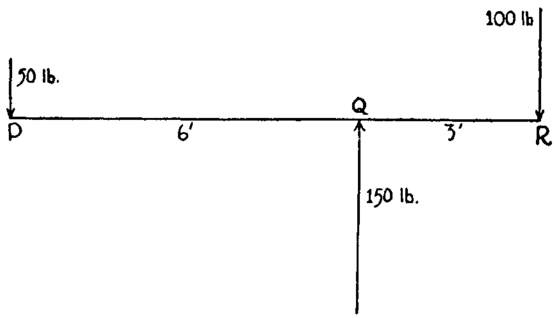


Figure 8-2.

If the body is in equilibrium, we are not limited in our choice of an axis; any point may be selected. Q is simply the most natural point to consider as the axis. Let us see if figure 8-2 will still represent equilibrium if we select R as the axis: it should. When R is the axis, the 100-pound force produces no torque because the perpendicular distance from R to the 100-pound force is zero; in other words, if the force acts directly on the axis, it will tend to produce no rotation about that point. We do, however, have two other torques: (50 lb.) (9 ft.) counterclockwise and (150 lb.) (3 ft.) clockwise, each of which is numerically 450 pound-feet. Hence, we still have equilibrium.

8-5. Rules for Solving an Equilibrium Problem. The engineer is often faced with the problem of the magnitude, direction, and point of application of the force that must be added to those already present to produce equilibrium. And he has found that in the solution of this type of problem it is convenient to follow a set of rules, as follows:

(1) Draw a diagram of the situation, putting in all of the forces in their proper directions, labeling the known forces with numbers and the unknown forces with letters. *All the forces must be applied to the same object.*

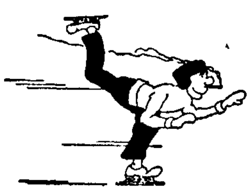
(2) Choose a convenient direction and a convenient axis.

(3) Resolve all the forces into components that shall be either parallel or perpendicular to the direction chosen, and henceforth use these components instead of the original forces.

(4) Write three equations. The first equates the components of forces in the given direction to the components in the opposite direction. The second deals similarly with the components perpendicular to the given direction. The third equation equates the clockwise moments of force to the counterclockwise moments about the selected axis.

(5) If there are no more than three unknowns, they may be found by solving simultaneously these three equations. If there are more than three unknowns, then other relations between the forces must be supplied, furnishing more equations.

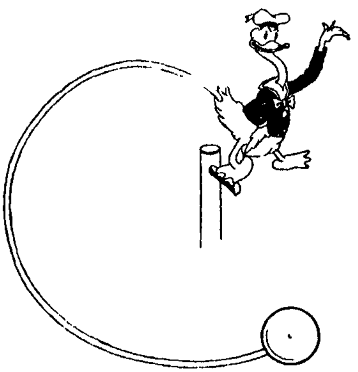
8-6. Center of Gravity. We have made several references to the existence of a center of gravity in a given body; it is now time to show how to locate it and to demonstrate its use. If we can suspend



the entire body by a single wire, the center of gravity will lie somewhere along the line which contains the wire. If we try the experiment again with the wire in a different place, we shall have two intersecting straight lines, and their intersection will be the desired center of gravity. While the body is suspended by the single wire, it is in equilibrium. The diagram representing this equilibrium contains just two equal and opposite forces:

one of these is the upward force exerted by the wire on the body, and the other is the downward pull of gravity on the body. In order to produce no torques, the forces must lie in the same line.

We draw the conclusion then that the effect of gravity, which can be considered to be a large number of small forces, one for each infinitesimal portion of the object, may also just as well be represented by a single arrow called the *weight* and drawn downward from a single point called the *center of gravity*. The position of the center of gravity then is defined by the fact that the sum of the clockwise torques due to the weights of each infinitesimal portion of the body about this center is equal to the corresponding sum of the counterclockwise torques. The center of gravity of a uniform symmetrical body is at its geometrical center. The intersection of the medians is the center of gravity of a triangle. If the body is made of parts, the center of gravity of each of which is known, the center of gravity of the whole may be found by solving an equilibrium problem, making use of the fact that when the body is supported at its center of gravity, it is in equilibrium. The center of gravity of a body like a doughnut is not in the material of the body at all, but in the hole.



The use to be made of the center of gravity concept is that the moment we know the weight of a body, we need draw but one arrow on our diagram to represent it—vertically down from the center of gravity—the length of the arrow being proportional to the given weight.

8-7. Problem Illustrating Equilibrium. A 50-foot 100-pound ladder, the center of gravity of which is at a point one third of its length from the bottom, stands with its base 30 feet from the foot of a perfectly smooth wall. A 200-pound man is two thirds of the way up the ladder. Find the forces exerted on the wall and the ground by the ladder, assuming equilibrium.

Following the rules given in section 8-5 (1) we draw a diagram (figure 8-3) in which the two known forces are drawn at the proper places and in the proper directions, and labeled respectively 100 pounds and 200 pounds. The unknown force exerted by the wall on the ladder must be exactly horizontal and toward the right, since a perfectly smooth wall is incapable of exerting any forces parallel to itself. This force is labeled with a letter, say F, and by Newton's third law, is equal and opposite to the desired force exerted by the ladder on the wall. Let us emphasize again that every force in the diagram must act *on the ladder*, so that it would be incorrect to replace force F by the force acting in the other direction, exerted *by* the ladder on the wall. The force exerted by the ground on the ladder slants up and to

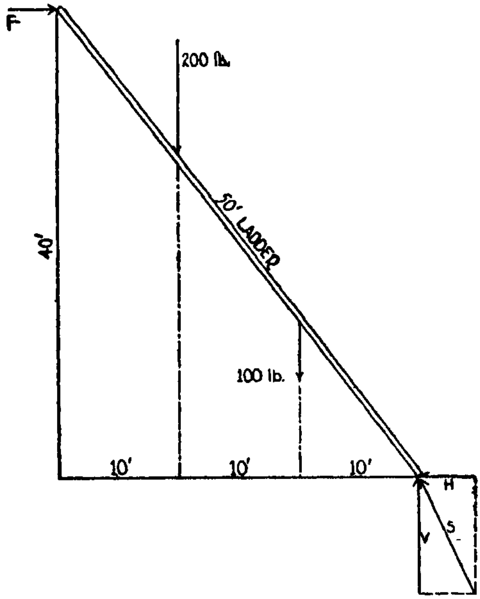


Figure 8-3.

the left, and is also, by Newton's third law, equal and opposite to the desired force exerted by the ladder on the ground. It has one component that prevents the ladder from sliding along the ground and one component that prevents the ladder from sinking into the ground. The slant force we shall call S. In accordance with rule (2), we choose the vertical direction as convenient because our two known forces are already in that direction, and the bottom of the ladder as a convenient axis because an unknown force acts there. Any forces that act at the axis will produce no torque, and it is advantageous to prevent the appearance of unknown forces in the torque equation. For this reason it is also common practice to choose as axis a point where at least two forces act, and if one or more of them are unknown, so much the better. (3) The only force that is not already either parallel or perpendicular to the direction chosen is the slant force S. So we resolve this into a vertical component V, and a horizontal component H. It will be noticed that all three of these forces S, V, and H are unknown. (4) We now write our three equations. The slant force S will not appear in these three equations; it is replaced by its two components V and H.

$$V = 100 + 200 \tag{a}$$

$$F = H \tag{b}$$

$$40\ F = (200)\ (20) + (100)\ (10) \tag{c}$$

Forty feet is the vertical distance of the upper end of the ladder above the ground, corresponding to side DE of figure 7-6, that is $40^2 = 50^2 - 30^2$. It is necessary to use the vertical distance here because the force F is horizontal, and by the definition of a moment of force (section 8-3), the distance and the force must be perpendicular to each other. (40) (F) is the only clockwise torque in the figure, the other two torques being counterclockwise. It will be noticed that in the case of all three of these torques, the procedure is first to draw a line containing the force, then to drop a perpendicular from the point selected as axis to the line containing the force. From equation (c), F is 125 pounds; from (a), V is 300 pounds; and from (b) H is 125 pounds. Knowing H and V, we can find from the Pythagorean theorem that S is 325 pounds (see figure 7-8). The answers to our problem are therefore 125 pounds and 325 pounds.

8-8. Problem Illustrating Center of Gravity. A certain bolt has a head measuring 1 by 1 by ½ inch that weighs a quarter ounce, and a shaft, nine inches long that weighs ten ounces. Compute the location of the center of gravity by finding just where a knife edge must be placed under the bolt so that the bolt will be in equilibrium.

Since the shaft is taken as having a uniform cross section, its center of gravity is 4.5 inches from the head of the bolt. Similarly the center of gravity of the head is at its geometrical center. We therefore draw arrows representing downward forces of 10.0 ounces and 2.25 ounces respectively at S and H in figure 8-4. To balance these two forces, there must be an upward force, equal to their sum and applied at the center of gravity of the bolt as a whole.

In choosing a suitable axis about which to compute our torques, it is possible to find arguments in favor of several positions. The natural axis is

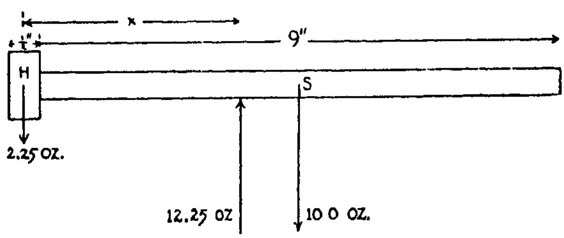


Figure 8-4.

the center of gravity, but since this is unknown, an unknown quantity will enter into each of the torques and make the equation unnecessarily complicated. If we take the right-hand end of the shaft as our axis, there will be three torques in the equation but there will be the advantage that when we solve for the unknown distance between the center of gravity and the end of the shaft, the result will need no further interpretation. If we take either point S or point H as an axis, there will be only two torques in the equation, but the result will have to be interpreted. We shall solve the problem with H as axis and leave it to the student to try some other point.

Let the distance from H to the center of gravity be called x. The moment of 2.25 ounces about H is zero. The moment of 12.25 ounces about

H is $12.25x$, and is counterclockwise. The moment of 10.0 ounces about H is (10.0) (4.75) and is clockwise. The equation is

$$12.25x = (10.0)\ (4.75)$$

Solving for x, we obtain $x = 3.88$ inches. But instead of announcing that the center of gravity is 3.88 inches from the center of the head, it will be much more convenient to say that it is $3.88 - 0.25$ or 3.63 inches from the head, or $9.00 - 3.63$ or 5.37 inches from the end of the shaft.

SUMMARY OF CHAPTER 8

Technical Terms Defined

Pure Translatory Motion. Motion such that any line drawn on the body remains parallel to its original position during the motion.

Pure Rotatory Motion. Only one line in the body, the axis of rotation, remains fixed. This line must contain the center of gravity. During the motion any line not parallel to this axis moves so as to make continually changing angles with its original position.

Moment of Force. Product of a force and a distance measured perpendicularly from the axis to the force. In our two-dimensional problems, torques will be either clockwise or counterclockwise.

Equilibrium. A situation such that the body is either at rest or moving with uniform speed in a constant direction.

Conditions for Equilibrium. (1) The sum of the upward forces equals the sum of the downward forces.

(2) The sum of the forces to the right equals the sum of the forces to the left.

(3) The sum of the clockwise torques equals the sum of the counterclockwise torques about a given axis.

Center of Gravity. A point in the body about which the gravitational torques are in equilibrium. In solving an equilibrium problem, the entire weight of the body may be considered as concentrated at this point.

PROBLEMS

8-1. Compute the three torques in figure 8-2 about a point two feet to the right of R. Are they in equilibrium?

8-2. In figure 8-1, consider the seesaw to consist of a uniform nine-foot plank weighing 60 pounds. Where should the 50-pound boy be placed if the pivot is to remain in the same place?

8-3. Compute the center of gravity of the bolt in figure 8-4 by using the right-hand end of the bolt as axis.

8-4. Find the center of gravity of a croquet mallet, considering it as consisting of two cylinders of the same material, the head 2.25 inches in diameter and 7.00 inches long, and the handle 0.875 inch in diameter and 26.0 inches long.

8-5. A 50-foot 200-pound ladder leans against a vertical wall making an angle of 30 degrees with the wall. The wall exerts an upward force of eight pounds on the ladder together with an unknown horizontal force. The center of gravity of the ladder is half-way up; also at the half-way point, a 100-pound boy stands. If the force of friction between the ladder and the ground is 30 pounds, find the additional horizontal force at the base necessary to prevent the ladder from slipping.

8-6. The load on a wheel 26 inches in diameter is 500 pounds. What horizontal force, applied at the axle, will be necessary to pull the wheel over a stone one inch high?

8-7. A gate weighs 25 pounds, has its center of gravity at its geometrical center, and is four feet square. Its hinges are three feet apart. If a 50-pound boy is swinging on the outer corner of the gate, find the horizontal component of the force on the upper hinge.

8-8. A 30-foot ladder leans against a smooth vertical wall making an angle of 30 degrees with the wall. A 200-pound man stands two thirds of the way up the ladder. The ladder weighs 100 pounds and has its center of gravity at the geometrical center. Compute (1) the horizontal force of the ladder on the wall, (2) the vertical component of the force which the ladder exerts on the ground, and (3) the necessary force of friction at the ground to prevent slipping.

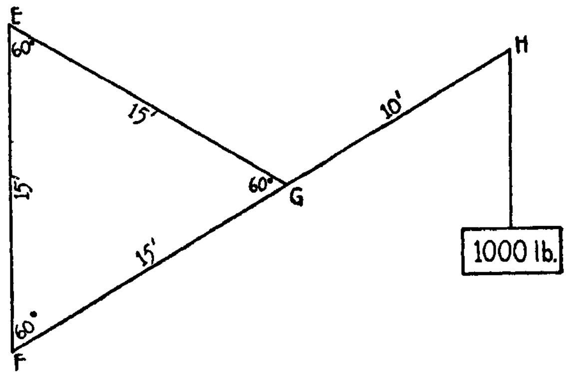
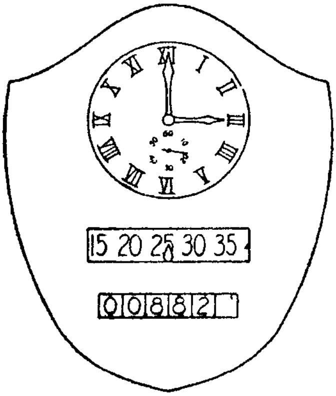


Figure 8-5.

8-9. A uniform 25-foot beam *FH* (see figure 8-5) is fastened to the wall *EF* at *F*. *EG* is a wire. *EFG* is an equilateral triangle 15 feet on a side. *FH* weighs 200 pounds. Find the tension in the wire *EG*, and the force exerted by the wall on *FH*.

8-10. How far may a 200-pound man climb a 100-pound, 26-foot ladder (center of gravity at the geometrical center) if the ladder stands with its base 10 feet from a vertical wall, the coefficient of friction between the ladder and the floor being 0.21?

CHAPTER 9



Acceleration

9-1. More General Conditions. Up to this time we have confined our attention either to cases of rest or of uniform motion in a straight line. Under either of these circumstances, we say that we have equilibrium. Now we must enlarge our discussion to include the numerous cases where translatory equilibrium is lacking; later we must see what happens when there is no rotatory equilibrium and then discuss the general case when we have neither.

9-2. Acceleration. Speed, it will be remembered, is the rate of change of position; its unit is feet per second, miles per hour, and so on. Velocity adds to speed the concept of direction, and therefore is a vector quantity. When there is a change in velocity, either because the direction or the speed changes, we have "accelerated motion." In order to visualize the physical situation, imagine yourself to be sitting in the front seat of an automobile, holding a watch, and looking at the speedometer. When the second hand of the watch points to 60, the speedometer reads, let us say, 30 miles per hour. Five seconds later the speedometer reads 45 miles per hour. The acceleration can be computed in this case by dividing the gain in speed of 15 miles per hour by the five seconds, and is therefore numerically three miles per hour per second. *Acceleration* is defined as the velocity gained per unit of time. In order to find the acceleration, the rule is to subtract the original speed from the final speed, and divide the difference by the time required to change the speed. The speed gained in one second (three miles per hour) may be expressed in other units. Three miles is the same as 15,840 feet; there are 3,600 seconds in an hour. (15,840 feet)/(3,600 seconds) is 4.40 feet per second. Therefore the above acceleration of three miles per hour per second may also be expressed as 4.40 feet per second per second. Since we have seconds in the denominator twice, it is also quite customary to express it as 4.40 feet per second squared, or 4.40 feet/second^2. The important thing to notice in all of these expressions is that units of time occur as factors in the denominator *twice*. This is the important difference between an acceleration and a velocity, where the unit of time occurs in the denominator but once.

9-3. Uniform Acceleration. With two exceptions, we shall in this text confine our attention to cases of acceleration where the gain in speed is uniform; otherwise the mathematics becomes complicated, and we need the calculus. The motion of a freely falling body and the motion of a body sliding down an inclined plane may be taken as illustrations of practically uniform acceleration. On the

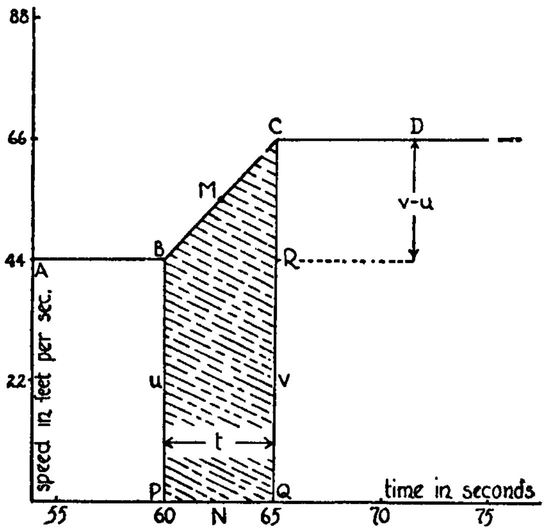


Figure 9-1.

other hand we must admit that in reality there is no such thing as absolutely uniform acceleration. Air resistance complicates the motion of a freely falling body so that it is really not uniformly accelerated, and even if we went to the trouble of constructing a perfect vacuum (which we cannot do) the fact that gravity varies inversely with the square of the distance from the center of the earth would make the acceleration increase slightly as the object fell. But we shall not worry about such refinements. Throughout a course in physics, the student will notice that many simplifying assumptions are made when a new idea is being introduced, such as weightless levers, frictionless planes, and so on. As the student advances into the subject, these simplifying assumptions are one by one removed. We can now deal with real levers which have weight, and when the coefficient of friction is given, the planes no longer need be perfectly smooth. After a study of calculus, it becomes possible to deal with variable accelerations. Figures 9-1 and 9-2 exemplify the distinction between uniform and variable acceleration; both figures may be considered as portraying graphically the illustration given in the preceding section. In figure 9-1 at point A, the speed is 30 miles per hour (or 44 feet per second) and the second hand is pointing to 55. At point B the speed is still 44 feet per second and the second hand now points to 60. Five seconds later the speed is 66 feet per second (45 miles per hour) and remains at that value for the rest of the time. Exactly the same remarks may be made about

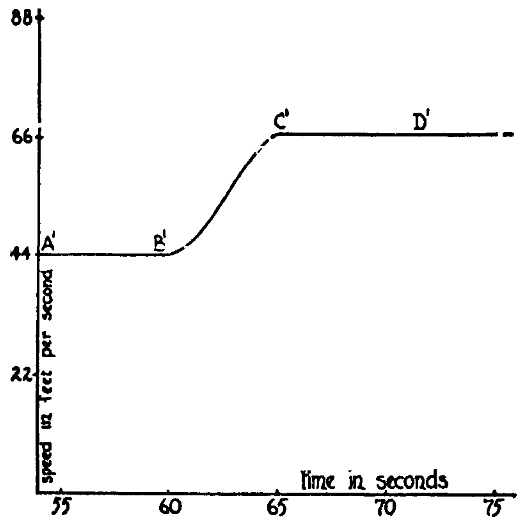


Figure 9-2.

points A', B', C', and so on of figure 9-2. The difference is that in figure 9-1 the speed increases *uniformly* from B to C as is shown by the straight line, while in figure 9-2 the increase is smoother but no longer uniform, and the line between B' and C' is no longer straight. However, even in figure 9-2 we may still say that the *average* rate of increase of speed is three miles per hour per second, or 4.40 feet/second^2. Accordingly, in our problems, when it is obvious that the acceleration is in fact far from uniform, we shall talk about the average acceleration and proceed as if the acceleration were quite uniform.

9-4. The Two Fundamental Equations. Limiting ourselves then to uniform acceleration, we shall never find more than five quantities involved in a single case of accelerated motion, namely: initial velocity, u; final velocity, v; the time necessary to change from one speed to the other, t; the space (one-dimensional) covered during the motion, s; and the acceleration itself, a. We have already discovered that in order to compute a, it is necessary only to subtract u from v and divide by t, assuming that v and u are expressed in the same units. Thus

$$a = \frac{v - u}{t} \tag{a}$$

or

$$v - u = at$$

The average of two quantities may be found by adding them together and dividing by two. Thus, in the case of the motion from B to C in figure 9-1, v is 66 feet per second and u is 44 feet per second. The average of 66 and 44 is $(66 + 44)/2$ or 55 feet per second. Average speed must not be confused with average acceleration. If we know the average speed of a body and the time the body is in motion, we can compute the distance covered by the body by multiplying the two. For instance, if for five seconds a body moves at the average rate of 55 feet per second, it will during that time cover (5) (55) feet or 275 feet. Expressed in terms of letters, this relationship is

$$s = t\,\frac{v + u}{2}$$

or

$$v + u = \frac{2s}{t} \tag{b}$$

9-5. Graphical Representation. The shaded area $BCQP$ in figure 9-1 represents the distance covered, because PB represents u; QC represents v; since $BCQP$ is a trapezoid, its area is the product of PQ (which is t) and the average of PB and QC (represented on the diagram by NM). Therefore $s = (PQ)(NM)$ or $(BR)(NM)$. In a similar fashion the acceleration, a, is $(CR)/(BR)$, because CR is $v - u$, and BR is the time, t. It is possible to draw the diagram for any problem in either uniformly or nonuniformly accelerated motion, and therefore to solve the problem graphically.

9-6. Derived Equations. The theory of algebraic equations tells us that if out of five quantities (a, s, t, u, and v), three of them are known, that is, *two* of them are unknown, then *two* equations such as (a) and (b) are sufficient to determine the unknowns. This is easy if the knowns happen to be u, v, and t, and the unknowns a and s; in this case the first form of each equation gives the quantity sought. But in the cases where the unknowns include any two of the three quantities u, v, and t, it will be necessary to solve the equations (a) and (b) simultaneously for the two unknowns. As an aid to the student, we shall now do this once for all, and in this way derive from equations (a) and (b) three more equations enabling us in the problems to avoid the solution of simultaneous equations.

In the first place, if we multiply the second forms of equations (a) and (b) by each other, we shall obtain

$$(v - u)\,(v + u) = (at)\left(\frac{2s}{t}\right) \qquad \text{(c)}$$

or

$$v^2 - u^2 = 2as$$

If, now, in (c) we substitute for v^2 the value $(u + at)^2$ or $u^2 + 2uat + a^2t^2$ from (a) we obtain

$$u^2 + 2uat + a^2t^2 - u^2 = 2as$$

or

$$s = ut + \tfrac{1}{2}\,at^2 \qquad \text{(d)}$$

Finally, in a similar manner we could replace u^2 in (c) by its equivalent $(v - at)^2$ or $v^2 - 2vat + a^2t^2$ from (a) and obtain

$$v^2 - (v^2 - 2vat + a^2t^2) = 2as$$

or

$$s = vt - \tfrac{1}{2}\,at^2 \qquad \text{(e)}$$

9-7. Summary of Equations. It will be noticed that each one of our five equations contains only four of the five variables and therefore omits one variable. It will therefore be more convenient to describe the equations in terms of the variable omitted than in terms of the variables contained. Let us now summarize the equations thus far derived in this chapter.

Variable omitted	Equation	No.
s	$v = u + at$	(a)
a	$s = t\,\dfrac{u + v}{2}$	(b)
t	$v^2 = u^2 + 2as$	(c)
v	$s = ut + \frac{1}{2}\,at^2$	(d)
u	$s = vt - \frac{1}{2}\,at^2$	(e)

It will be found that there is never any need of using equation (e); it may therefore be discarded at this point. The first four equations should, however, be memorized, unless it be preferred to solve acceleration problems directly from a consideration of figure 9-1 as indicated, and handle the simultaneous equations that arise thereby. The use of the first four equations will be illustrated presently. In the solution of acceleration problems, it is important to settle on a positive direction at the outset and remember that the opposite direction is negative. Negative time, however, denotes time measured backward from the beginning of the problem, and is usually unimportant.

9-8. The Acceleration of Gravity. When a body falls freely vertically as a result of gravity, its speed increases nearly uniformly each second, and we refer to this acceleration as the acceleration of gravity. The acceleration of gravity is represented by g and is equal numerically to 9.80 m./sec.^{2} or 32.2 ft./sec.^{2} This means, for example, that if at a certain instant during the fall a speedometer attached to the falling body read 100 feet per second, then just one second later the speedometer would read 132.2 feet per second. It is a fact discovered by Galileo about the year 1600 that all bodies, whether heavy or light, accelerate at about the same rate when dropped. His celebrated demonstration of this fact took place at the leaning tower of Pisa, and disproved notions which had been held by physicists for over nineteen centuries.

9-9. Hints Concerning the Solution of Problems Involving Uniform Acceleration. The five equations of section 9-7 are arranged in the order of their difficulty. The first contains two one-degree terms and one second-degree term, but none of the variables occur to the second power; the second equation is only slightly more complicated; the third equation contains the second powers, but no variable occurs more than once; the fourth and fifth equations are affected quadratics in t. In a simple problem involving only one object, there will be five variables, but no more than two of them will be unknown. Pick the two equations that omit the two unknown variables. Start with the equation nearest the top of the list. When this equation is solved, we shall then have four known variables and only one unknown; this can always be obtained either from equation (a) or (b).

There are ten possible combinations of five things taken two at a time, therefore there are ten possible types of acceleration problems. These combinations of unknowns are as follows: a, s;

s, t; s, u; s, v; a, t; a, u; a, v; t, u; t, v; and u, v. In the case of the first four, equation (a) may be used to solve for one variable and then equation (b) for the other. In the case of the next three, equation (b) may be used to solve for one variable and then equation (a) for the other. In the eighth and ninth cases, equation (c) may be used to solve for one variable and then equation (a) for the other. In the tenth case, equation (d) may be used to solve for u and then equation (a) for v. It will not be necessary to solve (d) as an affected quadratic, and it will not be necessary to use equation (e) at all.

9-10. Illustrative Problems. Several acceleration problems will now be worked as illustrations.

(A) If a stone is dropped from the top of a precipice 500 feet high, how long a time will elapse before it strikes the bottom and what will be its speed just before it lands?

It usually helps, when a problem is to be solved by algebraic methods, to make a table of the knowns and the unknowns, assigning letters to each. In this case

$$\begin{aligned} a &= 32.2 \text{ feet/second}^2 \\ s &= 500 \text{ feet} \\ t &= ? \\ u &= 0 \\ v &= ? \end{aligned}$$

When a body falls freely, the acceleration is that of gravity and may therefore be assumed to be known numerically; the only question is whether it shall be considered positive or negative. It is immaterial which choice shall be made, but once the choice is made, it settles the question of sign for the other variables. For example, in this problem, the moment that we assume that the acceleration of gravity is positive, everything else in the problem that is downward also becomes positive, and if there happened to be any upward distances or velocities in the problem, they would automatically become negative.

Since the unknowns are t and v, the problem belongs to the ninth case of section 9-9. We have our choice of using equation (c) which does not contain t and solving for v, or using equation (d) which does not contain v and solving for t. Since u is zero in this problem, the choice is of little importance; if u were not zero, it would be much easier to solve (c) for v than to solve (d) for t. We shall choose the former method. Substituting in equation (c) we obtain

$$2\,(32.2)\,(500) = v^2 - 0^2$$

Therefore $v^2 = 32{,}200$, and $v = 179.4$ feet/second. Any equation that contains t may now be used; equation (a) is the simplest.

$$179.4 = 0 + 32.2t$$

Solving for t gives us $t = 5.57$ seconds.

(B) If a stone is projected vertically upward from the top of a 500-foot precipice with a velocity of 50 feet per second, (a) how long will it take to reach the highest point in its path and how far above the top of the precipice will that be; (b) how long will it take to reach the foot of the precipice and what will its velocity then be?

(a) Tabulating the data, this time letting the upward direction be positive, we have

$$a = -32.2 \text{ feet/second}^2$$
$$s = ?$$
$$t = ?$$
$$u = 50 \text{ feet/second}$$
$$v = 0$$

This time the unknowns are s and t, so that the problem belongs to the second case of section 9-9. Therefore we first substitute into equation (a) obtaining

$$0 = 50 + (-32.2)\, t$$

Solving this equation for t gives $t =$ 1.553 seconds. Next we substitute into equation (b)

$$s = (1.553)\frac{(50 + 0)}{2}$$

or, $s = 38.8$ feet.

(b) We are now going to let the stone drop 538.8 feet to the bottom of the precipice and solve for the time and final velocity. In accordance with our practice of retaining only three significant figures, we shall round off the distance to 539 feet. The data, letting the downward direction be positive, are

$$a = 32.2 \text{ feet/second}^2$$
$$s = 539 \text{ feet}$$
$$t = ?$$
$$u = 0$$
$$v = ?$$

Since we have the same set of unknowns here as in problem (A) of this section, we shall merely record the results of substituting in equations (c) and (a). $v = 186.3$ feet/second; $t = 5.79$ seconds. Adding the time necessary to rise from the top of the precipice to the highest point, to that necessary to drop to the bottom gives us $1.55 + 5.79 = 7.34$ seconds.

(C) Now that we have all the data concerning the trip of the stone from the top of the precipice upward and then downward to the bottom, let us tabulate them, then, as a check, assume the initial and final velocities unknown, thus obtaining a problem belonging to the tenth case of section 9-9. Our problem will be stated as follows: A stone leaves a point 500 feet from the base of a precipice vertically and 7.34 seconds later lands at the bottom. Was the stone projected upward or downward, what was its initial speed, and what was the final speed just before landing?

The quantity s is not the total number of feet covered by the stone, but is the distance between initial and final positions, and is therefore 500 feet in this case, assuming that we take the downward direction as positive. Tabulating the data gives us

$$\begin{aligned} a &= 32.2 \text{ feet/second}^2 \\ s &= 500 \text{ feet} \\ t &= 7.34 \text{ seconds} \\ u &= ? \\ v &= ? \end{aligned}$$

The result of substituting into equation (d) is

$$\begin{aligned} 500 &= (u)\ (7.34) + \tfrac{1}{2}\ (32.2)\ (7.34)^2 \\ 500 &= 7.34u + 867 \end{aligned}$$

Solving for u gives us $u = -50$ feet/second. We must interpret the minus sign as upward since we chose the downward direction as positive. Substituting now in equation (a) gives us

$$v = -50 + (32.2)\ (7.34)$$

or $v = 186.3$ feet/second. This velocity is positive and therefore downward. And the results check those of problem (B) of this section.

SUMMARY OF CHAPTER 9

Technical Terms Defined

Acceleration. Rate of change of velocity, that is, the velocity gained per unit time.

Uniform Acceleration. The type of acceleration that would appear as a straight line on a velocity-time graph.

Average Acceleration. A fictitious uniform acceleration which could replace an actual acceleration and involve the same initial and final velocities in the same time interval.

Equilibrium In Terms of Acceleration. Translatory equilibrium may be defined as a case in which the linear acceleration is zero.

Acceleration Equations.

$$\begin{aligned} &\text{(a) } v = u + at \\ &\text{(b) } s = \frac{t}{2}(u + v) \\ &\text{(c) } v^2 = u^2 + 2as \\ &\text{(d) } s = ut + \tfrac{1}{2}\, at^2 \end{aligned}$$

where u = initial velocity, v = final velocity, s = distance between initial and final positions, t = time between initial and final positions, and a = acceleration.

PROBLEMS.

9-1. Galileo dropped a light object and a heavy object simultaneously from the top of the leaning tower of Pisa. Both fell with the same acceleration, 32.2 feet/ second^2. How long did it take for the objects to reach the ground, 180 feet below?

9-2. An automobile travels a distance of 100 feet while slowing down from a speed of 40 miles per hour to a speed of 25 miles per hour. Find the time it took to slow down, also the acceleration.

9-3. A block slides down an inclined plane with an acceleration of 16 feet/second.^{2} How far does it go during the third second from rest?

9-4. A block is sliding down an inclined plane with an acceleration of 400 centimeters per second squared. Find the initial and final speeds if it covers a distance of 2.5 meters in one second.

9-5. A ball is thrown from a third-story window to the ground 24 feet below. If it takes two seconds to arrive at the ground, compute the initial velocity, giving both magnitude and direction.

9-6. If a stone is projected vertically downward from the top of a 500-foot precipice with a velocity of 50 feet per second, how long will it take to reach the foot of the precipice and what will its velocity then be? Compare the answers of this problem and problem (B) of section 9-10 and explain.

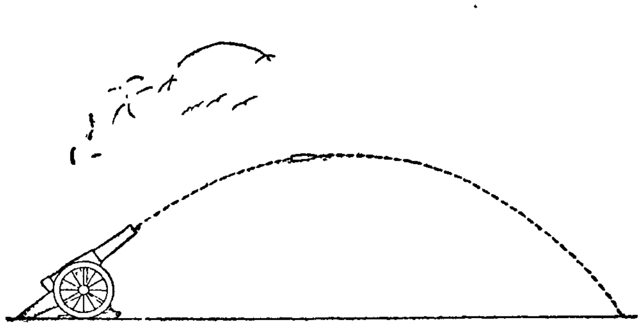
9-7. A ball is thrown upward with an initial velocity of 64.4 feet/second from a point 80.5 feet above the ground. Find (1) the time that elapses before it reaches the ground; (2) the velocity it then has; (3) the maximum height reached above the ground; (4) the time required to reach this height. (5) Where will it be at the end of three seconds?

9-8. A ball drops 16.1 feet and keeps rebounding in such a way that on each rebound it rises one per cent of the distance that it has just fallen. Show that it will bounce an infinite number of times and come completely to rest in just one and $\frac{2}{9}$ seconds.

9-9. The engineer of a passenger train which is going at the rate of 80 feet per second, sees a freight train 1,000 feet ahead traveling in the same direction at the constant rate of 10 feet per second on the same track. He applies the brakes which produce a deceleration of 2.4 feet/second.^{2} Will there be a collision, and if so, when?

9-10. Express 32.2 feet/second^2 in (1) centimeters/second^2, (2) miles/hour^2, (3) miles per hour per second, and (4) miles per second per hour.

CHAPTER 10



Projectiles; Centripetal Acceleration

10-1. Velocities and Accelerations Are Vector Quantities. Technically it is permissible to speak of a *speed*, not a velocity, of twenty miles per hour. A *velocity* of twenty miles per hour must be spoken of as proceeding in some definite direction, such as due north. We could speak of two opposite velocities, but it would be meaningless to speak of two opposite speeds. We can find the resultant of two velocities just as we can with any vectors, and we can resolve a given velocity into two components. An acceleration is also a vector quantity and has the same direction as the *change* in velocity which gives rise to it. When we speak of the acceleration of gravity, we should at the same time describe its direction as being vertically downward.

10-2. Projectiles. The problems of the previous chapter involved accelerations in the same line with the velocities; for example, vertical velocities and vertical accelerations. In practice, however, these cases are comparatively rare. The path of a baseball or projectile from a gun is almost never confined to a vertical direction. Nevertheless, any velocity may be resolved into two components, one of which is exactly vertical and the other horizontal, and the methods of the previous chapter may then be applied to the vertical and horizontal components separately. We call problems of this type "projectile problems."

10-3. A Simple Projectile Problem. Let us now consider a type of motion such as would be experienced by a bag of sand dropped from a dirigible which is flying eastward at the rate of 80 feet per second.

We shall as usual neglect the effect of air resistance. Since the effect of gravity is vertically downward, there will be nothing either to increase or decrease the horizontal component of the subsequent velocity of the bag of sand. As just indicated, all problems of this type may be separated into

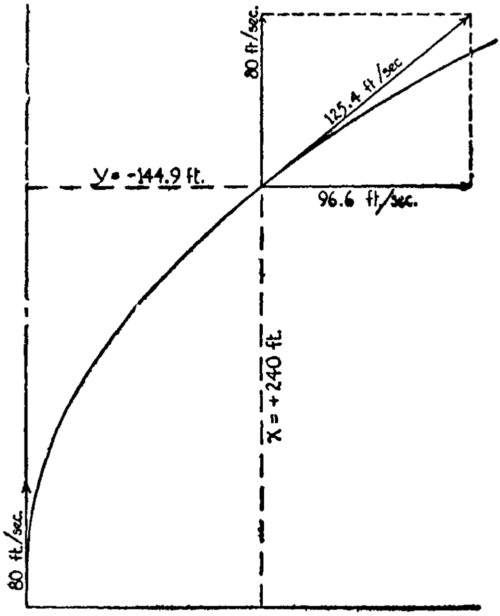


Figure 10-1.

two parts, one dealing with the vertical motion and the other dealing with the horizontal motion. Let the dirigible and the bag of sand be considered as being at the origin 0 of a set of coordinates (see figure 10-1) at the instant that the bag is dropped, where the X-axis is horizontally eastward and the Y-axis is vertically upward. Let the problem be to find the value of x and y after three seconds, also to find the speed of the bag of sand at that time. The bag may be considered as doing two things at once. It is a freely falling body as far as the vertical part of its motion is concerned, and as far as the horizontal part of its motion is concerned, it is drifting eastward at the rate of 80 feet per second, and in fact remains directly under the dirigible. Let us discuss the vertical part of the problem first. Since the bag is merely dropped and not thrown down, the initial velocity, u, is zero. $a = -32.2$ feet/second^2, $t = 3$ seconds. We want v and s (which we shall call y) in this problem. To find v, use the equation that does not contain s (equation (a), section 9-7). $v = 0 + (-32.2)(3)$. Therefore $v = -96.6$ ft./sec. s may be found from equation (b). $y = 3(-96.6 + 0)/2$. Therefore $y = -144.9$ feet. Now solve the horizontal part of the problem. u and v are both 80 ft./sec., $a = 0$, and $t = 3$. s will be called x. Either equation (b) or (d) will give us $x = 240$ feet. The resultant of a velocity vertically downward of 96.6 ft./sec. and a velocity horizontally eastward of 80 ft./sec. is found by the Pythagorean theorem to be 125.4 feet per second.

10-4. A More General Projectile Problem. A projectile is shot at an angle of 30 degrees above the horizontal with a muzzle velocity of 2,000 feet per second. When and where will it again return to the same horizontal level?

Again draw a set of coordinate axes (figure 10-2) and let the projectile start from the origin 0. It is again necessary to split the problem into a vertical and a horizontal part (since there is a vertical but no horizontal acceleration); therefore we begin by resolving the initial velocity into two components. The horizontal component is 1,732 ft./sec. and the vertical component is 1,000 ft./sec. (see figure 7-5). Solve first the vertical problem. $u = +\ 1{,}000$ ft./sec.; $v = -\ 1{,}000$ ft./sec.; $a = -\ 32.2$ ft./sec.^{2}; $s = y = 0$. t is unknown. Using equation (d) of section 9-7, we have $0 = 1{,}000t + \frac{1}{2}(-\ 32.2)t^2$. This may be written $0 = t\ (1{,}000 - 16.1t)$. There are two

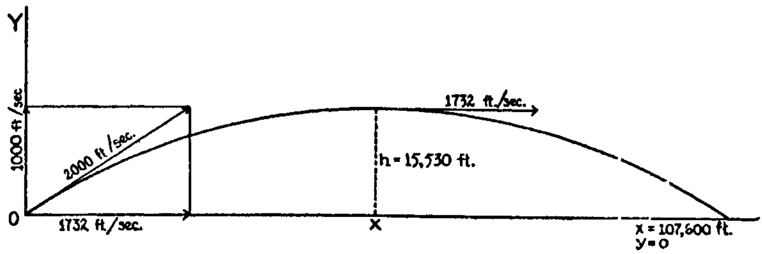
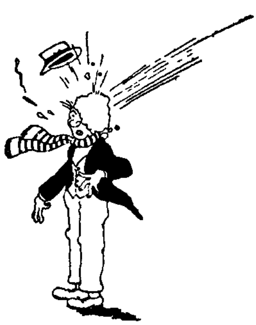


Figure 10-2.

solutions, $t = 0$, and $t = 1{,}000/16.1$ or $t = 62.1$ seconds. The first solution simply means that when $t = 0$, the projectile starts from the origin where $y = 0$. But it again returns to $y = 0$ when $t = 62.1$ seconds, which is one of the answers we seek.



This result could also be obtained by finding the time necessary to make half of the trip and then doubling it. The details would be as follows. For the vertical motion during the first half of the trip, $u = 1{,}000$ ft./sec., $v = 0$, and $a = 32.2$ ft./sec.^{2} From equation (a), section 9-7, we can find t. Substituting, we have $0 = 1{,}000 + (-\ 32.2)\ t$ and $t = 31.1$ seconds. Since this is the time for half the trip, the whole trip requires 62.2 seconds, which checks the previous work to the degree of precision to which we are working. In the horizontal problem, $u = v = 1{,}732$ ft./sec.; $a = 0$; $t = 62.2$ sec.; and $s\ (= x)$ is our unknown. Equation (d) gives us $x = (1{,}732)\ (62.2) + 0 = 107{,}600$ feet. The value of h in figure 10-2 may be found as follows. Again we are solving a vertical problem. $u =$

1,000 ft./sec.; $v = 0$; $a = -32.2$ ft./sec.² By this time we know that t is 31.1 seconds, but let us not make use of this information. This means that we shall use the equation that does not contain t, namely equation (c), section 9-7. This gives us $(2)(-32.2)(s) = 0^2 - 1{,}000^2$. Therefore $s = 1{,}000{,}000/64.4 = 15{,}530$ feet. This is the maximum height, h, attained by the projectile. The curve followed by the projectile in figure 10-2 is called a parabola.

10-5. Centripetal Acceleration. Thus far our accelerations have either been in the same direction as the velocity itself, or at least in the direction of some component of the velocity. But now we wish to discuss the case when the acceleration is always at right angles to the velocity and in which there is no component of the velocity in the direction of the acceleration. This is the case when the motion is in a circle; the acceleration is then called *radial*, or *central*, or *centripetal*. The *centripetal acceleration* does not change the component of the velocity in the direction of the motion, but creates a component at right angles, so that the resultant velocity steadily changes in direction but not in magnitude.

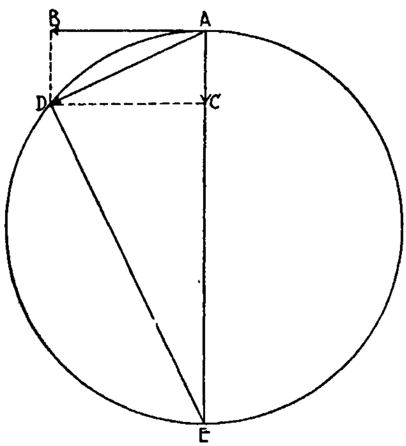


Figure 10-3.

Consider a body at A in figure 10-3 with a velocity in the direction of the vector AB. In time t, the body would travel a straight distance vt represented by the arrow AB, and leave the circumference of the circle. But if at each instant it were subject to an acceleration a toward the center of the circle, then in time t it would travel a distance $\frac{1}{2}at^2$ toward the center, according to equation (d) of section 9-7, since u in this direction is zero; this distance is represented in the figure by the arrow AC. The resultant of the displacements AB and AC is AD. The problem is to find the correct value of the acceleration a so that the point D shall lie on the circumference. Furthermore, the instant that A has moved to a different point on the circumference, such as D, a new diagram must be drawn with a new acceleration pointing from D toward the center. That is, the point D must be infinitely close to A, and the time t therefore must be infinitely small, so that the body shall *always* remain on the circumference of the circle.

In figure 10-3, $ABCD$ is a parallelogram, and ACD is a right triangle, since AB, and therefore DC, is perpendicular to the diameter, AE. (A tangent to a circle is perpendicular to the diameter through the point of tangency). Since the angle ADE is inscribed in a semicircle, it is also a right angle. Since the angles of triangle ADC and triangle DCE are equal, triangles ADC and DCE are similar and their sides are therefore proportional. It is therefore true that

$$\frac{AC}{CD} = \frac{CD}{CE}$$

$AC = \frac{1}{2}\, at^2$; $CD = AB = vt$; and $CE = 2r - \frac{1}{2}\, at^2$. Substituting these values into the proportion, we have

$$\frac{\frac{1}{2}\, at^2}{vt} = \frac{vt}{2r - \frac{1}{2}\, at^2}$$

Equating the product of the extremes to the product of the means gives us

$$rat^2 - \tfrac{1}{4}\, a^2\, t^4 = v^2\, t^2$$

Dividing through by t^2 simplifies the equation to

$$ra - \tfrac{1}{4}\, a^2\, t^2 = v^2$$

Remembering now that these relations hold only when t is infinitely small, we set t equal to zero and divide both sides of the equation by r and obtain

$$a = \frac{v^2}{r}$$

That is, the centripetal acceleration is always directed toward the center of the circle and has a magnitude found by dividing the square of the speed of the object in its circular path by the radius of the circle.

10-6. Problems Illustrating Centripetal Acceleration. (A) A locomotive is rounding a curve the radius of which is 500 feet, at a speed of 30 miles per hour. What is the centripetal acceleration?

We have seen that 30 miles per hour is the same as 44 feet per second. We therefore substitute $v = 44$ feet/second and $r = 500$ feet into the equation $a = v^2/r$ and obtain

$$a = \frac{44^2}{500}$$

Therefore the centripetal acceleration is 1,936/500 or 3.87 feet/second^2. If there is any doubt about the proper units in which to express the result of a series of algebraic operations, the best procedure is to put the units into the equation along with the numbers. In this case the numerator is (44 feet/second)^2 or 1,936 feet^2/second^2. Since we are dividing this numerator by the denominator consisting of 500 feet, the feet in the denominator partially cancel the feet^2 of the numerator, and the final unit is thus feet/second^2, as we should expect for a unit of acceleration in the English system. It is important for the stability of the locomotive that the centripetal acceleration be small compared with the acceleration of gravity, otherwise the track must be banked.

(B) An automobile goes over a slight convexity in the road at the rate of 60 miles per hour. What must be the radius of curvature of the hummock at its highest point so that gravity will just hold the car to the road?

According to the conditions of the problem the car is just about to leave the road for an instant and become a projectile. As a projectile it is subject to the acceleration of gravity, 32.2 feet/second^2 vertically downward; on the other hand, if it is barely to follow the curvature of the hummock in the road, the centripetal acceleration must be v^2/r. Therefore these two accelerations may be equated, and we may at the same time fill in the numerical value of the speed, which is 60 miles/hour or 88 feet/second. This gives us

$$32.2 = \frac{88^2}{r}$$

Solving for r, the radius of curvature of the hummock, we obtain $r = 88^2/32.2 = 7{,}740/32.2 = 240$ feet. It is doubtful if this would be called a hummock at all.

SUMMARY OF CHAPTER 10.

Since velocities are vector quantities, the velocity of a projectile may be resolved into vertical and horizontal components, thus splitting such a problem into two parts with nothing in common but the time element.

Since acclerations are likewise vector quantities, it is possible for the acceleration to be at right angles to the velocity.

Centripetal Acceleration. An acceleration toward the center of a circle accompanying a velocity tangent to the circumference. Its value is

$$\frac{v^2}{r}$$

PROBLEMS.

10-1. A projectile is discharged horizontally from a gun located on a hilltop, with a speed of 2,000 feet per second. Find the position of the projectile 10 seconds later.

10-2. In the preceding problem, find the horizontal and vertical components of the velocity of the projectile 10 seconds after discharge; also find the resultant velocity.

10-3. Find the position and velocity of a projectile 10 seconds after being discharged at an angle of 45 degrees above the horizontal with a muzzle velocity of 2,000 feet per second.

10-4. A certain long-range gun has a muzzle speed of 4,000 feet per second. Find the maximum height reached by the projectile, also the horizontal range, assuming the angle of elevation to be 45 degrees.

10-5. A body slides from rest 20 feet down a roof inclined 30 degrees to the horizontal with an acceleration of 16 feet per second^2, and then falls to the ground, 30 feet below. Just where will the body land?

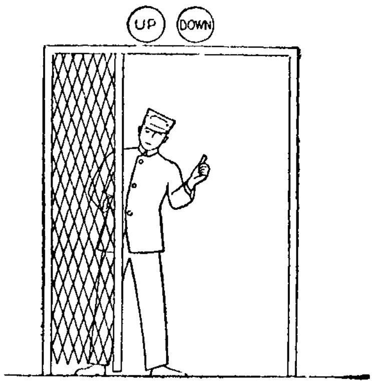
10-6. With what horizontal velocity must a boy throw a paper bag full of water to hit a cat 20 feet below his window and 10 feet from the base of the building? As usual, neglect wind resistance.

10-7. If friction will allow a centripetal acceleration of 25 feet per second^2, find the maximum speed with which an automobile can make a 90-degree turn with a radius of curvature of 20 feet.

10-8. Find the radial acceleration of an apple which is being whirled on the end of a string in a horizontal circle of 60 centimeters radius, if it takes 1.6 seconds to make a round trip.

10-9. A body moving in a circle of radius r feet, makes n round trips per second. Show that its speed, v, is $2\pi rn$ feet per second. Also show that its centripetal acceleration is $4\pi^2n^2r$.

CHAPTER 11



Newton's Second Law

11-1. The Cause of Acceleration. Accelerations are caused by unbalanced forces. Up to this point we have been rather fussy about having our forces balanced since we have usually desired equilibrium; but in the absence of equilibrium there is always accelerated motion. We shall find a similar situation when we come to discuss rotatory motion; when the torques are unbalanced there will be a loss of rotatory equilibrium and a consequent angular acceleration.

11-2. Newton's Second Law. Newton's first law (see section 2-2) tells us that when the forces acting on an object in a given direction add to zero, that the acceleration in that direction is also zero; that is, if the body is at rest it will remain at rest, and if it is in motion it will remain in motion with uniform velocity. *Newton's second law* states that if the forces acting on a body do not add vectorially to zero, then the body will change velocity in such a way that the acceleration will be proportional to the vector sum of the forces, and in that direction. The proportionality factor is the "mass." A word of caution is necessary in this connection; we are so accustomed in everyday life to find the forces that we exert balanced by friction (in which case the algebraic sum of the forces is zero) that we get into the habit of thinking of a force as producing a velocity rather than an acceleration. But when the sum of the forces is not equal to zero, then at least one of the forces is "unbalanced." We shall take the expressions "unbalanced force" and "vector sum of the forces" to mean exactly the same thing. Thus, if there is an unbalanced force, the velocity is never uniform, but there is always an acceleration in the direction of the unbalanced force.

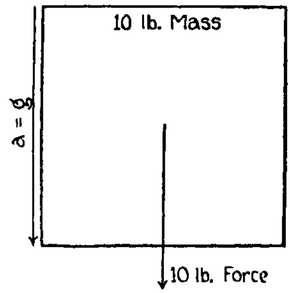


Figure 11-1.

11-3. Illustrations. Consider in figure 11-1 a 10-pound force acting on a 10-pound mass. In reality this represents nothing more than a 10-pound body in mid-air with no force acting other than the 10-pound pull of gravity, ordinarily called the weight. Under these conditions we know that the body will be falling with the acceleration of gravity, g (32.2 ft./sec.^{2}). We can make the general remark that when the sum of the forces acting on the body is numerically equal to the weight of the body, the acceleration will always be g (32.2 ft./sec.^{2}), not necessarily downward, but in the direction of the resultant force. The acceleration would still be g if we added a 5-pound force downward and a 5-pound force upward. The sum of the forces would still be 10 pounds downward and would be equal to the weight. If, on the other hand, we add to figure 11-1 a 5-pound upward force (figure 11-2), the vector sum of the forces (which in this case is also the algebraic sum) now becomes 5 pounds downward, or half of the weight. Under these conditions the acceleration is reduced to half the acceleration of gravity, 16.1 ft./sec.^{2}, still in the direction of the resultant of the forces (downward). In figure 11-3, the quantity of matter in the body is 10 pounds; the weight or force of gravity is also 10 pounds. The body is resting upon a horizontal surface which

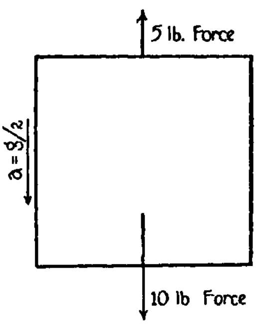


Figure 11-2.

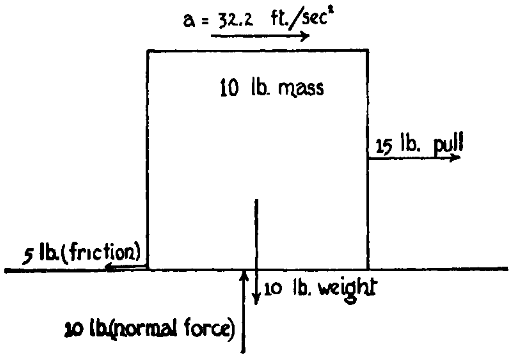


Figure 11-3.

supports it with an upward force of 10 pounds, called the *normal force.* A 15-pound force acts toward the right and is opposed by a 5-pound force of friction. The algebraic sum of the vertical forces is zero, and the algebraic sum of the horizontal forces is 10 pounds toward the right. The vector sum of all the forces is therefore 10 pounds toward the right, or numerically the same as the weight. The acceleration will therefore be 32.2 ft./sec.^{2} toward the right; that is, the body will behave as if it were "freely falling" toward the right instead of downward. For instance, if its velocity at a certain moment is 100 ft./sec. toward the right, then one second later the velocity will be 132.2 ft./sec. in the same direction.

11-4. Formulation of Newton's Second Law. The accelerations that a body will experience are proportional to the resultant forces that may act; this fact may be expressed algebraically as follows

$$\frac{F}{F'} = \frac{a}{a'}$$

This equation states that if a force, F, produces an acceleration, a, in a certain body, then a force, F', will produce an acceleration, a', where F and F' are proportional to a and a'. Fortunately we already know that one particular force, W, the weight of the body, will produce the particular acceleration, g, so that a special form of the equation is

$$\frac{F}{W} = \frac{a}{g} \tag{a}$$

The F in the numerator is understood to be the vector sum of all the forces that actually act on the body.

11-5. Illustrative Problems. (1) Assume that the automobile referred to in section 9-2 weighs 3,000 pounds and experiences resisting forces to the extent of 400 pounds. With what force are the drive wheels pushing backward on the road?

By Newton's third law, the road pushes forward on the drive wheels with a force equal and opposite to that asked for in this problem, and since all the forces involved in Newton's second law must act on the same body, we shall have to use this forward force. Call the force X. Since there is no vertical acceleration, the sum of the vertical forces must be zero and we need give them no further consideration. The sum of the horizontal forces acting on the car are then the unknown forward force, X, and the backward frictional forces which total $-$ 400 pounds. Thus F in equation (a) is $X - 400$. Applying the equation which expresses Newton's second law, we have $(X - 400)/3{,}000 = 4.40/32.2$. Solving for X we obtain 810 pounds. In this type of problem, it is usually simplest to adopt as positive the direction of the acceleration.

(2) Anyone who has tried stepping about in an elevator that is speeding up or slowing down has experienced peculiar sensations of unusual lightness or heaviness of body according to the direction of the acceleration. Find the force with which a 200-pound man pushes down on the floor of an elevator

(1) as it starts upward from the ground floor with an acceleration of 4 ft./sec.^{2}; (2) as it nears the top floor and experiences a deceleration of 4 ft./sec.^{2}

As in the preceding problem, instead of the force exerted by the man on the elevator floor, since we wish all our forces to act on the man, we shall consider the upward force exerted by the floor on the man, and call this force X in part (1) and X' in part (2) of our present problem. In part (1) the acceleration is upward, therefore this will be taken as the positive direction. There are two forces acting upon the man during the acceleration, the upward (positive) force X, and the downward force of gravity, $-$ 200 pounds. Therefore F of equation (a) of section 11-4 is $X - 200$. Our equation is then $(X - 200)/200 = 4/32.2$, and $X = 225$ pounds. In part (2) of our problem, the acceleration is downward, so we shall consider the downward direction as positive. The equation is therefore $(200 - X'/200 = 4/32.2$ and $X' = 175$ pounds. Instead of merely substituting values into equation (a) of section 11-4, as we have been doing, it is possible to set up the proportion directly by arguing somewhat as follows: if the man in part (2) of our problem were up in midair with no force acting upon him but gravity, his acceleration would be 32.2 ft./sec.^{2} but with a total force of $200 - X'$, he will experience an acceleration of 4 ft./sec.^{2} which has the same relation to 32.2 as $200 - X'$ has to his weight of 200 pounds. This shows immediately just why the quantity 32.2 always appears in these equations.

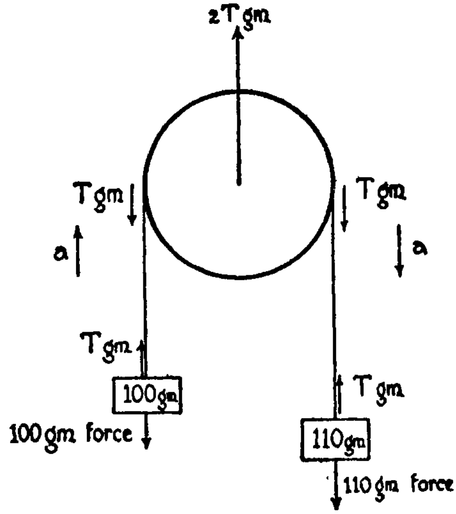


Figure 11-4.

(3) A device invented by George Atwood of Trinity College, Cambridge, England in 1784 is shown in figure 11-4, and is called Atwood's machine. It is essentially a device for "diluting" gravity. Over a pulley, so light that its weight may be neglected, is passed a light cord, one end of which is attached to a 100-gram weight, and the other end to a 110-gram weight. Find the acceleration of the moving system and the tension in the cord.

Our common sense tells us that the 100 grams will go up, the 110 grams down, and that therefore the tension in the cord is more than 100 grams and less than 110 grams. Since we are now using metric units, g is 980 cm./sec.^{2} Since the tension in the cord, T, is the force that the cord exerts on each weight, the forces acting on the 100-gram weight are T and — 100 grams; and the forces on the other weight are — T and $+$ 110 grams. Therefore the equation for the 100-gram weight is $(T - 100)/100 = a/980$, and for the 110-gram weight, $(110 - T)/100 = a/980$. Equating the two left-hand sides, we obtain an equation that can be solved for T and find that $T = 104.8$ grams. The first equation then reduces to $4.8/100 = a/980$, which gives $a = 46.9$ cm./sec.^{2}

The "light pulley" has zero translatory motion since its center of gravity remains at rest. It may therefore be assumed that the upward forces on the pulley balance the downward forces. Since the only forces acting down on the pulley are those exerted by the cord, namely, two forces of 104.8 grams each, it follows that the upward force on the center of the pulley is 209.6 grams, that is, less than the sum of the two weights. If the two weights were not subject to an acceleration, that is, if we clamped the system so that everything were in equilibrium, this upward force on the center of the pulley would become 210 grams.

11-6. Mass. Another way of writing the equation for Newton's second law follows directly from the proportion above:

$$F = \left(\frac{W}{g}\right)a$$

This combination of W in the numerator and g in the denominator has occurred before in the expression of kinetic energy (see section 3-6). It is customary to call it *mass*; that is, the *mass* of a body is technically its weight divided by the acceleration of gravity. The equation is a convenient description of the quantity of matter present in the body because in places where the weight is small, such as near the poles or at high altitudes, g is also small in the same proportion, so that although the weight of a body varies from place to place, the ratio W/g is constant; therefore the mass is, under ordinary circumstances, practically constant for a given body. Newton's second law may thus be written

$$\text{resultant of the forces} = (\text{mass})\ (\text{acceleration})$$

or

$$F = ma$$

In the case of a freely falling body, where the only force acting is the weight of the body, this equation becomes

$$\text{weight} = (\text{mass})\ (\text{acceleration of gravity})$$

or

$$W = mg$$

11-7. Inertia. We have seen that at any one point on the surface of the earth, mass is proportional to weight; is there any other property of matter to which mass may be related? Yes! There is the property of matter known as *inertia*, which is that property of matter which makes it necessary to apply a force when we wish the body to become accelerated. A negative way of describing the property of inertia is to say that if we do *not* apply a force we have *no* acceleration, and the body remains at rest if it is at rest, or if it is in motion, remains in motion with uniform velocity. The next question is naturally, "How can inertia be measured?" Since inertia is the property of matter that makes it necessary to apply a force to produce a given acceleration, inertia is to be measured as the ratio of the force acting to the acceleration produced; inertia $= F/a$. But we have seen that this is exactly what mass is. We have not merely related mass to inertia, we have actually identified the two, so that from here on, if we wish, we may use the terms mass and inertia interchangeably; all units of mass are also units of inertia.

11-8. Engineering Units and Absolute Units. From the point of view of the engineer, mass is a more or less artificial concept; he prefers to base all his mechanical units on three fundamental concepts, namely, length, time, and force. For example, velocity and acceleration units are derived from the ideas of length and time alone; a weight is a force; energy is the product of force and length; power involves all three, force, length, and time; mass likewise, being the ratio of weight to the acceleration of gravity, involves force, length, and time. On the other hand, theoretical physicists prefer to work with what are called "absolute units." For this purpose, the three fundamental concepts are mass, length, and time; that is, the idea of mass is to the theoretical physicist fundamental, and force is a derived concept.

Since this book is written from the engineering standpoint, not much will be said about mass, nevertheless it will be understood that wherever the ratio (W/g) occurs in an equation, it may be replaced by the single letter m, and after this is done, absolute units may be used in the equation instead of engineering or gravitational units, as they are often called. The next section may therefore be omitted by the student who is interested only in engineering physics.

11-9. Systems of Units. In engineering work it is not necessary to worry about units. If there is any doubt about whether the units used are correct or not, it is merely necessary to insert the units into the equations along with the numerical values, and if the units are correct, they may be cancelled in pairs from the entire equation. On the other hand, when an absolute system of units is being used, forces and masses both appear in the equations, and it is necessary that they be given *different* units.

To show how this works out, it will be convenient to discuss four different systems of units: two engineering systems and two absolute systems. These will be called respectively, (1) the English engineering system, (2) the metric engineering system, (3) the English absolute system, and (4) the M.K.S. system. There was formerly a fifth system called the c.g.s. system.

In 1935 the International Committee on Weights and Measures decreed that in January, 1940, this system should be replaced by the M.K.S. system, but scientists, like other people, are conservative, and it may be many years before the c.g.s. system falls into disuse. Therefore in the remainder of this section we shall include all five systems.

The *kilogram mass* is defined as the mass of a certain block of platinum called the *standard kilogram*, preserved at the International Bureau of Weights and Measures, near Paris, France. A *kilogram force* is the weight of a kilogram mass, and since this varies from place to place on the surface of the earth, the kilogram force is not a definite force. In the United States, by act of Congress, the *pound mass* is defined in terms of the mass of the standard kilogram, but in England the pound mass is the mass of a certain block of platinum preserved at the Standards Office in Westminster, London. Thus it is that the United States pound and the British pound, though intended to be alike, are actually slightly different. The *standard pound force* is the weight of the standard pound mass at sea level and at 45 degrees north latitude, and therefore, unlike the kilogram force, the pound force is a definite force. The acceleration of gravity, g, has been determined at sea level at 45 degrees north latitude to be 32.1740 feet per second^2.

In system (1), in the United States, the foot, the unit of length, is defined as 1,200/3,937 of the standard meter; the second, the unit of time, is 1/86,400 of a mean solar day; and the pound force is the weight of the United States pound mass at sea level and at 45 degrees north latitude. Although there is no fundamental unit of mass in this system, one may be derived from the other units in accordance with the relation $W = mg$. Thus, if we take the unit of mass in this system to be 32.1740 times the mass of the United States pound and call it "one slug," then we can substitute into the equation $W = mg$ the values: $W = 32.1740$ pounds of force, $m = 1.00000$ slug, and $g = 32.1740$ feet/second^2, and the numerical values check. But in order to make the units check, it is necessary to think of the pound force and the slug as related by the equation: pound force equals slug-feet per second squared.

In system (2) the kilogram is taken as the unit of force, the meter as the unit of length, and the second as the unit of time. A derived unit of mass may be created and called the *metric slug*. Using the arguments of the previous paragraph, the mass of the metric slug would have to be 9.80 times the mass of a kilogram.

System (3) is used considerably in textbooks. It has the same units of length and time as the English engineering system, but differs in that it has a fundamental unit of mass, the *pound mass*, and a derived unit of force, the *poundal*. The poundal is the force that is necessary to give an acceleration of one foot per second per second to a mass of one pound. In this system, weight, being a force, must be measured in poundals. To find the weight of a pound mass in poundals, substitute in the equation $W = mg$ as follows: $m = 1.000$ pound, and $g = 32.2$ feet/second^2, therefore W in poundals equals 32.2. If we try to find the weight of a pound mass in poundals at a place where g is slightly different, the result will be slightly different, namely, the new value of g.

System (4), the M.K.S. system, uses the meter as the unit of length; the second, or 1/86,400 of the mean solar day, as the unit of time; and the mass of the standard kilogram as the unit of mass. The *standard meter* is defined as the distance between two fine lines engraved on a platinum-iridium bar kept at the above mentioned International Bureau of Weights and Measures at Sevres, near Paris, France. The derived unit of force is the *newton*, defined as the force which will produce an acceleration of one meter per second per second in a mass of one kilogram. By the arguments of the previous paragraph, we discover that the weight of a kilogram mass is 9.80 newtons at a place where the acceleration of gravity is 9.80 meters per second per second. This system will be found to fit nicely into the practical electrical system of units. The newton-meter is a joule; the newton-meter per second is a watt. Other electrical units belonging to this system such as the volt, ampere, ohm, and so on will be discussed later.

In the c.g.s. system the centimeter, the unit of length, is one hundredth of a meter; the gram, the unit of mass, is one thousandth of a kilogram; and the dyne, the unit of force, is one hundred-thousandth of a newton. It is like a toy system, useful chiefly in dealing with small quantities. Yet, ironically enough, its units are too large when we deal with atomic entities.

The approximate relative magnitudes of these units of mass and force, jumbling them together rather indiscriminately, may be seen from the following table:

980 dynes	=	1 gram
454 grams	=	1 pound
1,000 grams	=	1 kilogram
9.80 newtons	=	1 kilogram
0.80 kilograms	=	1 metric slug
32.2 poundals	=	1 pound
32.2 pounds	=	1 slug

The various units discussed in this section may be arranged as in the following table, where the derived units appear in parentheses.

System	*Time*	*Length*	*Mass*	*Force*	*Weight*
(1) English engineering	second	foot	(slug)	pound	pound
(2) Metric engineering	second	meter	(metric slug)	kilogram	kilogram
(3) English absolute	second	foot	pound	(poundal)	(poundal)
(4) Kilogram-meter-second	second	meter	kilogram	(newton)	(newton)
(5) C.g.s.	second	cm.	gram	(dyne)	(dyne)

11-10. Kinetic Energy. (See section 3-6.) By equation (c) in section 9-7, if a body drops a vertical distance, h, *from rest*, with the acceleration of gravity, g, the square of its velocity will be $2gh$ downward. In this case $h = v^2/2g$. Utilizing the law of conservation of energy, we can say that the potential energy (Wh, or mgh) at the height h, will be converted into an equal amount of kinetic energy, $Wv^2/2g$ or $mv^2/2$, after dropping the distance h. The kinetic energy depends only on the numerical values of the mass and the velocity, and will have the same value whatever the direction of the velocity. It is therefore not a vector quantity.

11-11. Illustrative Problems (4). As an illustration of the equation, $F = ma$, let us do again problem (1) which has already been solved in section 11-5. Since the weight of the automobile is 3,000 pounds, its mass is 3,000/32.2 or 93.2 slugs. The sum of the forces still is correctly expressed as $X - 400$ pounds. And the acceleration is 4.4 feet/second². Therefore when we substitute in the equation, we have $X - 400 = (93.2)\ (4.4)$, and the solution is still 810 pounds.

(5) On a smooth roof inclined 30 degrees to the horizontal, an object is placed 32 feet from the eaves and released. The eaves are 80 feet above the ground. How long does it take the object (a) to slide to the eaves, and (b) to reach the ground? (c) Where will it strike the ground, and (d) what velocity will it have just before it strikes?

(a) When the roof is described as "smooth" the interpretation is that the coefficient of friction is zero. Referring to figure 11-5, the weight is resolved into two components, one perpendicular to the roof and the other parallel. Since we are dealing with a 30-degree right triangle, the component, F, parallel to the roof is just half of the weight, and this is the only force parallel to the motion; that is, it is the *sum* of the forces in this case. When we substitute values into the equation

$$\frac{F}{W} = \frac{a}{g}$$

we see that since on the left the numerator is just half of the denomiminator; the same will have to be true on the right; therefore, $a = 16.1$ feet/second². During the motion down to the eaves we have the following data

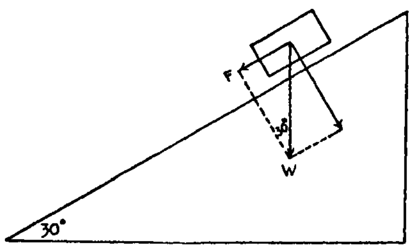


Figure 11-5.

a = 16.1 feet/second²
s = 32 feet
t = ?
u = 0
v = ?

Equation (c) of section 9-7 does not contain t; we can therefore use it to solve for v. Substituting in the data, we obtain

$$(2)\ (16.1)\ (32) = v^2 - 0^2$$

Solving gives us $v^2 = 1{,}030$, and $v = 32.1$ feet per second. Equation (a) of section 9-7 now gives us t

$$32.1 = 0 + 16.1\,t$$

Solving, we find that $t = 1.994$ seconds.

(b) From this point on, we have a projectile problem on our hands; it is therefore necessary to consider separately the vertical and the horizontal part of the motion. First resolve the velocity at the eaves, 32.1 feet per second, 30 degrees below the horizontal, into vertical and horizontal components. By figure 7-5, the vertical component is (0.5) (32.1) or 16.0 feet/second downward, and the horizontal component is (0.866) (32.1) or 27.8 feet/second. The data for the vertical part of the problem line up as follows.

$$\begin{aligned} a &= 32.2 \text{ feet/second}^2 \\ s &= 80 \text{ feet} \\ t &= ? \\ u &= 16.0 \text{ feet/second} \\ v &= ? \end{aligned}$$

Since we have the same set of unknowns as in part (a), the procedure will be the same. Equation (c), section 9-7, gives us

$$(2)\ (32.2)\ (80) = v^2 - (16.0)^2$$

$v^2 = 5{,}150 + 256 = 5{,}410$, and $v = 73.6$ feet/second. Then equation (a), section 9-7, becomes

$$73.6 = 16.0 + 32.2\ t$$

From this, $t = (73.6 - 16.0)/32.2 = 57.6/32.2 = 1.789$ seconds.

(c) In order to find where the object will strike the ground, it is only necessary to multiply the horizontal component of the velocity as the object leaves the roof, 27.8 feet/second, by the time it is in the air, 1.789 seconds. The product is 49.7 feet, the horizontal distance from the building.

(d) The velocity of the body just as it reaches the ground is the resultant of the horizontal component, 27.8 feet per second, and the vertical component, 73.6 feet/second. Squaring these and adding, we have 773 + 5,410 = 6,180. Extracting the square root, we find the resultant to be 78.6 feet/second. The result could also have been obtained graphically by drawing the figure to scale and measuring the diagonal; the result may in this way be obtained to the same degree of accuracy as by the use of the slide-rule. If the figure has been drawn, we can measure the angle between the resultant and the vertical with a protractor and find it to be 20° 42′. The angle may also be found trigonometrically either from the fact that its tangent is 27.8/73.6 (or 0.378) or from the fact that its sine is 27.8/78.6 or (0.354).

(6) What centripetal force is necessary to make a 16.1-pound body revolve in a horizontal circle of one foot radius at the rate of 0.5 revolutions per second? If the body is supported against gravity and caused to revolve in this way by means of a cord attached to it, find the tension and length of the cord.

In order to find the centripetal force we need first the centripetal acceleration, the expression for which is v^2/r. In one second the body will make just half a revolution. Since the circumference of the circle which constitutes its path is (2π) (1) or 6.28 feet, half of this divided by one second, the time it takes for a half revolution, is 3.14 feet per second, its speed. Therefore v^2/r is $(3.14)^2/(1)$ or 9.86 feet/second². We could have obtained the same result by utilizing a formula developed in problem 10-9, that is, that the centripetal acceleration is $4\,\pi^2 n^2 r$. In this case the expression becomes $(4)\ (3.14)^2\ (0.5)^2\ (1)$, which also gives us 9.86 feet/second². Utilizing equation (a) of section 11-4 in which W is 16.1 pounds, a is 9.86 feet/second², and g is 32.2 feet/second², we have

$$\frac{F}{16.1} = \frac{9.86}{32.2}$$

Solving for the resultant of the forces gives us $F = 4.93$ pounds, and since in this case, there is only one force in the direction of the center of the circle, 4.93 pounds is the desired centripetal force. The negative of this force, called *centrifugal force*, is a fictitious force which would have to be applied to hold the body in any of its instantaneous positions if the whirling motion stopped.

In this problem, the body is caused to remain in its horizontal circle by means of a cord, one end of which is held at a point directly above the center of the circle and the other end of which is attached to the revolving body. The tension in this cord, which is always in a slanting position, represents a slant force exerted on the revolving body. One component of this slant force supports the weight of the body, 16.1 pounds, and the other component supplies the centripetal force of 4.93 pounds towards the center of the circle. The resultant of these two components is the tension we seek. $(16.1)^2 + (4.93)^2 = (16.84)^2$. Therefore the tension in the cord is 16.84 pounds. If the student draws the diagram, he will see that it contains two similar triangles in which the length of the supporting cord is to the radius of the circle as the tension in the cord is to the centripetal force. If x is the length of the cord, we have $x/1 = 16.84/4.93$. Therefore the length of the cord must be 3.42 feet.

SUMMARY OF CHAPTER 11

Technical Terms Defined

Mass. Quantity of matter in a body. Represented in engineering expressions by the ratio of the weight to the acceleration of gravity.

Inertia. A property that matter possesses which makes it necessary to apply an unbalanced force to a body in order to produce an acceleration of that body. It is indistinguishable from mass and is measured in mass units.

Translatory Kinetic Energy. The energy that a body possesses by virtue of its translatory motion. It is found by multiplying half the mass of the body by its velocity squared.

Newton's Second Law. Unbalanced forces produce accelerations in a body; the accelerations are proportional to the resultant force.

The equation:

$$F = \frac{W}{g}\,a$$

PROBLEMS

11-1. A 20-pound body rests on a smooth horizontal surface. If a certain unbalanced horizontal force moves it 8 feet from rest in one second, find both the acceleration and the force.

11-2. A 40-pound body rests on a rough horizontal surface such that the coefficient of friction is 0.2. What acceleration will a horizontal force of 20 pounds produce on the body, and how far will it move the body from rest in two seconds?

11-3. A 20-pound body rests on a smooth plane inclined at such an angle that a force of 10 pounds, acting up the plane, is necessary to produce equilibrium. What force must replace the 10-pound force to cause the 20-pound body to move up the plane 8 feet from rest in one second with accelerated motion?

11-4. On a roof inclined 30 degrees to the horizontal, an object is placed 32 feet from the eaves and released. The eaves are 80 feet above the ground. If the coefficient of friction between the object and the roof is 0.1, how long does it take the object (1) to slide to the eaves and (2) to reach the ground? (3) Where will it strike the ground and (4) what velocity will it have on striking?

11-5. A 161-pound man stands in an elevator while the elevator has a downward acceleration of 12 feet/second.^{2} With what force do his feet push on the floor of the elevator? Is it necessary for the elevator to be moving downward for the above situation to occur? If the elevator has an upward acceleration of 16.1 feet/second^2, how hard will his feet push on the elevator floor?

11-6. A 16-pound body and a 48-pound body are placed side by side on a smooth horizontal surface and a horizontal force of 32 pounds is applied to the 16-pound body so as to set both in motion. (1) What acceleration will be produced? (2) What force will the 16-pound body exert on the 48-pound body? (3) What force will the 48-pound body exert on the 16-pound body?

11-7. If the coefficient of friction in problem 11-6 is changed from zero to 0.1, what do the answers become?

11-8. A and B are two objects that each weigh 10 pounds. Show, if A rests on a smooth horizontal bench 8 feet high, 24 feet from the edge, and is connected by a 24-foot cord to B, which is just falling off the edge of the bench, that B will reach the floor in one second from rest, and A will reach the edge of the bench in two seconds, striking the floor in 2.71 seconds, assuming the acceleration of gravity to be 32.0 feet/second^2.

11-9. A 48-pound weight rests on a horizontal bench which is 4 feet high with the weight 12 feet from the edge. A 12-foot cord connects this weight with a 16-pound weight which is just ready to fall from the edge of the bench. Assuming no friction, how long after the bodies start from rest will it be when (1) the 16-pound weight strikes the floor, (2) the 48-pound weight reaches the edge of the bench, and (3) the 48-pound weight reaches the floor? (4) What are the velocities of the 48-pound weight at these three times?

11-10. Solve the preceding problem, assuming a coefficient of friction of 0.25 between the 48-pound weight and the bench.

11-11. A 16-pound weight and a 48-pound weight hang on opposite ends of a cord which passes over a light frictionless pulley. Compute (1) the acceleration of the bodies, (2) the speed they will acquire in one second from rest, and (3) the distance each will move during that second. (4) Find the tension in the cord.

11-12. If, instead of moving vertically, the 48-pound weight of the preceding problem moves down a smooth plane inclined at 30 degrees to the horizontal, with the cord now pulling parallel to the plane, what acceleration will the 16-pound weight (still hanging vertically) now have?

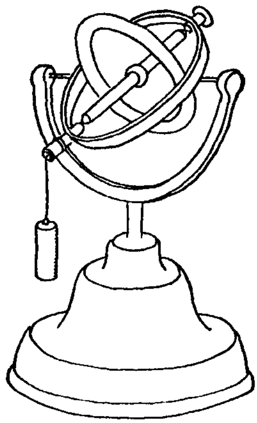
11-13. If in the preceding problem the coefficient of friction between the 48-pound weight and the plane is assumed to be 0.175, what will the resulting acceleration of the 16-pound weight become?

11-14. Prove (1) that 1 newton = 1 kilogram-meter/second^2 and (2) that 1 slug = 1 pound-second^2/foot.

11-15. If a 3,220-pound automobile is rounding a curve of 100 yards horizontal radius while traveling at a speed of 30 miles per hour, compute (1) the necessary centripetal force, (2) the vertical components of the force which the road exerts upon the automobile, and (3) the resultant force which the road exerts upon the automobile. For maximum stability (that is, no tendency for the automobile to slip either to the inside or to the outside edge of the curve) the surface of the road should be perpendicular to this resultant. This makes the angle of banking (the angle between the surface of the road and the horizontal) equal to the angle between the resultant force found in (3) and the vertical. Find (4) the angle of banking.

11-16. A pail of water is rotating about a vertical axis through its center at such a rate that four inches from the axis the surface of the water slants at an angle of 45 degrees with the horizontal. How many rotations per second is the pail making?

CHAPTER 12



Angular Acceleration; Gyroscope

12-1. Units of Angle. We are now about to apply the laws of accelerated motion to rotatory motion. In this connection it is rather startling to discover that there are seven distinct units of angle all in common use. The system of degrees, minutes, and seconds is well known. The quadrant (90 degrees) and the revolution (360 degrees) likewise need no introduction. Many modern military instruments are now graduated in mils, where the mil is 1/1600 of a quadrant. This is particularly convenient for range finders; for example, if an automobile known to be 15 feet long subtends at the eye an angle of 5 mils, it is about 3,000 feet away. If the angle subtended is 2.5 mils, it is 6,000 feet away. The rule is to divide the known length of the object by the number of mils subtended and multiply by 1,000.

The mathematician's favorite unit of angle is the radian, which is about 57.3 degrees. This peculiar number is obtained by dividing 180 degrees by π ($\pi = 3.14159\ldots\ldots$). The reason for this procedure is due to the following argument. Let two lines, AB and CD, intersect at O. With center at O, describe an arc intersecting the two lines at P and Q (see figure 12-1). If the magnitude of the angle QOP (angle θ) is such that the length of QP (arc s) is equal to OQ (radius r), then the angle is said to be one radian. Therefore 3.14

radians (π radians) equal 180 degrees. The relations between the various units of angle are as follows:

60 seconds (″)	=	1 minute (′)
60 minutes	=	1 degree (°)
57.3 degrees	=	1 radian
1,600 mils	=	1 quadrant
90 degrees	=	1 quadrant
360 degrees	=	1 revolution
2 π radians	=	1 revolution

The mil is slightly less than one thousandth of a radian. It is customary to use the revolution as an angular unit when the angle is large; for example, an angle of 3,600 degrees is 10 revolutions or 20 π radians (62.8 radians). If a wheel is turning at the rate of 60 revolutions per second, then in one minute a revolution counter would indicate an angle of 3,600 revolutions.

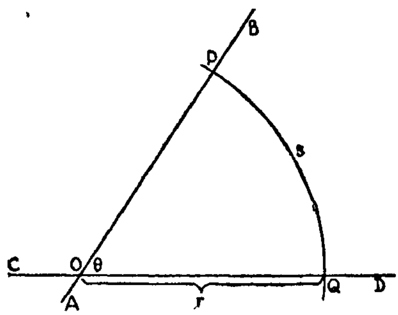


Figure 12-1.

12-2. Angular Speed. A point is merely a position in space without dimensions, and hence cannot rotate. A line, however, is capable of rotating about any one of its points, and in so doing, creates an angle between its original and final positions. The rate at which a line describes an angle is called its *angular speed.* It is expressed in terms of a unit formed by dividing a unit of angle by a unit of time; for example, 10 radians per second, 3,600 revolutions per minute, 15 degrees per hour, the last being the angular speed of the earth's rotation.

12-3. Rotatory Motion. In section 8-1 rotatory motion was described. In pure rotatory motion, the center of gravity lies on the axis of rotation; every point in this line is at rest. Consider now any plane in the body at right angles to the axis. If a line be drawn in this plane in such a way as to intersect the axis, then during the rotation, this line moves with an angular speed which we can identify with the angular speed of the rotating body.

A spinning top usually has a very complicated motion, but at times the axis of spin remains stationary. The schoolboy describes the top in this condition as "sleeping," and a sleeping top constitutes a good illustration of pure rotatory motion. On the other hand a projectile shot from a rifled gun barrel moves with a combination of rotatory motion about its center of gravity and translation of the center of gravity. The projectile problems which we have solved were concerned only with the motion of the center of gravity.

12-4. Angular Velocity. In rotatory motion, the same distinction exists between angular speed, a scalar quantity, and angular velocity, a vector, that we have discovered between linear speed and linear velocity. We have just seen that a point not on the axis of a rotating body moves in a plane, following a circular path. If the center of this path, the orientation of the plane in space, the sense of the rotation, and the rate of rotation be specified, we know all there is to be known about the rotation. All these things may be represented by means of a single arrow, so that angular velocity may therefore be considered as an ordinary vector quantity. Just how a single arrow is capable of representing completely the angular velocity is rather interesting. This could not be done if the arrow were to lie in the plane of the rotation, because a single line, lying in a plane, cannot completely determine the position of the plane. But through a given point, only one plane can be passed *perpendicularly* to a given line, so that if it be understood that the arrow shall be drawn from the center of the circular orbit along the axis of rotation, the orientation of the plane will be completely determined by the arrow. But the arrow must also show the sense of the rotation, that is, whether it is clockwise or counterclockwise as viewed from the head of the arrow. We therefore make the convention that the arrow shall be so drawn that the rotation shall appear counterclockwise as viewed from the head of the arrow (see figure 12-2). And as usual, the length of the arrow is proportional to the speed of rotation. As an aid to memory, we may let the fingers of the right hand represent the rotation, in which case the thumb will indicate the proper direction for the vector representing the angular velocity.

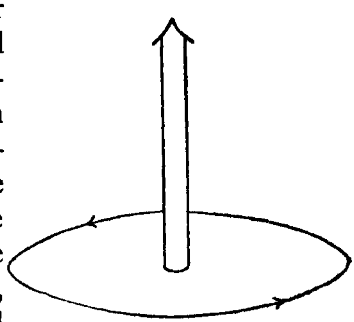


Figure 12-2.

12-5. Equations of Angular Acceleration. In this book we shall confine our discussion of angular acceleration to cases in which the acceleration is uniform. Therefore an *angular acceleration* may be defined as the gain in angular velocity in unit time, or in other words angular acceleration is the ratio of the change in angular velocity to the length of time required for the change. As in our discussion of linear acceleration, we shall require a set of letters representing the five quantities involved; for this purpose it is customary to use two more Greek letters. We have already used the Greek letter *theta* (θ) to designate an angle. *Alpha* (α) will stand for angular acceleration, and omega (ω) for final angular speed. For initial angular speed, we shall use ω_0; t will stand for time. Since the equations are similar to the ones we have already derived, we shall merely list them; the list should be compared with the one in section 9-7.

$$\omega = \omega_0 + \alpha t \qquad \text{(a)}$$

$$\theta = t\,\frac{\omega_0 + \omega}{2} \qquad \text{(b)}$$

$$2\,\alpha\,\theta = \omega^2 - \omega_0{}^2 \qquad \text{(c)}$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha\,t^2 \qquad \text{(d)}$$

As before, one variable is omitted from each equation, θ, α, t, and ω respectively.

12-6. Illustrative Problem. A certain flywheel slows down from 4,800 r.p.m. to 3,600 r.p.m. in ten seconds. Find the angular acceleration and the number of revolutions made in slowing down.

Use equation (a) to find the acceleration, because it does not contain θ. Before substituting in this equation, the time units must be made to agree; at present we have a mixture of minutes and seconds. Since the problem specifies that the angle be expressed in revolutions, it is not necessary in this case to change to radians. But merely to illustrate the process, we shall solve the problem both in revolutions and in radians. In terms of revolutions, we have $\omega_0 = 80$ revolutions per second, $\omega = 60$ revolutions per second, and $t = 10$ seconds. Therefore equation (a) becomes $60 = 80 - \alpha\,10$. Solving for α we find that the angular acceleration is -2 revolutions per second squared. From equation (b) we can find the number of revolutions; $\theta = 10\ (60 + 80)/2 = 700$ revolutions. The negative acceleration means a retardation, or *deceleration* as it is sometimes called.

If the angular acceleration is to be used in some subsequent equation, it is *necessary* that it be expressed in radians/second^2. Therefore we shall solve the problem again, using the radian as the unit of angle. Since each revolution is $2\,\pi$ radians, then ω_0 is $(80)\ (2\pi)$ radians per second = 520 radians/second, and $\omega = (60)\ (2\pi) = 377$ radians per second. Equation (a) now takes the form, $377 = 502 - \alpha\,10$, and α is now -12.50 radians/second^2. We could also have changed -2 revolutions/second^2 directly into radians/second^2 by multiplying by 2π, obtaining -12.56 radians/second^2, which checks with the other value within the limits of accuracy to which we are working.

12-7. Relations Between Linear Magnitudes on the Circumference and the Corresponding Angular Magnitudes at the Center. From the definition of the radian (see section 12-1 and figure 12-1) we have seen that the angle θ is the ratio of the arc s to the radius r. When $s = r$, the ratio is unity and θ is one radian. Therefore, since $\theta = s/r$

$$s = r\theta$$

Similarly, the linear velocity of a point moving along an arc is related to the angular velocity of the line connecting the moving point with the center by the equation

$$v = r\omega$$

And the linear acceleration of a point on the circumference in a direction tangent to the circumference is related to the angular acceleration of the line connecting the moving point with the center by the equation

$$a = r\alpha$$

Very frequently angular velocity is represented by the letter n when the unit is revolutions per second; if it is also understood that ω is in radians per second, then, since there are 2π radians in a revolution, it is true that

$$\omega = 2\,\pi n$$

And since $v = r\omega$, then

$$v = 2\,\pi r n$$

Furthermore, since centripetal, or radial, acceleration, that is, the linear acceleration of a point on the circumference toward the center (see section 10-5) is equal to v^2/r, it is also equal to $4\,\pi^2r^2n^2/r$, or

$$a = 4\,\pi^2n^2r$$

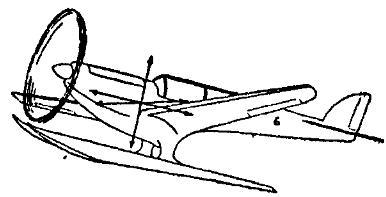
12-8. Illustration Involving Three Types of Acceleration. A flywheel, 10 feet in diameter, starts from rest and in 10 seconds acquires an angular speed of 1,200 r.p.m. Find the *tangential* acceleration of a point on the circumference, the *centripetal* acceleration of the same point at the end of 10 seconds, and the *angular* acceleration.

Both the tangential and centripetal accelerations are *linear* and are expressed in feet/second²; the angular acceleration will come out in radians/second². By the end of the 10 seconds, n = 1,200 r.p.m. or 20 revolutions/second. $\omega = 2\pi n$ and is therefore 40π radians per second, or 125.6 radians/second, and $v = r\omega$ or (5) (125.6) which is 628 feet/second. v could also be found from $v = 2\pi rn$ = (2) (3.14) (5) (20) = 628 feet/second. Since it took 10 seconds to acquire this speed of 628 feet/second from rest, then the acceleration in the direction of the tangent is 62.8 feet/second², from equation (a), section 9-7.

The centripetal acceleration is the component of linear acceleration at right angles to the tangent, along the radius toward the center. If it were not for this acceleration, the point under discussion would move in a straight line and not follow the circumference of the wheel at all. In order to find the radial acceleration, it is necessary to substitute in either the formula, v^2/r, section 10-5, or $4\pi^2n^2r$ of the previous section. The first gives $628^2/5$ or 78,800 feet/second^2; the second gives 4 (3.14^2) (20^2) 5 or 78,800 feet/second^2, enormously greater than the tangential component.

A point may have linear acceleration, but only a line may have angular acceleration; therefore, to solve the third part of the problem, connect the point in question with the center of rotation. This line describes an angle as the wheel turns. In this case the angular velocity of the radius is not uniform but increases from ω_0 = zero to ω = 125.6 radians/second in 10 seconds time, therefore the gain in angular velocity per second, or angular acceleration, is 125.6/10 or 12.56 radians/second^2 from equation (a), section 12-5. As a check we can apply the equation $a = r\alpha$ connecting the tangential and angular accelerations. This gives 62.8 = (5) (12.56), which is obviously correct.

12-9. The Gyroscope. Some rather interesting applications of angular velocity vectors occur in dealing with gyroscopes. A gyroscope is simply a flywheel mounted in such a way that it is free to turn about three different axes, at right angles to each other. The front wheel of a bicycle may be taken as an illustration. Let us consider what ought to happen if the bicycle rider turns the handle bars to the right while the wheel is turning. In accordance with section 12-4, the angular velocity of the wheel, due to its rotation on its own axis, is represented by a relatively long horizontal arrow, pointing to the left. The other angular velocity of the wheel, due to the rotation of the handle bars to the right, is represented by a relatively short arrow pointing nearly vertically down. Draw the parallelogram of these two vectors and find the resultant. It will be found that this resultant will be represented by a long arrow pointing downward to the left. In order that this may represent the resultant angular velocity of the spinning wheel, the bicycle must tip to the left. This would be more clearly recognized if one should remove the front wheel from the frame, set it spinning, and attempt to give it a twist such as the handle bars would if turned to the right. As a result, the wheel will almost tear itself out of one's hands in setting itself to correspond to a bicycle leaning to the left! As a matter of fact, gyrostatic action constitutes the usual reason for a bicycle rider steering to the right; he is starting to fall to the right and wishes to create a torque which will neutralize the tendency by rotating the bicycle to the left.



With the spinning bicycle wheel still held in his hands, if the experimenter whirls himself upon his heel completely around to the right, the bicycle wheel will continue turning to the left till the vector representing the rotation of the wheel coincides in direction with the vector representing the rotation on the heel. This experiment illustrates in part the principle of the gyrocompass. A properly mounted rotating flywheel on the rotating earth behaves just as the spinning bicycle wheel does in the hands of a spinning person; that is, it tends to set its axes parallel to the axis of the rotating earth, north and south. Following the same principle, the student can see how a gyrostatic stabilizer properly placed in a ship is able to convert some of the rolling motion into a pitching motion at right angles to the roll. Since the ship is so much longer than it is wide, the pitch is less objectionable.

SUMMARY OF CHAPTER 12

Technical Terms Defined

Mil. An angle equal to one sixteen-hundredth of a quadrant.

Radian. An angle subtended by an arc which is equal in length to its radius; 180 degrees divided by π.

Angular Speed. The magnitude of the time rate of change of an angle. A scalar quantity.

Angular Velocity. A complete description of the angular motion of a body including not only the magnitude of the angular speed but the position of the axis, also the sense of the rotation. A vector quantity.

Angular Acceleration. Rate of change of angular velocity. This is also a vector quantity.

Gyroscope. A flywheel mounted so that it is free to rotate about three different axes at right angles to each other. Its angular velocities may be combined vectorially.

PROBLEMS

12-1. An automobile, fifteen feet long, is five thousand feet distant from an observer. What angle does it subtend at the observer in radians? In mils?

12-2. Find the percentage difference between a mil and one thousandth of a radian.

12-3. A cylinder rotating about its axis with constant angular acceleration makes two complete revolutions from rest in two seconds. Compute (1) the magnitude of the angular acceleration, (2) the angular speed at the end of two seconds, and (3) the average angular speed. (4) What other information is necessary in order to compute the linear speed of a point on the cylindrical surface?

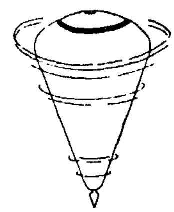
12-4. If a uniformly accelerated rotating body increases its speed from one revolution per second to two revolutions per second in making three revolutions, compute (1) the average speed, (2) the time required for this increase in speed, and (3) the magnitude of the constant acceleration. Express these answers in terms of both revolutions and radians.

12-5. A rotating body has a constant angular deceleration of 6.28 radians per second squared and makes eight revolutions in two seconds. Compute, in terms of both revolutions and radians, (1) the initial angular speed and (2) the final angular speed.

12-6. If in the preceding problem a point in the rotating body is two feet from the axis of rotation, compute (1) the linear distance it moves during the eight revolutions, (2) its linear tangential acceleration, (3) its initial linear speed, (4) its initial radial acceleration, and (5) its initial resultant linear acceleration.

12-7. Show that centripetal force may be equated to $mv\omega$.

12-8. The flywheel of an automobile is turning clockwise as viewed from the front of the car (if one could see it). What will the gyrostatic tendency of the flywheel on its bearings be while the car is turning a corner to the right?



12-9. What is the gyrostatic effect when a boy pushes the top of a rolling hoop to the right?

12-10. Show how a gyrostatic stabilizer in a ship should be mounted.

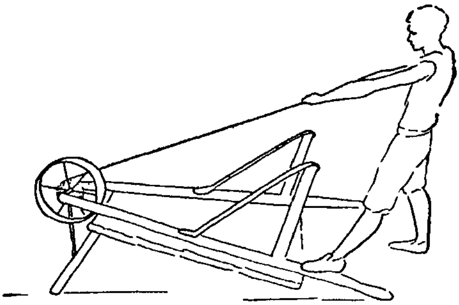
12-11. The slow wabbling of a top is called "precession." If a top is spinning to the right as viewed from above, will the precession be to the right or to the left?

12-12. Imagine the motion of the point farthest forward on the bicycle tire used in the illustration in section 12-9. Due to the rotation of the wheel, this point has a large linear velocity vertically downward. Due to the rotation of the handle bars to the right, this point also has a small horizontal velocity to the right. Find the resultant of the two linear velocities. What position of the wheel will account for this new (resultant) velocity?

12-13. If a cylinder three feet in diameter rolls four feet down an inclined plane in one second from rest, compute (1) the linear acceleration of the axis of the cylinder relative to the plane, (2) its final linear velocity relative to the plane, (3) the final linear velocity of the plane relative to the axis of the cylinder, and (4) the angular acceleration of the cylinder; (5) find also the final angular speed of the cylinder.

12-14. In part (3) of the preceding problem, the magnitude of the final linear velocity of the plane relative to the cylinder is equal to the final linear tangential speed of every point on the cylindrical surface with reference to its axis. Find (1) the speed of a point on the cylindrical surface directly opposite to a point of contact, relative to the plane at this same instant and (2) the corresponding angular speed of a diameter connecting these two points. How does the answer to 12-13 (5) compare with 12-14 (2)?

CHAPTER 13



Dynamics of Rotation

13-1. Moment of Inertia. Newton's second law applied to linear motion states that the sum of the forces applied to a body is proportional to the linear acceleration produced, where the proportionality constant is called the mass or the inertia (see section 11-6). We can also apply Newton's second law to angular motion; it then becomes: the sum of the torques, or moments of force, applied to a body is proportional to the angular acceleration produced, and the proportionality constant is called the *moment of inertia*. A moment of force is the product of a force and a distance, but a moment of inertia is proportional to the product of an inertia and the *square* of a distance. Moment of inertia will be represented by the letter I, and the resultant torque by the letter L, so that the equation for Newton's second law when applied to rotation becomes

$$L = I\alpha$$

The fact just mentioned, that

$$I = kmr^2$$

can be proved from a consideration of units, or quite directly by the argument in the following section.

13-2. Derivation of Formula of Moment of Inertia. If we consider the simplest possible case, it will involve a somewhat hypothetical object. This object will consist of a stiff, weightless rod of length r, connecting a mass, W/g, which occupies no space, to an axis. See figure 13-1. Throughout the discussion, for brevity, we shall use m instead of W/g. In the case of this simplified object, the proportionality constant, k, turns out to be unity, as will be shown. If the force F were applied directly to the mass m, an acceleration would be produced in accordance with the formula, $F = ma$, so that the acceleration would be F/m. If the force were applied at the axis, no acceleration at all would be produced. But if the force is applied at a point P, which is s/r of the distance from the axis to the mass, then the acceleration will be s/r of the value F/m, that is, Fs/mr. The angular acceleration of the whole rod is related to the linear acceleration of the lower end of the rod by the formula, $\alpha = a/r$ (see section 12-7), therefore the *angular* acceleration will be Fs/mr^2. From the previous section, α also equals L/I or the sum of the torques divided by the moment of inertia. We recognize Fs as the only torque acting about the axis chosen (product of the force by the perpendicular distance from the axis; see section 8-3), therefore mr^2 is the *moment of inertia* of the mass m, when it is r units from the axis, and the constant k of the preceding paragraph is unity. Another case when most of the mass is at a distance r from the axis of rotation is the flywheel; here an effort is made to concentrate the mass in the rim as much as possible, so that for most purposes we may say

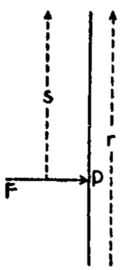


Figure 13-1.

$$I_{\textit{flywheel}} = mr^2$$

or

$$I_{\textit{flywheel}} = \frac{W}{g}r^2$$

A solid cylinder has matter all the way between the axis and the circumference; the moment of inertia of the matter at the axis is zero while that at the circumference is mr^2, so that it is not surprising to find that, taking the cylinder as a whole, the expression becomes

$$I_{\textit{cylinder}} = \tfrac{1}{2}mr^2$$

Since the length of the cylinder does not enter into this formula, the moment of inertia of a disk is likewise $\frac{1}{2}\ mr^2$.

The moment of inertia of a sphere is still less because a still smaller proportion of the matter lies at the distance of the extreme radius. In this case the formula is

$$I_{\textit{sphere}} = \tfrac{2}{5}mr^2$$

The formula for the hollow cylinder involves both the inner radius r_1, and the outer radius r_2, and is

$$I_{\textit{hollow cylinder}} = \tfrac{1}{2}m(r_1^2 + r_2^2)$$

If we imagine a case (like a stove pipe) where r_1 becomes very close in value to r_2, then $(r_1^2 + r_2^2)/2$ becomes approximately $2\,r^2/2$ or just r^2 and we get back to the expression for the flywheel where all the mass is in the rim.

If we make a long slim rod of length l (such as a meter bar), rotate about an axis perpendicular to the rod, we obtain

$$I_{rod} = \tfrac{1}{12}\, ml^2$$

13-3. Units of Moment of Inertia. We could also convince ourselves that the length must be squared in the expression $I = kmr^2$ by a consideration of the units involved. In the equation, $L = I\,\alpha$, we may express L in pound-feet and α in radians per second^2. But when an angle is expressed in radians, it is actually a ratio of an arc to the radius, and therefore a pure number. (See the first equation of section 12-7.) Therefore if $I = L/\alpha$, its unit must be pound-foot-second^2, since it is not necessary to say pound-foot-second^2/radian. Consider now the units involved in kmr^2 or $k(W/g)r^2$; k is a pure number, W is in pounds, reciprocal g is in seconds^2/foot, and r^2 is in feet^2. Taking the product of these, after making one cancellation, we have again pound-foot-second^2.

If we use one of the systems of units mentioned in section 11-9, we could express moment of inertia in slug-feet^2 or kilogram-meters^2.

It is often convenient to consider moments of inertia of areas instead of masses. When this is done, the m or W/g in the foregoing formulas become cross-sectional areas perpendicular to the axis of rotation, and the unit of area moment of inertia is foot^4 or meter^4 instead of slug-foot^2 or kilogram-meter^2.

13-4. Illustrative Problem. As an illustration of Newton's second law applied to rotation, assume the following problem: a grindstone, two feet in diameter and weighing 100 pounds, is equipped with a crank of six inch radius. If a steady force of 25 pounds is maintained on this crank at right angles to the radius, find the resulting angular acceleration.

The sum of the torques about the axis of rotation in this problem is (25 pounds) (0.5 feet) or 12.5 pound-feet. The moment of inertia is given by the formula $Wr^2/2g$, or, numerically, $(100)\,(1)^2/(2)\,(32.2)$ or 1.553 pound-feet-second^2. The angular acceleration, α, is L/I and is therefore 12.5/1.553 or 8 radians per second^2. We could have expressed the angular acceleration as simply 8/second^2. That is, dividing 12.5 pound-feet by 1.553 pound-foot-seconds^2 gives 8 seconds^{-2}, 8/second^2, all three being the same.

13-5. Work and Energy of Rotation. The product of the torque and the angle expressed in radians will give the work done by the torque on the rotating object, and the kinetic energy of the rotating object may be found by using the formula

$$k.e. = \tfrac{1}{2} I\omega^2$$

This kinetic energy of rotation is to be added to whatever kinetic energy of translation may be present, to obtain the total kinetic energy.

13-6. Illustrative Problem. A sphere (figure 13-2), radius two feet, weight 500 pounds, rolls down a 100-foot plane inclined 30 degrees with the

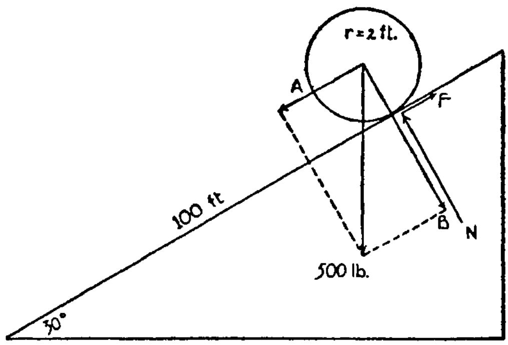


Figure 13-2.

horizontal. If it starts from rest at the top of the plane, what is its angular speed at the bottom of the plane?

By comparison with figure 7-5, we see that the vertical height of one end of the plane compared with the other is 50 feet. Therefore the potential energy of the sphere at the top of the plane is Wh, or in this case, (500 pounds) (50 feet) or 25,000 foot-pounds. Since the sphere is at rest at the top of the plane, it has neither translatory kinetic energy, $(1/2)\ (W/g)v^2$, nor rotatory kinetic energy, $(1/2)\ (I)\ \omega^2$. That is, at the top of the plane, the total energy is 25,000 foot-pounds.

At the bottom of the inclined plane, if we rule out the production of heat energy by friction, the total mechanical energy will also be 25,000 foot-pounds, and, since the potential energy at the bottom is zero, the 25,000 foot-pounds will be divided between the two kinds of kinetic energy, rotatory and translatory. In the expression for the latter, v is unknown, and in the case of the rotatory kinetic energy, both I and ω are unknown. However, we can find I; furthermore v and ω are connected by the relation, $v = r\omega$. We may therefore say that the translatory kinetic energy is $(1/2)\ (500)/(32.2)\ (2\omega)^2$ or $31.1\omega^2$ foot-pounds. Since in the case of a sphere, $I = (2/5)\ (W/g)\ r^2$, in this problem, $I = (2/5)\ (500/32.2)\ 2^2$ or 24.8 pound-foot-seconds². Therefore the rotatory kinetic energy is $(1/2)\ (24.8)\ \omega^2$ or $12.4\ \omega^2$ foot-pounds. We are now in a position to say

$$25{,}000 = 31.1\ \omega^2 - 12.4\ \omega^2$$

Solving for ω^2 yields $\omega^2 = 575$, or $\omega = 24.0$ radians per second.

It is worth while to do this problem again by making use of both forms of Newton's second law, getting first the angular acceleration, then the final angular speed at the foot of the incline.

We may handle the translatory and rotatory aspects independently. First resolve the 500-pound vertical force (the weight) into component A parallel to the plane (250 pounds) and component B perpendicular to the plane (433 pounds). The normal force N will also be 433 pounds. Unless there is a backward force of friction F, the sphere will slide down the plane without any rotation, therefore the resultant force parallel to the incline is $250 - F$ pounds. The linear acceleration will be related to the angular acceleration by the equation $a = r\alpha$ in accordance with section 12-7. Therefore Newton's second law for translation gives us

$$250 - F = \frac{500}{32}\,(2\,\alpha)$$

This equation contains two unknowns so that it cannot be solved until we set up the corresponding equation for rotation.

The only torque about the center of gravity of the rolling sphere is that exerted by the friction, F. We have discovered that the moment of inertia of the sphere is 24.8 pound-foot-seconds^2, therefore the relation $L = I\,\alpha$ becomes

$$F\,2 = 24.8\,\alpha$$

Simplifying both of these equations gives

$$250 - F = 31.2\,\alpha$$

and

$$\mathrm{F} = 12.4\,\alpha$$

Adding them now gives

$$250 = 43.6\,\alpha$$

yielding

$$\alpha = 5.74 \text{ radians/second}^2$$

Since we now have $\omega_0 = 0$, $\theta = 100/2 = 50$ radians (see section 12-7) and $\alpha = 5.73$ radians/second^2, the use of equation (c) of section 12-5 gives

$$\omega^2 - 0^2 = 2\,(5.74)\,(50)$$

and again

$$\omega = 24.0 \text{ radians per second}$$

13-7. Moment of Inertia About Axis Other Than Center of Gravity. In pure rotation, the center of gravity is at rest and rotation takes place about an axis through the center of gravity. It is much more common, however, to have a combination of both types of motion, as for example, in the case of a rolling body. The point of contact between the sphere of the preceding section and the plane is sometimes called the *instantaneous center* and it is often convenient to take the torques and the moment of inertia about this instantaneous center.

The moment of inertia of an object about an axis not passing through its center of gravity may be found by adding the moment of inertia through the center of gravity to an expression formed by multiplying the mass by the square of the perpendicular distance between the two axes.

For example, the moment of inertia of the sphere of section 13-6 about the instantaneous center is

$$\frac{2}{5}\frac{W}{g}r^2 + \frac{W}{g}r^2 \quad \text{or} \quad \frac{7}{5}\frac{W}{g}r^2$$

As another example, find the moment of inertia of a meter stick about an axis perpendicular to the stick and passing through the end of the stick rather than the center. This will be

$$\frac{1}{12}\frac{W}{g}L^2 + \frac{W}{g}\left(\frac{L}{2}\right)^2 \quad \text{or} \quad \frac{1}{3}\frac{W}{g}L^2$$

13-8. Illustrative Problem. Consider again the problem of section 13-6, this time using an axis through the instantaneous center. Again resolve the weight, 500 pounds, as before. Of the four forces that we now have, three pass through the instantaneous center. Therefore the torque is now $A2$ or (250 pounds) (2 feet) or 500 pound-feet. The moment of inertia of the sphere about the instantaneous center is

$$\left(\frac{7}{5}\right)\left(\frac{500}{32}\right)(2^2) \text{ or } 87.5 \text{ pound-foot-seconds}^2$$

Therefore the equation $L = I\,\alpha$ gives

$$500 = 87.5\,\alpha$$

and α is again 5.73 radians/second^2. From this point on, the computation is the same as before.

SUMMARY OF CHAPTER 13

Technical Terms Defined

Moment of Inertia. The moment of inertia of a body about a given axis is the combination of the products of each elementary portion of mass in the body by the square of its distance from the axis.

$I_{\text{flywheel}} = mr^2 = \frac{W}{g}r^2$

$I_{\text{cylinder}} = I_{\text{disk}} = \frac{1}{2}mr^2 = \frac{1}{2}\frac{W}{g}r^2$

$I_{\text{hollow cylinder}} = \frac{1}{2}m\left(r_1^2 + r_2^2\right) = \frac{1}{2}\frac{W}{g}\left(r_1^2 + r_2^2\right)$

$I_{\text{sphere}} = \frac{2}{5}mr^2 = \frac{2}{5}\frac{W}{g}r^2$

$I_{\text{rod}} = \frac{1}{12}mL^2$

$I_{\text{any axis}} = I_{\text{center of gravity}} + mx^2$ where x is the perpendicular distance from the center of gravity to the new axis.

Sum of the Torques $= I\alpha$. Both torques and moment of inertia must be about the axis through the center of gravity or both must be about the instantaneous center (if there is one). Contrast this limitation of axes to either center of gravity or instantaneous center in the case of acceleration with the possibility of using any axis whatever in cases of equilibrium (zero acceleration).

Kinetic Energy of Rotation. $= \frac{1}{2} I\omega^2$.

PROBLEMS

13-1. A boy turns a wheelbarrow upside down so that the wheel is free to turn, ties a rope to a spoke of the wheel (moment of inertia = 0.4 pound-foot-seconds^2), and winds up the rope on the axle (which has a diameter of 2 inches and turns with the wheel). If the boy exerts a pull of 25 pounds and the rope is 4 feet long, how fast does he get the wheel to turning?

13-2. A 64-pound cylinder, 1 foot in diameter, is free to rotate on its axis and has a cord wrapped around its circumference on which a force of 16 pounds is applied. (1) Show that the moment of inertia of the cylinder is 0.25 pound-foot-seconds^2. Compute (2) the accelerating torque, (3) the angular acceleration, (4) the angle turned through in 2 seconds from rest, (5) the angular speed at the end of 2 seconds, and (6) the length of cord unwrapped in the 2 seconds.

13-3. An unknown weight suspended by a cord in which the tension is 16 pounds has a downward acceleration of 16 feet/second^2. Find the weight. Would this weight, hanging on the cord mentioned in the preceding problem, give the cylinder an angular acceleration greater or smaller than that of 13-2 (3)?

13-4. What weight hanging on the cord wrapped around the cylinder of problem 13-2 will descend 12.8 feet from rest in 2 seconds? What is the tension in the cord?

13-5. (1) Find the moment of inertia of an 800-gram cylinder, 10 centimeters in diameter. If the cylinder is free to rotate on its own axis, find (2) the weight which, hanging on a cord wrapped around the cylinder, will descend 90 centimeters from rest in 3 seconds. Compute (3) the linear acceleration of the weight, (4) the angular acceleration of the cylinder, (5) the necessary torque, and (6) the tension in the cord.

13-6. Show that the angular kinetic energy of a rolling cylinder is half as large as the linear kinetic energy.

13-7. Show that if a cylinder slides down a smooth inclined plane and then rolls down another plane just like it except that the second plane is sufficiently rough to cause rolling, the ratio of the sliding speed to the rolling speed is 3/2 at the foot of the plane.

13-8. Solve a problem like that of section 13-6 except that a cylinder is substituted for the sphere.

13-9. Using the data of section 13-6, compute the frictional force, F.

13-10. If the friction between the sphere and plane in section 13-6 is just enough to cause rotation without slipping will any heat be generated? Give reason for answer.

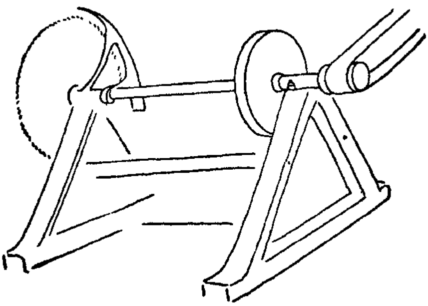
13-11. Collect all the equations dealing with rotation, and pair off each equation with another one similar to it dealing with translatory motion.

Then pair off each physical quantity met in rotation with the corresponding quantity in translation, e.g., moment of inertia corresponds to inertia.

13-12. A bridge table two and a half feet square and two feet high, with center of gravity in the center of the top and weighing 10 pounds, is dragged up a smooth 30-degree incline with an acceleration of 3.2 feet per second^2. If the force is exerted, parallel with the incline, on the table top, solve for this force, also for the normal forces on each leg.

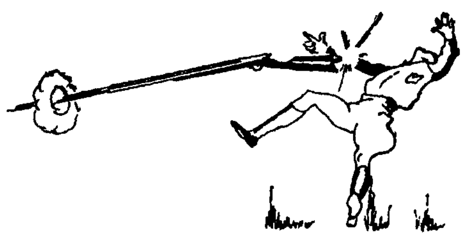
13-13. Repeat problem 13-12, this time assuming a coefficient of friction of 0.2. Is it permissible in these two problems to take the moments about *any* point?

CHAPTER 14



Conservation Laws

14-1. General Survey of the Field of Mechanics. We have nearly completed manufacturing physical concepts by the process of multiplication and division. After this chapter the task becomes the application of principles we have developed to special situations such as oscillation and wave motion. Newton's laws have served as the guiding principle throughout; we have also seen how the third law of Newton gave rise to the law of conservation of energy. In the present chapter, we shall examine another conservation law that also grows out of Newton's third law and pay our last respects to a deceased conservation law. These generalizations serve to bind together the seemingly heterogeneous parts of mechanics; we shall also find that the principle of energy pervades all the rest of physics as well.



14-2. Impulse and Momentum. The two physical quantities that remain to be defined are *impulse* and *momentum*. *Impulse* is the product of the sum of the forces acting and the time during which it acts. *Momentum* is the product of a mass, (W/g), and its velocity. Both impulse and momentum are vector quantities, and both are expressed in the same units. Furthermore we can show that when an impulse Ft acts, the *change* in momentum is equal to the impulse. For the sake of simplicity, let us assume that everything occurs along the same straight line; in such a case, vector addition and subtraction become identical with algebraic addition and subtraction. Newton's second law states that the sum of the forces is equal to the product of the mass acted upon and the acceleration produced, that is $F = (W/g)a$. Assuming as usual that the acceleration is uniform, $a = (v - u)/t$, so that $F = (W/g)(v - u)/t$. Multiplying both sides of this equation by t, we obtain

$$Ft = \frac{W}{g}v - \frac{W}{g}u$$

$(W/g)v$ may be called final momentum and $(W/g)u$, initial momentum, so that the difference is the change in momentum, and we have proved it equal to the impulse. Let us examine the units. Ft is naturally expressed in pound-seconds. Mass is expressed in pound-seconds^2/foot, and velocity is expressed in feet/second. Therefore the product of the mass by velocity will have the units (pound-seconds^2/foot) $\times$ (feet/second) or, cancelling, pound-seconds.

14-3. Illustrative Problem. A 500-gram body slides down a smooth plane inclined 30 degrees to the horizontal. Compute (1) the time required to move 980 centimeters from rest, (2) the impulse acting on the body during that time, (3) the momentum gained, and (4) the final velocity.

(1) As in figure 14-1 the weight, 500 grams, must be resolved into forces parallel and perpendicular to the inclined plane. Since the angles are 30, 60, and 90 degrees, the component parallel to the plane will be 250 grams

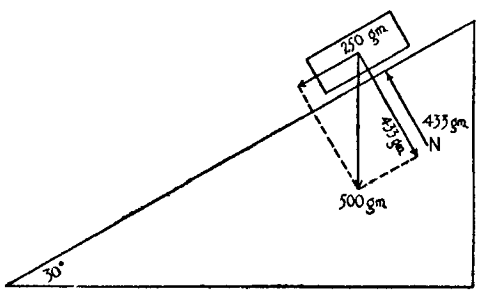


Figure 14-1.

and the component perpendicular to the plane will be 433 grams. The latter will just be balanced by the normal force, and therefore need not be further considered. Since there is no friction, the sum of the forces in the direction of the possible motion reduces to 250 grams. We can substitute enough numerical values into the equation $F = (W/g)a$ to solve for the acceleration. Substituting, we have $250 = (500/980)a$. Solving, we have $a = 490$ centimeters/second.[2] Using equation (d) of section 9-7, that is, $s = ut + \frac{1}{2}at^2$, we have $980 = (0)(t) + (1/2)(490)t^2$. Solving, we have $t = 2$ seconds.

(2) Since the impulse is Ft, we have immediately impulse = (250) (2) or 500 gram-seconds.

(3) Since the momentum gained is the same as the impulse, the answer is again 500 gram-seconds. The initial momentum in this case was zero gram-seconds, therefore the gain in momentum represents the final momentum.

(4) The final momentum is equal to $(W/g)v$. We therefore have $500 = (500/980)v$. Solving this equation for v gives us $v = 980$ centimeters per second. We may also obtain the final velocity by using equation (a) of section 9-7. That is, $v = u + at$, or in this case, $v = 0 + (490)(2)$ or again, $v = 980$ centimeters per second.

14-4. Conservation of Momentum. Momentum is transferred from one object to another without any gain or loss, so that we have what is known as the law of *conservation of momentum.* So far, no exceptions have been found to this law. Let us illustrate it with an example. Assume two elastic balls, No. 1 and No. 2, rolling along a smooth horizontal table, both in the same straight line, with No. 2 behind and gaining on No. 1. When No. 2 reaches No. 1, there is a collision which results in speeding up No. 1 and slowing down No. 2. We are interested in eight physical quantities which we shall designate with letters as follows. The masses of No. 1 and No. 2 will be represented by m_1 and m_2 respectively. Their initial velocities will be u_1 and u_2; and their final velocities will be v_1 and v_2. While the two elastic balls are in contact, No. 2 is exerting an average forward force on No. 1 equal to F for a short time, t. By Newton's third law, No. 1 is meanwhile exerting an equal and opposite force backward on No. 2 or $-F$ for the same length of time, t. The forward impulse on No. 1 is therefore Ft and the backward impulse on No. 2 is $-Ft$. In accordance with the equation derived in the previous paragraph

$$Ft = m_1v_1 - m_1u_1$$

and

$$-Ft = m_2v_2 - m_2u_2$$

If we change all the signs of the second equation, the left-hand side will become the same as the left-hand side of the first equation, therefore the right-hand sides can then be equated to each other, giving

$$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

Now transpose the negative terms so as to make everything positive

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

or in words, the total final momentum is the same as the total initial momentum, therefore during the collision or in general, on any occasion when no outside forces act on the system, momentum is neither lost nor gained. The equation is often expressed as

$$\Sigma\ (mv) = \Sigma\ (mu)$$

The character Σ (sigma) is often used by mathematicians to mean "the sum of." Therefore the law of conservation of momentum follows from Newton's second and third laws.

14-5. Conservation of Angular Momentum. Since exactly the same arguments may be made for rotating bodies, we can define *angular impulse* as the sum of the torques multiplied by the time Lt, and *angular momentum* as moment of inertia multiplied by angular velocity ($I\,\omega$); furthermore we can write

$$Lt = I\omega - I\omega_0$$

and derive from this the law of conservation of angular momentum which holds when no outside torques are acting

$$\Sigma\ (I\,\omega) = \Sigma\ (I\,\omega_0)$$

14-6. Illustrations. Consider the linear momenta involved in the case of the discharge of a gun. Before it goes off, the total momentum is obviously zero. The momentum of the projectile and the momentum of the powder gases are both positive, whereas the momentum of the gun itself after the discharge is negative; these three must add to zero. We often hear the statement made that the momentum of the gun backward is numerically equal to the momentum of the projectile forward; this, of course, is neglecting the momentum of the powder gases. As an illustration of conservation of angular momentum, imagine a boy standing on a piano stool and holding a heavy dumbbell in each hand. If the boy were given an angular velocity with his arms outstretched, and then left to himself, he would maintain this velocity indefinitely if there were no friction. But if the boy were to bend his arms so as to place the dumbbells close to his body, his moment of inertia would be decreased, and as a consequence, his angular velocity would increase in accordance with the law that the total angular momentum remains constant.

14-7. Variation of Mass With Speed. Until recent years it was supposed that mass was the one property of matter that was not subject to change. Any other property that could be named, such as weight, temperature, volume, color, shape, and so on, is subject to change, but it was supposed that the inertia, or mass, of a given object was absolutely a fixed quantity. Then it was discovered, first by theoretical considerations, and later by actual measurement, that when an object, such as an electron or, to go to the other extreme, the planet Mercury, moves with a high velocity, its mass increases somewhat. As an illustration, the planet Mercury has a speed of 36 miles per second when nearest the sun, and when farthest from the sun, its speed is 23 miles per second. Its mass is 3,300,000,000,000 tons *more* at the larger speed than it is at the smaller speed; this is out of a total of about 3.3×10^{20} tons. In recent years the speed of electrons has been computed to be 184,000 miles per second at times, and in these cases, the masses are always more than they are when the electrons are at rest. Light travels with a speed of 186,000 miles per second; material particles have never been observed to travel with a speed greater than this. Since it seems likely that the speed of light is actually the maximum possible speed for objects in this universe, we may argue somewhat as follows: as an object acquires speed in the neighborhood of 186,000 miles per second, it becomes more and more difficult to accelerate it. More exactly, using Newton's second law, the force necessary to produce unit acceleration increases, until, when the speed of light is reached, no force in the universe can accelerate it; that is, the necessary force becomes infinite. Since the ratio of the force to the acceleration is the mass, or inertia, then we have to say that the mass increases to an infinite value when the speed increases to 186,000 miles per second. But for any of our ordinary engineering projects, this increase of mass is absolutely negligible.

14-8. "Law of Conservation of Mass" No Longer Held To Be True. From the facts given in the preceding paragraph, we see that mass may be increased and decreased. This statement already contradicts the so-called "law of conservation of mass" which used to be found in textbooks a few years ago. In 1933, matter was first observed actually to be created out of "radiant energy," and converted back again into radiant energy, and this was theoretically predicted as a possibility several years previous to that. But in spite of these exceptions, we may still say that the amount of water in the universe seems to be *nearly* constant. Furthermore it is to be noted that matter itself must now be added to the list of forms of energy. An atom bomb represents a direct conversion of matter into energy.

14-9. Conservation of Energy. Another conservation law (which apparently has no exception) has already been discussed (see section 3-3), namely the law of conservation of energy, which now includes the defunct "law of conservation of mass." No further treatment of it is necessary at this point; it is mentioned here merely to make the list of conservation laws complete.

SUMMARY OF CHAPTER 14

Technical Terms Defined

Impulse. Product of the unbalanced force acting on a body by the brief time during which it acts.
Momentum. Product of mass of a body by its velocity.

Laws

The momentum of a system remains constant when no outside forces act.
The angular momentum of a system remains constant when no outside torques act.
Mass is now known to be merely another form of energy.

PROBLEMS

14-1. A 15-pound unbalanced force acts on a 48-pound body and moves it 20 feet from rest. Compute (1) the final speed, (2) the final momentum, and (3) the impulse which caused this momentum.

14-2. A 64-pound body is moving along a horizontal surface with an initial speed of 20 feet/second. It slows down under the action of friction alone. The coefficient of friction is 0.25. Compute (1) its initial momentum, (2) its momentum after it has moved 24 feet, and (3) the change in momentum. (4) Compute the impulse directly from the force and the time and compare (4) with (3).

14-3. How far will a body fall in one second from rest? What will its velocity be at the end of that second? Compute the loss in potential energy of a 32-pound body falling one second from rest, and the kinetic energy gained during that second. This is assuming that there is no air resistance. When air resistance is actually considered, the body does not fall so far in the second nor gain as much speed. Under the new conditions, is the gain in kinetic energy equal to the loss in potential energy? Why?

14-4. A watch spring is wound, thus storing potential energy. The spring is then dissolved in acid. What becomes of the potential energy?

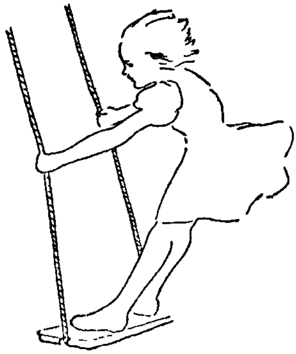
14-5. A 32-pound body has an initial velocity of 24 feet per second up a smooth plane inclined 30 degrees to the horizontal. Compute (1) the initial kinetic energy, (2) the amount of potential energy into which this kinetic energy could be converted, (3) the vertical height to which the body would rise in acquiring this potential energy, and (4) the distance along the inclined plane which corresponds to this height. Does this agree with the distance along the plane computed from the initial velocity and deceleration?

14-6. In problem 14-2, calculate the initial kinetic energy. What becomes of this energy? Calculate the work done in stopping the 64-pound body. Is this work positive or negative?

14-7. A 2-gram bullet is fired from a 4-kilogram gun; the powder gases weigh 0.6 grams. What was the total momentum before the bullet was fired? What is the total momentum as the bullet and powder gases leave the gun? If the bullet has a muzzle speed of 30,000 centimeters per second and the gun kicks with a speed of 27 centimeters per second, find the velocity of the powder gases.

14-8. If an 8-ounce ball is thrown vertically upward with a speed of 64 feet per second, and potential energy is measured with reference to the height where the ball leaves the thrower's hand, compute the kinetic and potential energies at (1) the point where the ball leaves the thrower's hand, (2) the point where it is one second later, and (3) the maximum height reached.

CHAPTER 15



Simple Harmonic Motion; Simple Pendulum; Compound Pendulum

15-1. Radial Acceleration. In section 10-5 we obtained an expression for the radial acceleration which exists when a body moves uniformly in a circle, and in section 12-7, we expressed this radial acceleration in terms of the number of revolutions per second. As will be remembered, the two expressions were $a = v^2/r$ and $a = 4\pi^2n^2r$. This is one of the two cases in this book in which the acceleration is not constant; to be sure, the magnitude is constant, but the direction varies, being always directed toward the center. We are now about to meet the other variable acceleration, and this time both the magnitude and direction vary.

15-2. Simple Harmonic Motion. If we watch a body that is moving uniformly in a circle in such a way that our eyes are in the plane of rotation, we see the rotation edgewise, and the particle no longer appears to be in uniform circular motion; it appears merely to move back and forth. An illustration of this may be found in the satellites of the planet Jupiter. Four of these satellites are large enough so that persons are occasionally found who can see them with the naked eye. Most of us, however, have to resort to opera glasses to make them visible. These satellites (or moons) run around the planet practically in circles, but since we see these circular orbits edgewise it appears to us that the moons simply go back and forth across the face of the planet to one end of the apparent route, and back behind the planet to the other end of the route. The inner one takes about two days for a round trip and the outer one about sixteen days. "Back and forth" motion of this type is called "simple harmonic motion;" that is, *simple harmonic motion* is uniform circular motion seen edgewise. The mathematician would say that it is the projection of uniform circular motion on the diameter of the circle.

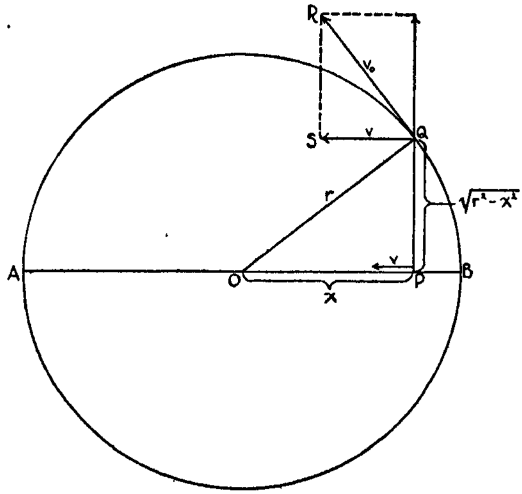


Figure 15-1.

15-3. The Velocity in Simple Harmonic Motion. In figure 15-1, the point Q is moving around the circle counterclockwise with a velocity v_0 which is equal to $2\pi rn$, and the point P is moving in simple harmonic motion back and forth along the diameter. If this motion is viewed from a point in the plane of the circle and at right angles to the diameter, AB, the component of the velocity that will be seen is v, and from the similar triangles QRS and QOP it is seen that $QS/QR = QP/QO$. Replacing these letters with the values from the figure, we have $v/v_0 = (\sqrt{r^2 - x^2})/r$. Since $v_0 = 2\pi rn$, the relation becomes

$$v = \pm 2\pi n \sqrt{r^2 - x^2}$$

The plus and minus sign is necessary because the velocity v may be either to the right or to the left; whereas 2, π, n, and the radical are essentially positive. From this equation it is clear that when the value of x is zero, the point P, which is always directly under Q, coincides with the point O, and the velocity v becomes $2\pi rn$. On the other hand, when the three points Q, P, and B coincide, x becomes r, the radical becomes zero, and therefore the velocity v is zero. From a common-sense standpoint, the point P will have to have a zero velocity at the points A and B, for it is there that it stops and reverses its motion; also the point P should have its maximum velocity at the exact center, and this is consistent with the fact that in the equation, v has its largest value $(2\pi rn)$ when x is zero.

15-4. The Acceleration in Simple Harmonic Motion. In figure 15-2, the acceleration a_0, which is $4\pi^2n^2r$, is shown directed toward the center of the circle from the point Q. But when the motion is viewed edgewise, it is the component of a_0 parallel to BA that becomes important. From the similar triangle relationship we have $a/a_0 = x/r$, and since $a_0 = 4\pi^2n^2r$ the equation reduces to

$$a = -4\pi^2n^2x$$

We see therefore that at the center, when $x = O$, that there is no acceleration; on the other hand we have a maximum value of the acceleration at A and at B, when $x = r$. This acceleration, like the

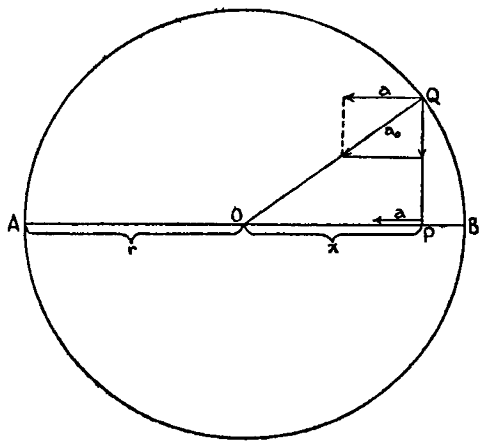


Figure 15-2.

centripetal acceleration, is always directed toward the center; therefore when x is on the right of O, the acceleration is toward the left, and when x is on the left, the acceleration is toward the right; in other words, a always has the opposite sign from that of x, hence the minus sign in the equation.

15-5. Technical Terms Associated With Simple Harmonic Motion. In figures 15-1 and 15-2, almost every quantity in the diagrams has a technical name. x, the distance between O and P, is called the *displacement*. It is positive when measured to the right and negative when measured to the left. The maximum value of x is r, the radius of the circle; but r is called the *amplitude*, in simple harmonic motion, and is always considered positive. The angle POQ is called the *phase angle*, and varies from zero to 360 degrees. If we have two points, P and P', which correspond to two angles such that there is a constant difference between the angles of, say, 90 degrees, then we say that P and P' are 90 degrees out

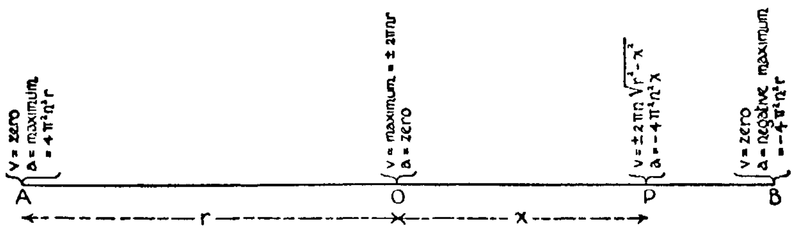


Figure 15-3.

of phase with each other. n is called the *frequency*; it is the number of round trips made in unit time. The reciprocal of the frequency is the *period* or the time necessary for one round trip; we shall call it T. We could therefore rewrite our two equations in terms of T instead of n, thus

$$v = \pm (2\pi \sqrt{r^2 - x^2})/T$$

and

$$a = -(4\pi^2 x)/T^2$$

which when solved for T is

$$T = 2\pi\sqrt{-x/a}$$

15-6. Force in Simple Harmonic Motion. We can apply Newton's second law to find the force necessary to produce simple harmonic motion.

$$F = \frac{W}{g} a$$

therefore

$$F = -\frac{W}{g} 4\pi^2 n^2 x$$

Put into words: whenever the force is proportional to the displacement but opposite in sign, it will produce simple harmonic motion.

We remember that in the case of an elastic body, since the stress is proportional to the strain (Hooke's law, section 6-5), in any particular case the stretching force, F, is numerically proportional to the stretch, x, and of the same sign. By Newton's third law, however, the elastic body will exert a restoring force which is numerically equal to, but the negative of, the stretching force; therefore the elastic body will cause whichever body is doing the stretching to tend to execute simple harmonic motion. We are therefore prepared to find that a weight suspended by a helical spring oscillates vertically nearly in simple harmonic motion. In the case of a given spring or other elastic body where W, g, 4, π, and n are all constant, the equation relating the force and the stretch may therefore be written

$$F = -kx$$

15-7. Illustrative Problem (1). Imagine a light, stiff, horizontal rod, with one end clamped in a vise, and a weight of 64.4 pounds, securely fastened to the other end of the rod. The stiffness of the rod is such that when the weight is pushed to one side a distance of 2.4 inches and released, it makes just two vibratory round trips per second. Find the speed of the weight as it passes through the central part of its path, the acceleration as it passes through the end of its path, the force necessary to displace the end of the rod 2.4 inches, and the force necessary to displace the end of the rod 4.8 inches.

By Hooke's law, the force will be proportional to the displacement, so we immediately draw two conclusions: one is that the weight will execute simple harmonic motion, so that our velocity and acceleration equations will hold good; the other is that the fourth answer to the problem will be twice the third answer. It is one of our simplifying assumptions that none of the energy of the vibrating rod will be converted into heat. A real rod will execute approximately simple harmonic motion, but not exactly. The rod will gradually come to rest and become slightly warmer. The correct name for the phenomenon, as it actually takes place, is *damped* harmonic motion. In *simple* harmonic motion the amplitude remains the same and the oscillations continue indefinitely. Assigning letters to our data, we have $r = 2.4$ inches or 0.2 feet, $n = 2$ per second, and $x =$ zero for the central part of the path and 0.2 feet for the end of the path. The velocity is therefore

$$\pm (2)\ (3.14)\ (2)\ \sqrt{0.2^2 - 0^2} \text{ or } \pm 2.51 \text{ feet per second}$$

It was however the speed but not the velocity asked for, so that we may discard the plus and minus sign, being indifferent as to the direction of the motion. The acceleration is $-$ (4) (3.14^2) (2^2) (0.2) or $-$ 31.6 feet/second^2. We have here taken the value of x on the right-hand end of the path so that the acceleration is toward the left and therefore negative. The force is equal to $(W/g)a$. W is 64.4 pounds, $g = 32.2$ feet/second^2, and $a =$

− 31.6 feet/second^2; the force therefore comes out − 63.2 pounds, exerted toward the left on the weight. As the question is worded, however, we wish to know the force necessary to displace the rod; this is equal numerically to the one just found, but opposite in direction or toward the right. If we wish twice the displacement we shall need twice as much force, or 126.4 pounds.

15-8. Illustrative Problem (2). It is found that a force of 126.4 pounds will displace one end of a rod 0.4 feet when the other end is clamped in a vise. If a weight of 64.4 pounds is attached to the free end and displaced 0.2 feet from the position of rest, what will be the frequency and the period of vibration? We now start by saying that since the forces must be proportional to the displacement in order to produce vibrations of the simple harmonic type, the force that corresponds to 0.2 feet is found by solving the proportion $F/126.4 = 0.2/0.4$, which gives us $F = 63.2$ pounds. The force that the rod exerts on the weight will be the negative of this. If now we substitute in the equation

$$F = -\frac{W}{g} 4\,\pi^2 n^2 x$$

the equation will become $-63.2 = -(64.4/32.2)\,4\,(3.14)^2\,(n)^2\,(0.2)$. Solving, we find that n is 2 round trips per second. The period of vibration is the reciprocal of the frequency and is in this case $\frac{1}{2}$ or 0.5 second per round trip. It is now clear that we did some unnecessary work when we stopped to find the force that went with the 0.2 feet because the use of the original displacement (0.4 feet) and the original force (126.4 pounds) in the equation would have given us the same value of n. That is, so long as Hooke's law holds, *the period of vibration of a given system is the same whether the amplitude is large or small.*

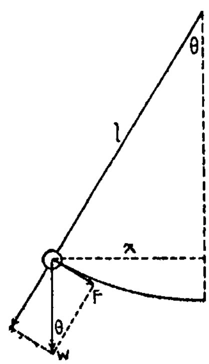


Figure 15-4.

15-9. The Simple Pendulum. A pendulum bob will almost obey the laws of simple harmonic motion if the vibrations are small. Consider the so-called simple pendulum shown in figure 15-4. It consists of a weightless inextensible string of length l, and a bob which has no volume but has a weight W. This will remind the reader of the fictitious object of which we determined the moment of inertia in section 13-2. Drop a perpendicular from the bob to the line of the vertical and call the perpendicular distance x, the displacement. The weight is always a force vertically downward; we therefore resolve it into two components, one which merely tends to stretch the string and which will not further concern us since this string of ours will not stretch, and the other component F, tangent to the arc which the bob swings through. Due to the similar triangle relationship, we can say $x/l = F/W$. Therefore $F = Wx/l$, that is, the force is proportional to x. But since F and x always have opposite senses, the equation to be complete must contain a minus sign, $F = -Wx/l$. If the vibrations are small, then F is *nearly* in line with x, and the pendulum will *approximately* execute simple harmonic motion. This means that F is nearly equal to $-(W/g)4\pi^2n^2x$ as well as being exactly equal to $-Wx/l$. Equating these two expressions for F and cancelling the minus signs, we have $Wx/l = (W/g)4\pi^2n^2x$. Cancelling W and x and multiplying through by gl

$$g = 4\pi^2n^2l$$

This gives us a convenient method of determining the acceleration of gravity, it being necessary merely to know the length of the pendulum, l, and the number of round trips, n, that the pendulum makes per second. We can replace n by $1/T$, since n and T are reciprocals. g is therefore also equal to $4\pi^2l/T^2$. If this equation is solved for T, it gives us

$$T = 2\pi\sqrt{l/g}$$

15-10. Illustrative Problem. A seconds pendulum keeps correct time at a certain temperature. If, as a result of a rise in temperature, its length increases by 0.02 per cent, how many seconds will it now lose per day?

The length of a seconds pendulum may be found by substituting into the equation $g = 4\pi^2l/T^2$, the values $g = 32.2$ feet/second², and $T = 2.00$ seconds. The period of a seconds pendulum is the time necessary for the pendulum to make a round trip, and since the pendulum ticks at one-second intervals, once on the way over and once on the way back, the total time for the round trip is two seconds. Thus the equation becomes

$$32.2 = \frac{4(3.14)^2l}{(2.00)^2}$$

Solving, we obtain $l = 3.26$ feet. If the increase of length is to be 0.02 per cent or two parts in ten thousand, then we must multiply 3.26 by 0.0002 to obtain the elongation. This gives us 0.000652 feet. An interesting point comes up in connection with the question of adding this elongation to the original length. Since we are working to slide-rule accuracy only, we are not at liberty to say that the unchanged length is known to be 3.260000 feet and therefore we cannot say that the changed length is 3.260652. But it is true that *if* the original length had been 3.260000 feet exactly, *then* the increased length would have been exactly 3.260652; at least the length has been increased in that proportion. Therefore, at this point the student has his choice of recomputing the problem with seven significant figures or of using an algebraic method; we shall show how it works both ways.

First, let us go back to the beginning and use seven significant figures in our computations. Since g is not known experimentally to that degree of precision, let us assume that $g = 32.20000$. T is now 2.000000, and $\pi = 3.141593$. The equation is now

$$32.20000 = \frac{(4.000000)\,(3.141593)^2\,l}{(2.000000)^2}$$

Solving, we obtain $l = 3.262542$ feet. Three figures of the elongation are enough, therefore we already have the elongation, $e = 0.000652$, and adding, we find that the new length is 3.263194. We now need the new value of T to go with our new length. Calling this T' the equation now becomes

$$32.20000 = \frac{(4.000000)\,(3.141593)^2\,(3.263194)}{(T')^2}$$

Solving, we obtain $T' = 2.000200$ seconds. In a day there are 86,400 seconds, therefore our clock will normally have time for 86,400/2.000000 or 43,200.00 periods. But under the conditions of the problem there will be time for only 86,400/2.000200 or 43,195.68 periods, a difference of 4.32 periods. Since the clock registers each period as two seconds, it will lose 8.64 seconds per day.

Let us now solve the problem again by an algebraic method which makes unnecessary so many significant figures. This method depends upon the fact that a number like 1.002 squared becomes 1.004004 which rounds off to 1.004 when reduced to slide-rule accuracy. That is, if x is small compared with unity, then x^2 will be negligible and we have $(1 + x^2) = 1 + 2x$ approximately, which we shall call case (1). Similarly, the square root of $1 + 2x$, which is written algebraically $(1 + 2x)^{0.5}$, is approximately equal to $1 + x$ [case (2)]. It is also true that $(1 + x)\,(1 - x) = 1 - x^2$, but since x^2 is negligible, we may write approximately $(1 + x)\,(1 - x) = 1$, and dividing both sides by $1 - x$ gives us case (3), namely, $1 - x = 1/(1 + x)$, or $(1 + x)^{-1} = 1 - x$. All three of these expressions may be included under one approximate equation, true only when x is small in relation to unity, namely

$$(1 \pm x)^n = 1 \pm nx$$

In the three cases just cited, the plus-or-minus sign is plus, and n has the three values, 2, $\frac{1}{2}$, and -1 respectively. Starting then from the point where the elongation was computed, we have the new length equal to $3.26 + 0.000652$, which may be written $3.26(1 + 0.000200)$. The process of solving for $(T')^2$ gives us the equation

$$32.2 = \frac{(4.00)^2\,(3.14)^2\,(3.26)\,(1 + 0.000200)}{(T')^2}$$

which gives us $(T')^2 = (4.00)\,(1 + 0.000200)$ and $T' = (2.00)\,(1 + 0.000100)$ by case (2). Dividing 86,400 by 2.00 gives us 43,200, and dividing 86,400 by $(2.00)\,(1 + 0.000100)$ gives $(43{,}200)\,(1 - 0.000100)$ by case (3), which may be written $43{,}200 - 4.32$. We therefore again have a difference of 4.32 periods or 8.64 seconds, the amount that the clock will lose per day.

15-11. The Physical or Compound Pendulum. A simple pendulum is obviously fictitious; it is impossible in practice to have a weightless string and a volumeless mass. But a real pendulum can always be found that will have the same period as any given simple pendulum, and we call this real pendulum a *physical pendulum* or a *compound pendulum.* Such a pendulum is shown in figure 15-5. S is the point of suspension, C is the center of gravity, and O is a point called the center of oscillation. The distance from S to O is the same as the length of a simple pendulum which would have the same period; in fact, the method of locating O is to measure down from S the computed distance l. It will be shown in the next section that the moment of inertia I, of the compound pendulum about the axis S is mhl, where h is the distance from the point of suspension to the center of gravity, so that l in the last equation of section 15-9 may be replaced by I/mh. This allows us to express the period T, as $2\pi\sqrt{I/mgh}$. We are able to determine variations in g by noting variations in the period of an actual pendulum at different locations.

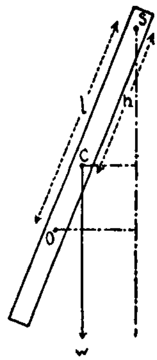
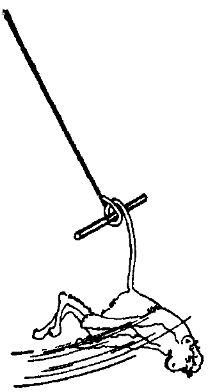


Figure 15-5.

The center of oscillation is sometimes called the *center of percussion.* If the object shown in figure 15-5 were a baseball bat and were grasped at the point S, the point O should be the best place for the baseball to meet the bat to obtain a satisfactory hit. If the ball strikes the bat between the points O and S, the batter's hands will be driven backward, and if the ball strikes on the other side of O, the batter's hands will be driven forward, but no jar at all will be felt when the contact is made at the point O.



15-12. Derivation of Fundamental Equation of the Compound Pendulum. In order to show that $I_s = mhl$, when I_s is the moment of inertia of the compound pendulum about the point of suspension S, and m is the mass W/g. of the pendulum with center of gravity at C, make the following assumptions. Let the angle between the axis of the compound pendulum in figure 15-5 and the vertical direction be the same as θ in figure 15-4; let the pendulum bob of the simple pendulum of figure 15-4 have the same mass (and weight) as the entire compound pendulum of figure 15-5; furthermore the length l, of the simple pendulum has been adjusted so that it will have the same period T, as the compound pendulum. The two pendulums will therefore be moving in the same way at all times and in particular, at a given instant, they will have the same angular acceleration about their respective points of suspension. We shall therefore set up the equation $L = I\alpha$ for each pendulum. The two torques will be different; in figure 15-4 it is Wx, while in figure 15-5 it is less, since the perpendicular dropped from C to the vertical line through S is h/l of the distance x. Thus, if we call L the torque of figure 15-4, we may express the other as hL/l. The moment of inertia of the simple pendulum is ml^2 since all the mass is at the lower end of l. Thus the values of $L = I\alpha$ become respectively

$$L = ml^2\,\alpha$$

and
$$\frac{hL}{l} = I_s\,\alpha$$

Dividing one equation by the other cancels the L's and the α's and gives

$$\frac{l}{h} = \frac{ml^2}{I_s}$$

Cancelling one l and cross multiplying gives us the desired result, $I_s = mhl$.

15-13. Use of Compound Pendulum Equation to Measure Moments of Inertia. We may make use of the relation given in section 13-7 to write

$$I_s = I_c + mh^2$$

Suppose then that it is desired to find the moment of inertia of a given object, of weight W, about its center of gravity. We already know how to locate the center of gravity (see section 8-6). Hang up the body by some other point, the distance of which from the center of gravity is h. Determine the time required to make a given number of complete vibrations, say fifty, from which T may be found. Knowing g at the locality of the experiment, l can be found by using the equation

$$T = 2\pi\sqrt{l/g}$$

I_s may now be found from the relation

$$I_s = Whl/g$$

Finally I_c is obtained from the first equation of this chapter.

15-14. Energy of a Body Executing Simple Harmonic Motion. The kinetic energy of a body executing simple harmonic motion may be readily computed by inserting the expression for its velocity derived in section 15-3 into the formula $\frac{1}{2}mv^2$. Thus we have

$$\tfrac{1}{2}m[4\pi^2n^2(r^2 - x^2)] \text{ or}$$

$$\text{kinetic energy} = 2m\pi^2n^2(r^2 - x^2)$$

Because of the law of conservation of energy, we can state that the total energy of an isolated vibrating body is a constant. When $x = r$, there is no kinetic energy because the velocity is zero. When $x = O$, we have the maximum velocity and at this point all the energy is kinetic. Therefore

$$\text{total energy} = 2m\pi^2n^2r^2$$

The difference between the total energy and the kinetic energy is potential energy. Subtracting the first equation of this section from the second, we have

$$\text{potential energy} = 2m\pi^2n^2x^2$$

A comparison of the two equations of section 15-6 gives

$$k = 4m\pi^2n^2$$

and using this k, we can express the potential energy as $\frac{1}{2}kx^2$. Thus another expression for the total energy of an oscillator is

$$\text{energy} = \tfrac{1}{2}\,mv^2 + \tfrac{1}{2}\,kx^2$$

SUMMARY OF CHAPTER 15

Technical Terms Defined

Simple Harmonic Motion. The projection on the diameter of uniform circular motion.

Displacement. Distance measured in a given direction from standard point taken as origin. In simple harmonic motion, the origin is the center of the path.

Amplitude. Maximum displacement from the center.

Phase Angle. A description of the position of a particle executing simple harmonic motion in terms of the angular position of the corresponding point moving on the circumference of the circle.

Frequency. Number of round trips per unit time.

Period. Time of one round trip.

Simple Pendulum. A volumeless bob suspended from a fixed point by means of a weightless inextensible cord.

Compound or Physical Pendulum. Any physical object suspended from some point other than its center of gravity and allowed to oscillate.

Center of Oscillation or Percussion. A point in a physical pendulum situated below the point of suspension by a distance equal to the length of a simple pendulum of the same period.

PROBLEMS

15-1. In the case of a body executing simple harmonic motion, what is meant by the terms (1) displacement, (2) amplitude, (3) period, (4) frequency, and (5) phase? (6) Which are variable and which are constant?

15-2. Describe simple harmonic motion in such a way as to indicate at what points the velocity and the acceleration have their minimum and maximum values. Is it possible for an object to have simultaneously a zero velocity and an acceleration that is not zero? Illustrate by means of a baseball moving freely under gravity.

15-3. A body oscillates in simple harmonic motion at the rate of three vibrations per second with an amplitude of two inches. Find its speed (1) at the center, (2) one inch from the center, and (3) two inches from the center.

15-4. Find the acceleration of the body in the preceding problem in the three positions mentioned.

15-5. If a 16-pound body is hanging on a helical spring that requires a downward force of 2 pounds to stretch it 3 inches, compute the period of the 16-pound body on the spring when pulled down 3 inches and released. Would the period be the same if the displacement were 2 inches instead of 3 inches? What requirement for simple harmonic motion proves this?

15-6. If the 16-pound body of the preceding problem is replaced by a 4-pound body, compute the period of the resulting simple harmonic motion.

15-7. A 50-gram weight is hanging on a helical spring. Another 50-gram weight stretches the spring 4.9 centimeters more. Compute the period if the two 50-gram weights are slightly displaced and allowed to vibrate.

15-8. What is the length of a seconds pendulum at a place where g is 980 cm./sec.^{2}?

15-9. If a pendulum, the length of which is 246 centimeters, has a period of 3.14 seconds in a certain place, what is the value of g in that locality?

15-10. The reading room of the central library of the Massachusetts Institute of Technology is immediately under the central dome. A long pendulum, consisting of a wire and heavy bob is suspended from the center of the dome to a table top in the center of the room to illustrate Foucault's famous experiment in which he demonstrated the rotation of the earth. It takes the pendulum just nine seconds to make a complete round trip. How long is the pendulum? (Look up Foucault's experiment in a reference book and write a 50-word description.)

15-11. How should the length of a pendulum be changed to halve its period? What is the effect on the period of a pendulum of carrying it up on a mountain where g is smaller than at the lower level?

15-12. If a pendulum clock keeps correct time at a place where the acceleration of gravity is 32.2 feet/second^2, will it gain or lose when taken to a place where the acceleration of gravity is 32.1 feet/second^2? How much will the gain or loss be per day?

15-13. A clock loses 5 minutes per week. What adjustment must be made to its pendulum to make it keep correct time?

15-14. An iron ring has an outer diameter of 10 inches, an inner diameter of 6 inches, and weighs 10 pounds. Find its moment of inertia (1) about its center and (2) about a point on the inner circumference. (3) If this ring is suspended from a peg at the inner circumference, what is the length of a simple pendulum that has the same period? (4) Find the period and frequency of this ring when thus used as a compound pendulum.

15-15. A physical pendulum consists of a lead sphere of specific gravity 11.34 and one inch in diameter, attached by a string, 12 inches long, to a point of support. Find (1) its moment of inertia about the center of the lead sphere, (2) its moment of inertia about the point of suspension, (3) length of the equivalent simple pendulum, (4) period, and (5) frequency. Would it be permissible to assume that (3) is 12.50 inches?

CHAPTER 16

Properties of Waves

16-1. Essential Characteristics of a Wave Transmitting Medium. Water waves represent a rather complicated type of wave, otherwise they would furnish an excellent starting point for this discussion. But all types of physical waves have one characteristic in common: that it is *motion* that is transferred and not matter. If we could watch one drop of water in a water wave, we should find that it goes round and round in a very limited region and does not follow the waves in the direction of propogation. Any medium which transmits a wave must possess at least two properties, elasticity and inertia. The individual particles in a wave execute simple harmonic motion, which implies elasticity in order to bring the particle back after it has been displaced, and also implies inertia sufficient to keep the particle going, after it reaches the equilibrium position, until the particle attains an equal displacement on the other side. If the medium possesses inertia, which is the same thing as mass, then it is possible to talk about its density, and, as it turns out, the speed V, with which the wave will be transmitted, is equal to the square root of the ratio of the elasticity of the medium to its density. That is

$$V^2 = \frac{\text{elasticity of medium}}{\text{density of medium}}$$

16-2. Transverse Waves. We shall find it convenient at the outset, in order to form a mental picture of a wave, to imagine a long block of jelly as in figure 16-1 with particles of sawdust imbedded at equidistant intervals, and the whole at rest. Now let the experimenter imbed his finger in the jelly at the left-hand end and commence to execute a vertical simple harmonic motion with it. Figures 16-2 and 16-3 represent two successive illustrations of the sawdust particles showing something of their consequent motion. In figure 16-2, particle A is at the top of its motion and about to start downward. In figure 16-3 it is still going down and particle B has now reached the top. The same remarks could be made about particles I and J that were made about A and B. The student will find it worth-while to draw the figure which would logically follow figure 16-3 in which particles C and K have risen to the top. It will then be clear that the *shape* of the wave is moving progressively toward the right although the individual particles are merely executing simple harmonic motion. A similar experiment could be carried out with a very long rope fastened at the right-hand end

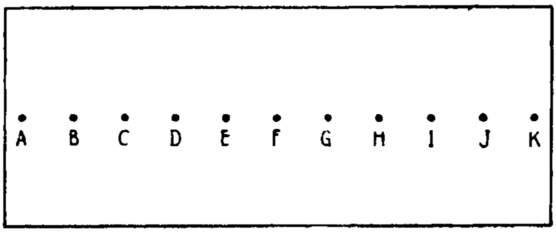


Figure 16-1.

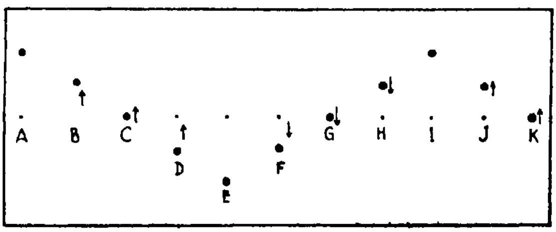


Figure 16-2.

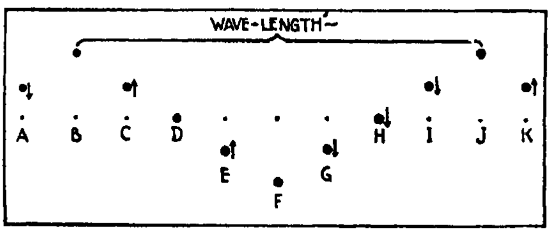


Figure 16-3.

and moved up and down at the left-hand end. When the direction of propagation of the wave is at right angles to the direction of the motion of the individual particles, as in figures 16-2 and 16-3, we say that the wave is *transverse*. From about the years 1800 to about 1925, light was considered to be an example of a transverse wave. We now think that light consists of streams of projectiles called photons, loosely controlled by sets of *non-physical* transverse waves called psi-functions. The "control" which psi-functions exert over photons, electrons, and so on, reminds us of the mathematical probability function which controls the proportion of bullets which will lie within each circle of a target. This topic will be understood when we study the chapter on light.

16-3. Longitudinal Waves. If the experimenter moves his finger in the jelly so as to execute a horizontal simple harmonic motion, a set of particles originally equidistant would soon appear, if a snapshot could be taken of them, as in figure 16-4. This figure will be self-explanatory in view of what has already been said. The individual particles execute simple harmonic motion while the series

of compressions and rarefactions travel from left to right. When the motion of the wave is parallel with the motion of the individual particles, as in figure 16-4, we say that the wave is *longitudinal* or *compressional*. Sound is an example of a compressional wave.

16-4. Technical Terms. All the expressions which were defined in connection with simple harmonic motion, namely, *amplitude, displacement, phase, frequency, period*, are also used in connection with wave motion with the same meanings. It is possible to give additional meanings to some of them. For instance, the *period* is not only the time necessary for one of the particles to make a complete round trip, but it is also the time that it takes the wave to travel from a particle such as A, to the next particle that is going through the same motion at the same time, which is I in this illustration. The distance from A to I is called a *wave length*, and is represented by the Greek letter λ (lambda). Therefore we could have defined the period as the time necessary for the wave to travel

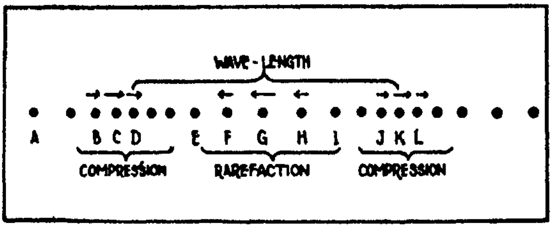


Figure 16-4.

one wave length. The *intensity* of a wave is proportional to the product of the square of the amplitude and the square of the frequency. The *speed* of the wave is the ratio of the wave length to the period, that is, $V = \lambda/T$. Since $n = 1/T$, we have also

$$V = \lambda n$$

16-5. Reflection. In section 16-2 we specified a long rope and a long block of jelly to prevent reflection from the further end. Waves are always reflected when they reach a boundary between two mediums. This statement is true whether or not the wave finds it possible to enter the new medium. If it can, then the energy is divided between the reflected wave and the wave that passes on. The path of a wave is called a *ray*. If the ray meets the boundary at right angles, it is reflected straight back upon itself; if not, the situation is as shown in figure 16-5. The angle θ, between the incident ray and the normal, must be the same as the angle θ', be-

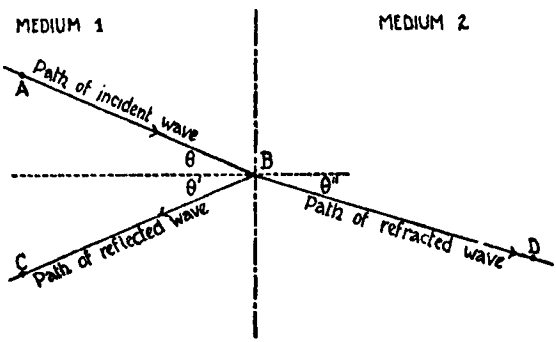


Figure 16-5.

tween the reflected ray and the normal. If the boundary is a rough surface, rather than a smooth plane surface, then at each small portion of the surface the law of reflection will hold true, but the total effect will be to produce rays going in all directions. We call this a case of *diffuse reflection.*

16-6. Refraction. The wave that passes on into the new medium is said to be *refracted.* Although it follows the same *general* direction as the incident ray, it will not be in *exactly* the same direction. The angle θ'', will be different from θ. The speed of the reflected wave is the same as that of the incident wave because both waves are in the same medium. But the speed V'', of the refracted wave, will in general be different from that of the incident wave V, because the medium is different. The following proportion holds true connecting the speeds and the angles

$$\frac{V}{V''} = \frac{\sin\theta}{\sin\theta''}$$

(The sine of an angle is defined in appendix 6.) This equation may be made evident as follows. In figure 16-6 the wave reflected is omitted for the sake of simplicity (although actually it is always present) and the incident and refracted waves are both given a finite breadth. The lines BD and FE are called "wave fronts" technically (although they are not the fronts of waves); a wave travels at right angles to its wave fronts except under circumstances so unusual that we shall not discuss them here. A *wave front* may

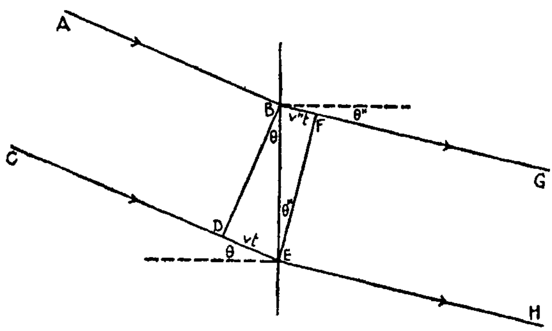


Figure 16-6.

be defined as a surface containing points all in the same phase at the same time. If anything happens to change the direction of the wave fronts, it will automatically change the direction of the ray. Assume the speed of the wave in medium 1 to be V and the speed in medium 2 to be V''. One edge of the wave travels the distance DE in the same time the other edge takes to travel the distance BF. If t represents this time, then $DE = Vt$ and $BF = V''t$. If BE is called k, then sin θ is Vt/k and sin θ'' is $V''t/k$. (See appendix 6.) If we divide sin θ by sin θ'', the k's and t's will cancel and we shall obtain

$$\frac{\sin \theta}{\sin \theta''} = \frac{V}{V''} = \mu$$

This ratio μ, is known as the index of refraction.

16-7. Illustrative Problem. In shallow water the speed of a water wave depends, among other things, upon the depth of the water. If a water wave suddenly passes from water eight feet deep to water two feet deep, its velocity will drop from 16 feet/second to 8 feet/second. If the path of the water wave makes an angle of 30 degrees with the normal to the boundary between the two depths while in the deeper water, find the corresponding angle in the shallow water. Find also the index of refraction.

Substitute in the equation, $\sin \theta/\sin \theta'' = V/V''$, letting $\theta = 30$ degrees, $V = 16$ feet/second, and $V'' = 8$ feet/second. From appendix 7 we see that $\sin 30° = 0.500$, so that the equation becomes

$$\frac{0.500}{\sin \theta''} = \frac{16}{8}$$

Solving, we obtain $\sin \theta'' = 0.250$. From appendix 7, we see that θ'' is slightly less than 15 degrees. A more complete table of sines or a slide rule will show that θ'' is 14° 29′. The index of refraction $\mu = V/V'' = 16/8 = 2.00$

16-8. Diffraction. Although water waves travel in nearly straight lines in a given medium, the waves have some tendency to bend around corners. The longer the wave length, the greater is this tendency; it may easily be observed in the case of water waves. This tendency of waves to bend around corners is called *diffraction.* In practice, in a diffraction experiment, a wave is made to go through a narrow opening in which case there will be two corners for it to bend around, and the amount of diffraction will, in addition to being proportional to the wave length λ, also be inversely proportional to the width of the opening w. The equation is

$$\sin \theta = \frac{\lambda}{w}$$

where the angle θ, represents the maximum deviation from the original direction. Rays will be present with all possible deviation angles between θ and zero.

16-9. Interference. Waves have one property that is not shared by any other type of motion; this property is called technically *interference.* If two waves are traveling in nearly the same direction so that they are able to cross each other's paths, the vibrations of one wave will add algebraically to the vibrations of the other wave. The two waves may be out of phase with each other by any number of degrees. If they happen to be 180 degrees out of phase with each other and have the same amplitude, they will completely neutralize each other at the point where they cross. On the other hand it is possible for the two waves to be in phase with each other (0 degrees phase difference) in which case they reinforce each other. There is no loss of energy during interference, but merely a redistribution. Examples of this phenomenon will be found when we discuss sound and light.

16-10. Polarization. The waves pictured in figures 16-2 and 16-3, also the wave motion in the rope, that is, transverse waves, require two dimensions for their description; we may think of them as existing in planes. When a collection of transverse waves traveling in the same direction exist in parallel planes, we say that the waves are *polarized.* A group of transverse waves could travel in the same direction and lie in such planes that no two planes are parallel to each other. Such a group would be described as completely unpolarized. Longitudinal waves cannot be polarized.

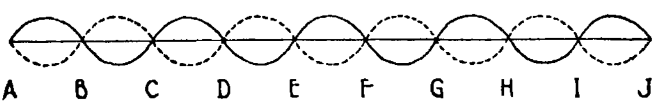


Figure 16-7.

16-11. Stationary Waves. If we should continue our experiment with the vibrating rope (section 16-2) long enough to allow the reflected wave to combine with the incident wave, it would be observed that certain points on the rope remain stationary. These points are called nodes, and are represented in figure 16-7 by the points A, B, C, D, E, and so on. The points between these nodes move up and down, and in figure 16-7 three different positions of the rope are shown, the straight line, the full curved line, and the dashed curved line. A point half way between two nodes is called an *antinode.* The distance from a node to the second node beyond, for example, from A to C, is a wave length. This combination of two wave motions going in opposite directions is called a *stationary wave.*

SUMMARY OF CHAPTER 16

Technical Terms Defined

Wave. A vibrational disturbance propagated through an elastic medium.

Transverse Waves. Waves in which the motion of the individual particles is perpendicular to the direction of propagation of the wave.

Longitudinal or Compressional Waves. Waves in which the motion of the individual particles is parallel to the direction of propagation of the wave.

Wave Length. Distance along the axis of the wave from a given particle to the next one that is in phase with it.

Period. Time required for a wave to travel one wave length.

Reflection. The reversal of the general direction of a wave upon meeting a surface boundary separating two different mediums.

Refraction. The slight change in the direction of a wave as it passes through a surface separating two different mediums.

Wave Front. A continuous surface in a vibrating medium which contains a set of points all in the same phase at the same time.

Index of Refraction. A constant characteristic of a medium. It represents the ratio between the velocity of a wave in a standard medium and the velocity of the wave in the given medium.

Diffraction. Bending experienced by waves while passing an edge or especially while passing through a slit.

Polarization. The removal from a set of transverse waves of all except those with the vibrations in a given direction.

Stationary Waves. A combination of a wave with its reflection. The effect is to produce regions of no vibration called nodes, half a wave length apart.

PROBLEMS

16-1. The velocity of a certain wave is 1,150 feet per second and its frequency is 440 vibrations per second. What is its wave length?

16-2. The wave length of a certain wave is 0.0000589 centimeter, and its velocity is 30,000,000,000 centimeters per second. What is its frequency?

16-3. Figure 16-6 is called a Huyghens construction, after the Dutch physicist of that name who lived from 1629 to 1695. Make a similar construction for the case of reflection, and show that the angle of reflection is equal to the angle of incidence.

16-4. If two mirrors are held at right angles to each other, how many reflections will be formed of an object held near the intersection of the two mirrors? Draw lines to illustrate the various possible paths of the waves emanating from the object to the mirrors and back.

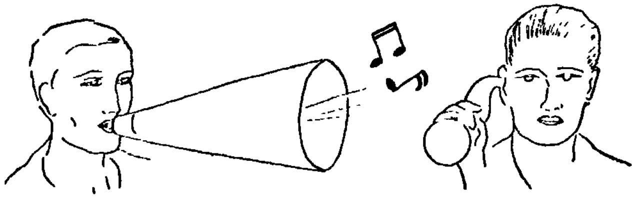
16-5. Solve a problem similar to that in section 16-7, except that the water wave passes from the shallow water to the deep water with an angle of incidence of (1) 25 degrees; (2) 35 degrees. The second case is described technically as *total reflection.*

16-6. On the basis of the data in the problem of section 16-7, would you expect the surf on a shelving beach to come in parallel to the shore, or at an angle?

16-7. Give a reason for the wave length in figure 16-7 extending from A to C rather than from A to B.

16-8. In figure 16-5, assume that the wave travels at the rate of 28 feet/second in medium 1 and 20 feet/second in medium 2. Determine whether it would take a longer or a shorter time for the wave to go from A to D by way of B or by a straight path

CHAPTER 17



Sound

17-1. Definitions. A psychologist and a physicist define sound differently. According to the psychologist, sound is a sensation perceived through the ear. The psychologist would go so far as to say that in a desert where there was no one to hear, there would be no sound! On the other hand, the physicist defines sound as a longitudinal wave motion in the medium (usually air) which is in contact with the ear. Needless to say, we shall use the latter definition and study these waves.

17-2. No Sound In A Vacuum. Since sound waves exist in a material medium, it follows that we must not expect sound to pass through a vacuum. If a bell could be supported under the receiver of an air pump in such a way that it did not touch anything, we should be unable to hear it even while it was ringing. In fact, in an actual experiment where an alarm clock is supported by a felt pad under the receiver of an air pump, there is a very noticeable difference in intensity after the air is pumped out.

17-3. Speed of Sound. Everyone has had the experience of watching a distant person swinging an ax or a hammer. Since light travels much faster than sound, it is sometimes possible for this person to get in one whole swing before the sound reaches the observer. In this case an extra blow will be heard at the end after the hammer stops moving. Whereas light travels at the rate of 186,000 miles per second, sound goes only 1,088 feet per second at 32°F. and 1,129 feet/second at normal room temperature (68°F.). For many purposes the approximate value of 1,100 feet/second is a convenient one to use. During a thunder storm it is often interesting to use a still greater approximation for the speed of sound and allow five seconds to the mile. One can tell in this way with the aid of a watch how near the disturbance is. The speed of sound has been measured by several methods. The method of Kundt's tube will be discussed in section 17-15; another procedure is a bit more obvious. The distance between two hilltops is carefully determined by the surveyor's methods. A cannon is placed on one hilltop. When the cannon is fired, the interval between the flash and the report is measured from the other hilltop. The required speed is the ratio between the distance and the time interval. We have seen that the speed of any wave motion depends on two properties of the medium, the elasticity and the density; the speed is equal to the square root of the ratio between the elasticity (bulk modulus) and the density. It will therefore be true that every substance will transmit sound at a rate peculiar to itself. Thus the speed of sound in water is about four times as fast, and in iron about fifteen times as fast as in air. If two experimenters placed themselves at opposite ends of a long steel rail, one of them would hear twice a single tap made by the other, once through the steel and again through the air.

17-4. Dependence of Speed of Sound On Temperature. The reason is now apparent for the fact that sound has a speed at 32°F. different from the value at 68°F. Any agency that is capable of affecting either the density or the elasticity of air can be expected to affect also the speed of sound in air. To the two values given in the previous section may be added: speed of sound is 1,266 feet per second at 212°F., 1,814 feet per second at 932°F., and 2,297 feet per second at 1,832°F. At ordinary temperatures, the speed increases about 1.14 feet per second for each Fahrenheit degree rise in temperature. On the other hand, a change in pressure does not perceptibly change the speed of sound because an increase of pressure increases both the elasticity and the density in practically the same proportion. Let us suppose, for example, that the barometer goes up enough to add one per cent to the value of the bulk modulus; then it will be found that the density also increases by one per cent, and there is therefore no change in the ratio of the elasticity to the density.

17-5. Pitch, Loudness, and Quality. Three characteristics of sound can be related to wave properties discussed in the previous chapter.

The *pitch* of a sound is directly connected with the frequency of the wave. The greater the frequency the higher the pitch. The human ear is capable of distinguishing pitches varying from 16 to 30,000 vibrations per second; some ears have a wider range than others. If one musical note has twice the frequency of another, the first is said to be an octave higher than the second. Occasionally a pipe organ is built with one or two notes below the sixteen-per-second limit just mentioned; these notes have to be felt rather than heard, and yet they seem to improve the general effect.

The *loudness* of a sound depends upon physiological factors, also upon the amount of energy per unit of area which reaches the ear per unit of time; the latter in turn is proportional to the product of the amplitude squared by the frequency squared. Thus from purely physical considerations, it is easier to hear a high pitched note than one of low pitch, assuming that both sounds are well within the audible range. For instance, an orchestra needs but one piccolo whereas several contra-basses are necessary. The amplitude of a sound wave is inversely proportional to the distance from the source; therefore the intensity of a sound is inversely proportional to the *square* of the distance from the source. This statement may also be shown to be true by the following argument: Imagine two concentric spherical geometric surfaces, the radius of one being half that of the other. The areas will then be in the ratio of one to four. If a sound starts at the center and is transmitted in all directions, the same quantity of sound that passes through one surface must also pass through the other surface, but the quantity of sound that passes through *unit area* of the inner spherical surface must be just four times as much as that passing through unit area of the outer surface, because the ratio of the areas is just one to four. Therefore the sound will have four times the intensity at the inner surface. By this argument, the inverse square intensity law would apply to energy of projectiles as much as it would to waves; we shall therefore expect to see that the inverse square law also applies to light. The ear is sensitive to such an enormous range of loudness values that it has become customary to adopt a unit of loudness which is logarithmic, the *bel*. If the standard intensity is called I_0, and the intensity I which we wish to measure is such that

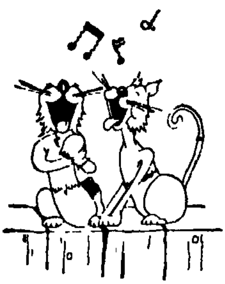
$$I = (10)\ I_0$$

then I is said to have a loudness of x bels with reference to I_0. A loudness of x bels is the same as a loudness of $10x$ *decibels*, the latter being the more usual unit. Like potential energy, the reference point may be taken anywhere, but the custom is becoming more common to take I_0 as the intensity which at a frequency of 1,000 vibrations per second represents an amount of energy per second per square centimeter at the eardrum of 10^{-9} ergs. (See section 3-12). If we solve the equation just given for x, we obtain

$$x = \log_{10} (I/I_0)$$

A three-place logarithm table will be found in appendix 8.

The third characteristic of a sound is its *quality*. The quality of the sound depends upon the shape of the wave. Another way of saying the same thing is the following: the quality of a sound depends upon the number and relative amplitudes of the harmonics present.



17-6. Harmonics. If we have a given pitch, which means a sound of a certain frequency, or, since wave length is equal to velocity divided by frequency, a sound of a certain wave length, then a harmonic is another sound the wave length of which is one half, or one third, or some aliquot part of the original wave length. The sound with the original wave length is called the fundamental tone or first harmonic. In figure 17-1, waves A and C represent sounds of the same wave length and therefore of the same pitch. B represents the second harmonic of A; a musician would say that it was one octave higher than the fundamental. Although A and C have the same pitch, they are different in quality because the

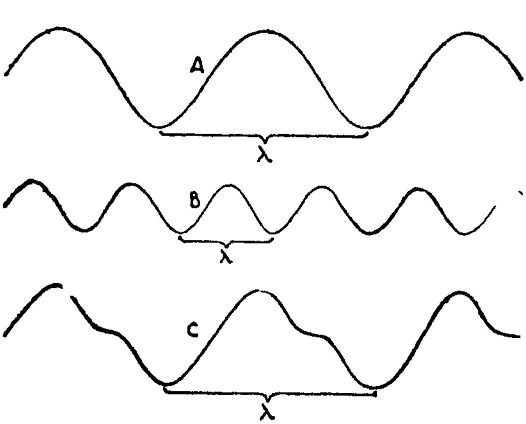


Figure 17-1.

waves have different shapes. A represents the type of tone emitted by a tuning fork or an open organ pipe when sounded gently, while C

sounds like an open organ pipe blown a trifle more vigorously. C is the result of adding the ordinates of A and B; therefore we can describe C as being the combination of A and its second harmonic in such a way as to make the amplitudes take the ratio of two to one. The French mathematician Fourier (1772-1837) discovered that any wave, no matter how complicated its shape, is simply the combination of its fundamental and some of its harmonics, each with the proper amplitude and phase relation. A certain physicist once amused himself by drawing a wave the shape of which was the same as the profile of his wife's face, and then determining the amplitudes of the harmonics necessary to give that shape!

17-7. The Doppler Effect. An interesting relation between either the velocity of the source of the sound or the velocity of the listener and the pitch of the sound is known as the *Doppler effect.* Everyone has noticed how the pitch of a factory whistle suddenly seems to drop as one rides by it in an express train, or how the pitch of an automobile horn suddenly appears to a stationary observer to drop as it passes him. In the first case the observer is moving toward or away from the stationary source. In moving toward the source, he encounters the waves a little faster than he would if he were stationary, the apparent frequency is increased, and he therefore hears a slightly higher pitch than the whistle is actually emitting. In the case of the approaching automobile horn, each wave is emitted from a point slightly nearer than the point from which the preceding wave was emitted, thus shortening the waves and raising the pitch. It is, of course, possible for both source and observer to move at once. The mathematical relations involved are expressed by the following equation

$$n' = \frac{V - u}{V - v} n$$

where n' is the frequency observed, n the frequency emitted, V the velocity of sound, u the velocity of the observer, and v the velocity of the source, the positive direction for all three velocities being the same, say for example toward the right.

17-8. Illustrations of the Use of Doppler's Equation. (1) Let the source be stationary (v = zero) and let the observer be moving toward the source with one fourth the velocity of sound. u is therefore equal to $-V/4$, and the equation becomes $n' = (V + V/4)\,n/V$, or, $n' = (5/4)\,n$. A musician would interpret this result by saying that the pitch observed was a major third above that emitted. (2) Let the observer be stationary and let the source move away from the observer with the velocity of sound. In this case, $u = 0$, $v = -V$, so that the equation becomes $n' = Vn/2V$, or $n' = n/2$. This time the musician would say that the observed pitch was just one octave below the emitted pitch. (3) If the velocity of the source and the velocity of the observer are in the same direction and equal in magnitude, it will be found that $n' = n$; no change in pitch. (4) The speeds of most projectiles are greater than that of sound. During World War II, the boys used to refer to certain shells as "whiz-bangs" because they heard them whizzing by before they heard the bang of the original detonation. If we try to compute the observed frequency with a stationary observer and a velocity of the source toward the observer equal to twice the velocity of sound, we shall have $u = 0$ and $v = 2V$. Our equation now becomes $n' = Vn/(V - 2V)$, and when we solve it, we get a negative value for n' which has no physical meaning.

17-9. Illustrative Problem. Two automobiles are traveling in opposite directions, on the same road, one at 20 miles per hour and the other at 40 miles per hour. The horn of the former is sounding (frequency = 440 per second). What frequency does the man in the second car hear before and after meeting?

Since nothing is said about the temperature, we shall round off the velocity of sound to 1,100 feet per second and change it to miles per hour to correspond to the other two velocities, obtaining $V = 750$ miles/hour; considering it positive settles the question of the signs of the velocities. We also have $u = -40$ miles/hour before meeting, the minus sign representing the fact that the observer is moving in a direction opposite to the motion of the sound wave. And $v = +20$ miles/hour, plus because both the source and the sound wave are traveling in the same direction, toward the observer. $n = 440$/second. If we substitute the units into the equation, the velocity units will all cancel, therefore the frequency unit on the left will be determined by the frequency unit on the right. The equation therefore becomes

$$n' = \frac{750 - (-40)}{750 - (+20)} (440)$$

n' is therefore (79/73) (440) or 476 vibrations per second.

After meeting, if we again regard the 750 miles/hour as positive, since the sound is now overtaking the observer, we must consider $u = +40$ miles/hour and similarly, $v = -20$ miles/hour. The new equation is therefore

$$n' = \frac{750 - (+40)}{750 - (-20)} (440)$$

This time $n' = (71/77)\ (440) = 406$ vibrations per second. That is, while the true frequency is 440 per second, the apparent frequency changes from 476 per second to 406 per second at the instant of passing, or nearly three semitones.

17-10. Reflection of Sound.

Since sound is a wave motion, it is subject to reflection at a boundary of mediums. Several applications of this will immediately occur to the reader. An echo is a case of simple reflection of sound. When the enclosure is so arranged that the sound is subject to more than one reflection, we say that we have a case of reverberation. Reverberation is observed in large auditoriums, under large bridge arches, especially over a water surface, and above all during a thunderstorm within the region occupied by the falling rain. In this case there is a distinct difference in density between the part of the atmosphere above the clouds where fair weather prevails with very little water vapor mixed with the rest of the air, and below the clouds, where not only is the relative humidity* one hundred per cent, but where there is a considerable quantity of liquid water as well; therefore the sound of the thunder is reflected back and forth between earth and cloud a great many times before being completely absorbed. The speaking tube is another application of the principle of reflection; the sound is forced to follow the course of the tube because every time it reaches the boundary of the tube it is reflected back in again. In bodies of water, "sounding" is often accomplished by noting the time required by sound to be reflected from the bottom.

17-11. Sound Represents Energy. Sound should be considered as a form of energy, along with light, heat, electrical, mechanical, and chemical energy. Three things can happen to a sound wave: (1) it is gradually converted into heat energy as it passes along through a given medium, (2) upon reaching the boundary of the given medium, some of the sound, as we have seen, is reflected, and (3) some passes through the boundary and obeys the laws of refraction. But in any case, the total energy remains constant.

17-12. Time of Reverberation. In designing large auditoriums, an important consideration is the so-called time of *reverberation.* This is directly proportional to the volume of the hall and inversely proportional to the total absorption of the walls. If the measurements are in feet, the equation is

$$t = 0.05 \frac{V}{A}$$

where t is the time in seconds of reverberation, V the volume† (in cubic feet) and A the absorption of the walls of the hall. An open window in the room is equivalent to almost perfect absorption because practically no sound returns from the window into the room.

* That is, the proportion of water vapor in the air is just as great as is possible at the given temperature.

† Notice that V is volume in this section while in section 17-7, V represents the velocity of the wave. It is unfortunate that there are not more letters in the alphabet.

It is therefore customary to express the absorption as equivalent to that of so many square feet of open window area. For example, if the absorption coefficient of a certain carpet is 0.2, which means that it absorbs one fifth as well as an open window of the same area, then A for that carpet is its area in square feet times 0.2.

The reverberation time should not be too long, thereby causing confusion between successive syllables of a speech, nor should it be too short, thereby rendering the room "dead" for musical performances. About 2.25 seconds is a reasonable value for a large hall used chiefly for music, while about 1.25 seconds is better for an ordinary-sized theater. Formerly the reverberation time of a radio broadcasting studio used to be reduced somewhat below normal, so that when the additional reverberation at the receiving end was added, the total would come out right. But radios are used mostly in small rooms, automobiles, and even outdoors, that is, in places where the reverberation time is practically zero, therefore it is the modern custom to supply the necessary reverberation at the studio.

17-13. Diffraction of Sound. The wave lengths of ordinary sounds are sufficiently great to produce a large amount of diffraction. Occasionally it is desirable to reduce the diffraction. It will be remembered (see section 16-8) that the amount of diffraction or "bending around corners" is proportional to the wave length, and what is more important in this case, inversely proportional to a linear dimension of the opening out of which the sound proceeds. This means that it is nearly as easy to understand a speaker when he has his back turned as when he faces you. On the other hand, if the speaker can do something to increase effectively the size of his mouth, such as using a megaphone, a smaller percentage of the sound energy will be diffracted and a greater percentage will be directed straight ahead. The writer noticed an illustration of diffraction of sound a number of years ago in a railroad station. A steam radiator was hissing; hissing means high frequency and therefore a short wave length. Suddenly the hissing stopped a moment and then continued. On looking up from the newspaper, it was seen that a very large woman had walked by the radiator and had momentarily cast a "sound shadow." If the sound had been of greater wave length (lower pitch), it would have bent around the obstruction.

17-14. Interference of Sound. At a given instant the two prongs of a tuning fork are vibrating in opposite directions. A tuning fork designed to vibrate at the rate of 440 per second (the musician's A) produces a wave length of 2.50 feet. If the fork be set vibrating and held so that one prong completely screens the other, or be turned at right angles to this position so that each prong vibrates at right angles to the line connecting the fork to the ear, the sound is loud. If, on the other hand, the fork be held obliquely, so that one prong just does not screen the other, two sets of waves will reach the air, one practically 180 degrees out of phase with the other (since the distance between the prongs is small compared with 2.50 feet); the result of these two sets of waves will be comparative silence. This furnishes us with a good example of interference of waves. We get another type of interference when two sound waves of the same wave length travel in opposite directions through the same medium. At certain points called *nodes* (see section 16-11), the vibrations cancel each other. Examples of this effect will be found in the case of Kundt's tube, described in the next section, in organ pipes, and in violin strings; *stationary waves* or *standing waves* is the technical term used to describe the resulting condition of vibration. A third type of interference obtained with sound waves is encountered when two waves of slightly different frequencies are produced simultaneously. There will now be times when the waves annul each other, alternating with times when the waves reinforce each other. This succession of variations in intensity is called *beats*; the number of beats per second is equal to the difference in frequency of the two waves.

17-15. Kundt's Tube. The fact has been noted in section 16-11 that in the case of standing waves the distance from a given node to the second node beyond is one wave length. If, in addition, we also know the frequency, we can use the equation $V = n\lambda$ (section 16-4) to compute the speed of the wave. This is the so-called *Kundt's tube* method. A glass tube, containing a sprinkling of powdered cork along its entire length, is placed in a horizontal position with one end closed. A brass rod, carrying a disk at one end, is clamped at its center by means of a vise, so that the disk fits loosely into the glass tube. Thus the center of the brass rod is a node and each end is an antinode. The disk should be near the open end of the glass tube, and when its exact position has been located experimentally, the disk, although it represents an antinode of the motion of the brass rod, will be nearly an antinode of the vibrating air column. See figures 16-7. The two ends of the brass rod are set vibrating longitudinally in opposite directions by grasping the rod with a piece of chamois dusted with powdered rosin, and pulling so that the chamois slides along the rod. The result is a shriek of high frequency and consequently short wave length, the pitch depending on the length of the brass rod, a known quantity in the case of our experiment. The experimental procedure is to make the brass rod vibrate repeatedly; the motion of the disk sets up stationary waves in the glass tube when the proper position of the disk is found; the stationary waves leave the cork powder at rest at the nodes and set it into vibration at the antinodes, so that the configuration of the cork dust clearly reveals the position of the nodes, and consequently the wave length of the sound waves, by means of which the speed can be computed.

17-16. Illustrative Problem. A certain Kundt's tube apparatus uses a brass rod with a frequency of 1,680 vibrations per second. If, after adjusting the position of the disk, the little piles of cork dust are just four inches apart, find the speed of sound in air, also the temperature.

The wave length corresponding to this particular frequency is twice the distance between two successive nodes, or eight inches, or 0.667 foot. We therefore have, $\lambda = 0.667$ foot, and $n = 1{,}680$ per second. Since the speed of the wave is the product of these two, it follows that $V = (0.667)(1{,}680) = 1{,}120$ feet per second. $1{,}120 - 1{,}087 = 32$ feet per second more than the value of the speed of sound at 32°F., and corresponds to a temperature of 60°F. since the speed increases 1.14 feet per second for each Fahrenheit degree the temperature rises.

17-17. Organ Pipes. The motion of the air in a *closed* organ pipe is exactly the same as in Kundt's tube, the closed end being a node, and the end containing the reed being an antinode. This means that the closed organ pipe is one quarter of a wave length long, or three quarters, or five quarters, and so on. Another way of saying the same thing is as follows: The fundamental tone of a closed organ pipe has a wave length four times the length of the pipe; the second harmonic (which is often called the first overtone) has a wave length four thirds the length of the pipe, and so on. In the case of the *open* organ pipe, both ends are antinodes. Thus the length of the pipe is one half a wave length, a whole wave length, three halves wave lengths, and so on; and the fundamental and other harmonics have wave lengths of twice the length of the pipe, the length itself, two thirds of the length, and so on.

17-18. Illustrative Problem. Find the wave length of the fundamental and of the first two overtones emitted by an eight-foot closed organ pipe; by an eight-foot open organ pipe.

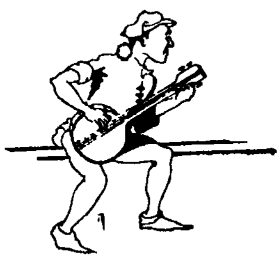
Since one end of a closed organ pipe is a node and the other end is an antinode, the shortest length of a pipe relative to a wave length is one quarter of a wave length; the next shortest, three quarters; and the next, five quarters. Thus we have

for the fundamental $8 = \lambda/4$ and $\lambda = 32$ feet
for the first overtone $8 = 3\,\lambda/4$ and $\lambda = 10.67$ feet
for the second overtone $8 = 5\,\lambda/4$ and $\lambda = 6.40$ feet

Since both ends of an open organ pipe are antinodes, the shortest length of a pipe relative to a wave length is one half a wave length, the next shortest, one wave length, and the next, three halves of a wave length. Therefore,

for the fundamental $8 = \lambda/2$ and $\lambda = 16$ feet
for the first overtone $8 = \lambda$ or $\lambda = 8$ feet
for the second overtone $8 = 3\,\lambda/2$ and $\lambda = 5.33$ feet

17-19. Violin Strings. There are three ways of varying the pitch of a violin string. The frequency is inversely proportional to the length of the string, directly proportional to the square root of the tension, and inversely proportional to the square root of the linear density (mass per unit length). In accordance with the last fact, the *G* string, from which are obtained the notes of the lowest frequency, is loaded to increase the linear density. The loading consists of winding the string with copper, silver, or some other metal wire. Tuning is accomplished by changing the tensions, and tones of different pitch are obtained in playing by changing the effective length of the string, which is accomplished by pressing the string against the finger board at various points with the fingers.



SUMMARY OF CHAPTER 17

Technical Terms Defined

Sound. Compressional waves in the medium (usually air) which is in contact with the ear. Sound is a form of energy.

Pitch. A physiological effect which depends on the frequency of the sound.

Loudness. A physiological effect which depends on both the amplitude of the sound wave and its frequency.

Quality. A physiological effect which depends on the shape of the sound wave.

Harmonic and Overtone. A sound wave the frequency of which is an integral multiple of a given fundamental wave. The fundamental is the first harmonic. Sometimes the second harmonic is called the first overtone, etc.

Doppler Effect. The effect whereby the apparent pitch of a sound depends on the relative velocity of source and observer.

Reverberation Time. The length of time that a sound of given intensity persists in a given auditorium.

PROBLEMS

17-1. At what temperature will the speed of sound be just 1,100 feet per second?

17-2. Find the speed of sound at 50°F; at 100°F.

17-3. If the man described in section 17-3 is swinging his hammer at the rate of three swings every two seconds, how far away is he?

17-4. In the cannon method of determining the speed of sound, what percentage correction should be made for the speed of light? How will this correction compare with the error introduced by the experimenter's "reaction time?"

17-5. If the steel rail in section 17-3 is 30 feet long, what is the time interval at one end between the sounds of a single tap at the other end?

17-6. How many octaves can the average man hear?

17-7. A certain sound is just loud enough to be heard one inch away. How far away could one be and still hear a sound one million times as loud?

17-8. A whistle with a frequency of 600 vibrations per second is blowing close beside a railroad track. What frequency will a passenger in a train going 60 miles per hour hear as he approaches the whistle? As he recedes from the whistle? Assume the velocity of sound to be 1,100 feet per second.

17-9. The problem of section 17-9 resulted in a sudden drop in pitch; what condition would have to be realized to result in an apparent increase of frequency?

17-10. A certain auditorium has a volume of 150,000 cubic feet and a total absorption equivalent to 1,800 square feet of open window. Find the reverberation time. A compact audience fills the hall, adding 4,200 square feet to the absorption. What is now the time of reverberation?

17-11. There are three tuning forks lying on a table. The frequencies of two of them are stamped upon them as 256 and 267 vibrations per second. The third fork makes four beats per second when sounded with the 256 fork and seven beats per second when sounded with the 267 fork. What is the frequency of the third fork?

17-12. Remembering that the two ends of the metal rod in the Kundt's tube apparatus are antinodes, we may use the experiment to determine the speed of sound in the metal. If the speed of sound in the air is 330 meters/second, if the average distance between adjacent cork dust piles is 8.56 centimeters, and if the metal rod is 80 centimeters long, find the speed of sound in the metal.

17-13. If the ratio between the frequencies of any two adjacent notes on a piano, that is, notes a semitone apart, is 1.0595, show that the ratio between the frequencies of notes a whole tone apart is 1.1225.

17-14. A closed organ pipe is one foot long and emits a musical tone the frequency of which is 270 per second. What is the speed of sound in air?

17-15. An open organ pipe is 2.25 feet long. Find the frequency of the third overtone of this pipe.

17-16. What is the effect on the pitch of keeping the length and tension constant in a violin string and increasing the linear density four times? What is the effect on the pitch of keeping the length and linear density constant and increasing the tension four times? What is the effect on the pitch of keeping the tension and linear density constant and decreasing the length to one fourth of the original value? What is the effect on the pitch of increasing both the tension and the linear density four times and at the same time reducing the length to one fourth of its original value?

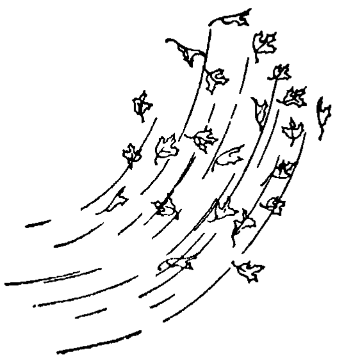
CHAPTER 18

Heat and Temperature

The Two Laws of Thermodynamics

18-1. Heat as a Form of Energy. Heat is another form of energy. This is equivalent to saying that, given a quantity of heat, we should be able to obtain work from it. As an illustration of this statement, we may cite the fact that with heat obtained from the burning of coal we are able to drive a steam engine. But it is impossible to convert a given quantity of heat *entirely* into work, or, in fact, into any other form of energy. The general statement may be made that whenever an attempt is made to convert any form of energy into some other form of energy, heat is one of the by-products. The only exception to this statement is the case when we attempt to change some other form of energy into heat, as in an electric stove. In this case there is no by-product; the process is 100 per cent efficient; nothing but heat is produced! From these statements the deduction may be made that gradually all other forms of energy are being reduced to heat and that eventually there will be no other form of energy in existence.

18-2. Theoretical Basis of Temperature. If a given substance such as paper, water, salt, sugar, or mercury could be sufficiently divided, a point would be reached when further subdivision would result in a change in the nature of the substance. The smallest particle of a given nature that can exist is called a *molecule*. To be sure, a molecule is a group of atoms, and before we get through, we shall also have something to say about the internal structure of atoms, but for the purposes of this chapter, it will be sufficient to regard the molecule as the smallest particle of a given nature. If the molecules of a body were all at rest relative to one another, the body would be completely cold and we should say that its temperature was at the bottom of the thermometric scale. On a Fahrenheit thermometer this would be 459 degrees below zero. On another type of thermometric scale used in scientific work, the corresponding reading would be minus 273 degrees centigrade. Heat energy represents a random chaotic motion of the molecules of the body involved, relative to each other, and the higher the temperature, the greater is this irregular motion. In a liquid it is possible to detect motion of this kind in particles considerably larger than molecules by means of a compound microscope. Any emulsion consisting of very fine particles in suspension may be used for this purpose; smoke will also do; and it will be observed that the motion (called the *Brownian movement*) is incessant as long as a given temperature is maintained. This means that the molecules must be perfectly elastic. If the temperature is increased, the particles will move about more rapidly; if the temperature is reduced, they will move more slowly.



18-3. Conversion of Energy of Motion Into Heat. A moving body may be made to do work. Examples of this may be found in the fact that the kinetic energy of a moving flywheel on a buzz-saw may be made to saw off a stick of wood after the power has been cut off. Or a moving stream of water may be made to turn a turbine. Since energy can neither be created nor destroyed, what becomes of this energy when the body stops moving? In every case of this kind, *some* heat is always produced, and in many cases the energy is *entirely* converted into heat. As an illustration of the latter case, carried to a theoretical extreme, imagine a piece of lead pipe with no heat in it (absolutely cold) but moving as a whole at a high rate of speed. The piece of lead is to be thought of as consisting of molecules, all of which are moving in the same direction, that is, in the direction in which the piece of lead as a whole is going. Since the lead is absolutely cold, the molecules have no random motion relative to one another. Suppose now that the lead strikes a hard, smooth portion of the earth, large enough to be "immovable," and also absolutely cold. What will become of the energy of the moving lead? The answer is that the individual molecules will maintain the same average speed that they had before, but the motion will be quite chaotic, random, irregular. The energy of the motion has been changed to heat.

18-4. Orderly Motion Tends to Become Chaotic, But Chaotic Motion Does Not Tend to Become Orderly. Another idealized illustration of the same thing is as follows: Imagine a billiard table with a row of perfectly elastic billiard balls, all moving parallel with one another at a certain instant, but not necessarily parallel with the edges of the table. If the table could be absolutely without friction and the edges perfectly elastic, there would be nothing to stop this motion, therefore under our suppositions the motion would continue forever. But the changes in direction due to the reboundings would result in a chaotic motion of the billiard balls instead of the original parallel motion. The question now arises, "How long will it be before by chance all the billiard balls will again be moving parallel with one another at some given instant?" With a given number of billiard balls and given dimensions of the table, it is possible to compute the answer to this question, and it will come out a surprisingly large number of years. Let us also ask the corresponding question about the piece of lead in the preceding paragraph, or of any rock lying by the wayside. What is the chance that each of the molecules, moving with terrific speeds but in all possible directions, will at some future time all happen to go in the same direction at once? The answer is about one chance in infinity, which is the mathematician's way of saying *no chance at all.* Orderly motion tends to become chaotic, but chaotic motions do not of themselves tend to become orderly. It should now be clear why it is impossible to change the heat of a piece of lead completely back into energy of motion of the piece of lead as a whole, or in other words, why a body left to itself will never become completely cold.

18-5. Distinction Between Heat and Temperature. Temperature means the degree of hotness of a body and is not connected with the mass of the body, whereas the amount of heat in a body cannot be computed unless we know both the mass or weight of the body as well as its temperature and the material of which the body is composed. Thus if two bodies are made of the same material and weigh the same, the one with the higher temperature will contain the more heat. But if two bodies are made of the same material, have the same temperature, but do not weigh the same, the heavier body will contain more heat than the lighter. In fact there is actually more heat in a tub of lukewarm water than in a teacup of boiling water.

18-6. Properties That Depend on Temperature. Temperature is proportional to the average energy of translatory motion (translatory kinetic energy) of the molecules of the object under consideration. When the temperature changes, various other properties of the object also change; it is from a measurement of these other properties that we are enabled to determine the temperature. Some of these other properties are length, volume, electrical resistance, vapor pressure, character of radiation emitted, and thermoelectric effect. Most thermometers depend upon the fact that when the temperature changes, volumes change.

18-7. Temperature Scales. We use a number of temperature scales. The two scales most convenient for everyday usage involve negative values for low temperatures, so that for scientific work, it is convenient to have in addition two absolute scales, each of which has zero for its lowest value. The ordinary Fahrenheit scale has as its zero point the lowest temperature conveniently obtained with a mixture of salt and ice, while the 100° point is about as high as the temperature would rise on a very hot summer day. The centigrade scale has the freezing point of water for its zero and the boiling point of water for the 100° position. On the Fahrenheit scale, water freezes at 32° and boils at 212°, that is, there are 180 degrees between the two points. Figure 18-1 indicates the relative readings of the four scales which have been mentioned. The upper

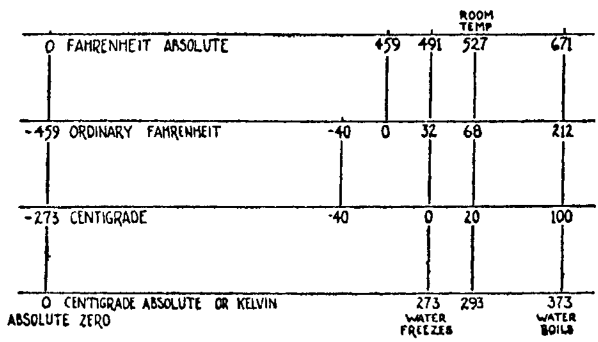


Figure 18-1.

and lower ones are the two absolute scales and read zero at the lowest theoretical temperature. Although this temperature has never been reached, deHaas, a Dutch physicist, has succeeded in getting to within 0.0044° of it. The existence of the absolute zero is indicated by a number of physical facts, chief among which is the behavior of gases; and there is very little doubt that it is located 273.15° below the zero of the centigrade scale. We shall round this off to 273°. Thus, on the centigrade absolute scale which is often called the Kelvin scale, water freezes at 273° and boils at 373°.

18-8. How to Change from One Scale to Another. Since there are 180 Fahrenheit degrees between the freezing and boiling points of water and only 100 centigrade degrees in this same interval, each Fahrenheit degree is 100/180 or 5/9 as large as a centigrade degree. Thus the boiling point of water on the Fahrenheit absolute scale is 9/5 of 373° or 671°. Similarly the freezing point of water is 491° F. Abs., which is 9/5 of 273°K. Readings on the Fahrenheit absolute scale are 459° greater than those on the ordinary Fahrenheit scale, while readings on the Kelvin scale are 273° more than readings on the ordinary centigrade scale. To complete the system of changes, it is necessary to devise a method of changing from the ordinary Fahrenheit to the ordinary centigrade scale. Suppose it is desired to express normal room temperature 68°F. as a centigrade reading. First, by subtracting 32 from 68, we see that normal room temperature is 36 degrees above the freezing point of water. 5/9 of 36 is 20, so that these 36 Fahrenheit degrees correspond to 20 centigrade degrees, and since on the centigrade scale water freezes at 0°, 20°C. is the result we are looking for. If we express this result algebraically, we find that

$$t_C = \frac{5}{9}(t_F - 32)$$

expresses the numerical operations that we have just performed, where t_C represents the temperature on the centigrade scale and t_F on the Fahrenheit scale. If, however, our problem was to change 20°C. into the corresponding Fahrenheit reading, the argument would run as follows. 20°C. is 20 degrees above the freezing point on the centigrade scale. Since the Fahrenheit degrees are smaller, there will be more of them in the same interval, namely 9/5 of 20 or 36 degrees. The Fahrenheit temperature which is 36 degrees above the freezing point (32°F.) is 36 + 32 or 68°F. This process may be summarized by the equation

$$t_F = \frac{9}{5}t_C + 32$$

which after all is nothing but the previous equation solved for t_F.

18-9. Illustrative Problem. (1) Change 15°F. to centrigrade, Fahrenheit absolute, and Kelvin readings. (2) Change 15°C. to the corresponding values on the other three scales.

(1) To change Fahrenheit readings to centrigrade, use the equation

$$t_C = \frac{5}{9}(t_F - 32)$$

where $t_F = 15$ degrees. Therefore $t_C = (5/9)(15 - 32) = (5)(-17)/9 = -9.44$°C. Since the zero point on the Kelvin scale is 273 degrees below that on the centigrade scale, the Kelvin temperature corresponding to 15° F. or -9.44° C. is $-9.44 + 273$ or 263.56°. The corresponding temperature on the Fahrenheit absolute scale is $15 + 459$ or 474°. As a check on these results we may notice that five ninths of 474 is 263.3. Since it is understood that problem results are to be reported to slide rule accuracy only, the answers to this problem should be reported: -9.44° C., 474° F. Abs., and 264° K.

(2) This time we start with

$$t_F = \frac{9}{5}t_C + 32$$

where $t_C = 15$°. Therefore $t_F = (9/5)(15) + 32 = 135/5 + 32 = 27 + 32 = 59$° F. Note that in this equation the parentheses do not include the 32. The Fahrenheit absolute reading corresponding to 59° F. is $59 + 459$ or 518° F. Abs. The Kelvin reading corresponding to 15° C. is $15 + 273$ or 288° K. As a check, we note that $(9/5)(288) = 518$.

18-10. The First Two Laws of Thermodynamics. The law of conservation of energy states that energy can neither be created nor destroyed. Since heat is a form of energy, when a certain quantity of heat disappears from one body it will be found to have transferred itself to some other body, providing that it has not been changed into some other form of energy. This statement of the law of conservation of energy applied to heat is called the *first law of thermodynamics*. The *second law of thermodynamics* goes on and states that when a quantity of heat transfers itself from one body to another by natural processes, it will always be found that the first body is at a higher temperature than the second. That is, heat will not of itself pass from a cold body to a hot body in such a way as to make the cold body colder and the hot body hotter.

18-11. Generalization of the Second Law. The second law of thermodynamics is capable of generalization in such a way as to apply to things other than heat. It has already been stated that heat represents a chaotic motion of the molecules of a body. If we now return to the illustrations in sections 18-3 and 18-4, we may say that the second law states that it is more natural for the molecules of the piece of lead mentioned there to change from the first condition into the second condition (orderly motion to chaotic motion) than it is for them to change from chaos to order. If all the molecules of a rock, each one of which has a velocity measured in miles per second in perfectly random directions, should suddenly all commence to move in the same direction at the same time, it might be extremely unfortunate for the innocent bystander! That this new statement of the second law is equivalent to that made in the preceding paragraph will be evident when we consider that there is less randomness of molecular motion in two bodies, one of which has one temperature and the other another, than if they both have the same temperature. Perhaps this will be seen more clearly if we exaggerate to the extreme case where *all* the heat is transferred from one body to the other, so that the first is left completely cold and the second is made quite hot. If the bodies are of equal weight, just half of the chaos has now been made into complete order. But this is just the sort of thing that the second law says cannot happen *by itself* because it involves making the hot body hotter and the cold body colder till the limit is reached. Thus the second law of thermodynamics may be generalized to read, whenever inanimate objects are left to themselves the tendency is always from order to chaos, and *never* in the other direction. If one should leave some papers piled neatly upon his desk, leave the room, return and find them scattered all over the floor, he would be justified in saying that some inanimate agency such as the wind was responsible. If, however, he should leave the room in this disarray, return a second time, and find the papers piled neatly upon his desk in their original order, he would never be justified in saying that a second gust of wind was responsible. It would be necessary to say that the second case involved an act of intelligence, and should therefore not be discussed in a course in physics.

18-12. Entropy. Efficiency of a Heat Engine. In a more extended course we should learn that *entropy* is a quantitative measure of the amount of randomness in a system, and by applying both the first and second laws of thermodynamics, we could discover that the maximum efficiency of a heat engine, say a steam or gasoline engine, operating between two given temperatures, is equal to the difference between those two temperatures divided by the higher temperature expressed on the absolute scale. For example, if the steam in a steam engine is at a temperature of 300°F. and the condenser is maintained at a temperature of 200°F., then the thermal efficiency is $(300 - 200)/(300 + 459)$ or $100/759$ or 0.1318 which is 13.18 per cent. Due to other losses, the thermal efficiency must be regarded as the maximim efficiency possible at the given temperatures and therefore as an upper limit to the actual efficiency.

SUMMARY OF CHAPTER 18

Technical Terms Defined

Heat. A form of energy which consists of the combined energy of separate molecules of a body.

Temperature. A quantity proportional to the average translatory kinetic energy of a single molecule of the body concerned.

Absolute Temperature. Temperature expressed on a scale such that the lowest possible value is zero.

Laws

First Law of Thermodynamics. A special form of the law of conservation of energy which holds when heat energy is involved.

Second Law of Thermodynamics. Heat will not of itself pass from a given body to another body of higher temperature.

Second Law Generalized. In inanimate nature, order tends to chaos. Order itself is a sign of intelligence, which is not one of the things studied in physics.

PROBLEMS

18-1. Change the following temperatures to centigrade: 86°F., 500°F., 5000°F., 0°F., −40°F., −273°F.

18-2. Change the temperatures of the preceding problem to Kelvin.

18-3. Change the temperatures of problem 18-1 to Fahrenheit absolute.

18-4. What simple relation exists between the answers of problem 18-2 and 18-3?

18-5. Change the following temperatures to Fahrenheit: 5000°C., 500°C., 50°C., −40°C., −200°C.

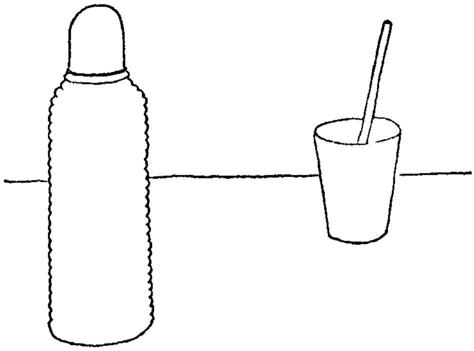
18-6. Find the Kelvin temperature corresponding to 574°F.

18-7. Using the data in figure 18-1, make four graphs on the same chart in which Kelvin temperatures are plotted along the X-axis while along the Y-axis we have (1) Fahrenheit absolute, (2) ordinary Fahrenheit, (3) ordinary centigrade, and (4) Kelvin temperatures plotted. Do any of these four lines cross each other, and if so, where?

18-8. A turbine operates with saturated mercury vapor from a boiler at 840°F., exhausting into a condenser at 350°F. Find the maximum efficiency possible with an ideal heat engine operating under these conditions.

18-9. Experiments made recently indicate that the efficiency of the human body operating between the temperatures of 98.6°F. and 68.0°F. is about fifteen per cent, and is furthermore greater than zero when the temperature of the surroundings is equal to the body temperature. Compute both thermal efficiencies. Have you an explanation of the discrepancy?

CHAPTER 19



Heat Transfer

19-1. Three General Methods of Heat Transfer. The engineer is frequently faced with the problem of transferring a quantity of heat from one place to another, also with the opposite problem of insulating buildings to prevent heat transfer. A steam heating plant is an excellent illustration of the first situation. If we trace the path of the heat in a plant from the time it is produced by the combustion of the fuel, we shall find that three different methods are utilized in delivering the heat ultimately at the place where we want it: conduction, convection, and radiation. If we generalize these three processes, we shall find that they include all possible methods of transferring heat.

19-2. Conduction. If we heat one corner of a solid object, we shall find very soon that some of this heat is transferred to the neighboring portions of the solid. Interpreting this in terms of molecules, we can say that if the molecules of one portion of a solid object are set into agitation, this motion will gradually be communicated to the neighboring molecules. A similar situation might be imagined as follows: suppose a large crowd of very hot-tempered, pugnacious men is standing quietly in a room. If a fight starts at one point of the room, we can imagine it spreading in about the same way that the molecular action spreads, except that there is a law of conservation of energy and unfortunately no corresponding law of conservation of belligerency. The rate at which heat is conducted through a solid depends upon three factors: (1) the differences in temperature between the two surfaces, (2) the material of which the solid is made, and (3) the ratio of its cross section to its thickness. The conductivity of silver is nearly 6,000 times as much as that of asbestos.

In a steam heating plant, the heat passes by conduction through the shell of the boiler into the water, and later passes by conduction from the hot steam through the material of the radiator to the outer surface of the radiator.

19-3. Computation of Transfer of Heat By Conduction. If we let H stand for the heat that gets through a slab of material of conductivity C, of thickness d, and cross-sectional area A, in time t, when the temperature of the hot side of the slab is t_2 and that of the cooler side is t_1, we can make the statement that

$$H = \frac{t(t_2 - t_1)AC}{d}$$

That is, the heat conducted through the slab is proportional to the four factors time, temperature difference, area, and conductivity, and inversely proportional to the thickness of the slabs.

We have seen that there are two common units by which heat energy is measured, the British thermal unit and the Calorie. (See section 3-12). The combination of units that may be used in this equation are rather numerous, the unit of C taking up the slack, so to speak. For example, if H is in Calories, t in seconds, t_2 and t_1 in centigrade degrees, A in square meters, and d in meters, we can express C in Calories per second per degree centigrade per meter, and the units on the right side of the equation will reduce to Calories. Or if we are using English units, H is in British thermal units, t in seconds, t_2 and t_1 in degrees Fahrenheit, A in square feet and d in feet. This time C is expressed in B.t.u. per second per degree Fahrenheit per foot. However there is nothing to prevent the English engineer from expressing H in British thermal units, t in days, t_2 and t_1 in Fahrenheit degrees, A in square feet, and d in inches, and he often does it! In this case C must be expressed in B.t.u.-inches per day per square foot per degree Fahrenheit.

19-4. Numerical Values of Heat Conductivities. A table of heat conductivities follows in the first two sets of units just mentioned.

Substance	*Cal./sec.-deg. C-meter*	*B.t.u./sec.-deg. F-foot*
Air	0.00000054	0.00000036
Aluminum	0.0422	0.0284
Asbestos	0.000019	0.000013
Brass	0.0204	0.0137
Brick	0.00012	0.00008
Copper	0.0975	0.0656
Cork slabs	0.000010	0.0000067
Flannel	0.0000035	0.0000024
Glass	0.00018	0.00012
Granite	0.000510	0.000343
Hair felt	0.000009	0.000006
Ice	0.00057	0.00038
Iron	0.0166	0.0112
Lead	0.00836	0.00562
Magnesia pipe covering	0.000016	0.000011
Marble	0.000470	0.000316
Sand	0.000093	0.000063
Sawdust	0.000015	0.000010
Silver	0.1096	0.0737
Slate	0.000272	0.000183
Snow	0.000026	0.000017
Tin	0.01519	0.01021
Water	0.000138	0.000093
Wood across grain	0.000009	0.000006
Wood along grain	0.00003	0.00002
Zinc	0.0284	0.0191

Silver and copper are the best conductors of heat known, whereas substances like hair felt, asbestos, and dry sawdust are among the best insulators, often because they imprison a quantity of air, which is an extremely poor conductor.

19-5. Illustrative Problem. Calculate the amount of heat that will escape from a house in 24 hours through a glass window of 2 square yards area, one eighth of an inch thick if the temperatures are 70°F and 10°F outside. If the heat of combustion of coal is 13,900 B.t.u. per pound, how much coal must be burned per day on account of this one window?

Using the symbols of section 19-3, t is 86,400 seconds, t_2 is 70°, t_1 is 10°, A is 18 ft.^{2}, C is 0.00012 B.t.u./sec-deg. F-foot, and d is 0.01042 foot. Substituting these values into the equation of that section gives

$$H = \frac{(86{,}400)\,(70 - 10)\,(18)\,(0.00012)}{0.01042} = 1{,}075{,}000 \text{ B.t.u.}$$

Dividing this by 13,900 B.t.u./lb. gives 77.3 pounds of coal per day. This assumes that none of the heat of combustion escapes up the chimney.

19-6. More Complicated Cases. In most practical problems, the same heat flows through several types of material. For example, the wall of a house could consist of several inches of brick, an air space, and an inch of wood. In this type of problem, a separate equation must be set up for each material involved. One of the unknowns will be the heat H, which is the same in each equation. The time and cross-sectional areas will likewise be the same in each equation. The additional unknowns will be the intermediate temperatures at the boundaries of the various materials.

19-7. Convection. Conduction of heat can take place in liquids and gases as well as in solids, but in these two cases the situation is complicated by the fact that in fluids (gases and liquids) convection is much more important than conduction. A hot fluid usually weighs less, volume for volume, than that same fluid at a lower temperature. This means that we shall generally find the hottest air near the ceiling and the coldest air near the floor. If we heat a portion of the air near the floor, it will rise, and if we cool some of the air near the ceiling it will drop toward the floor. In certain types of automobiles formerly on the market, the water was not forced through the cooling system by a pump but depended on the fact that when cooled in the radiator it tended to drop, and when heated in the engine it tended to rise. Thus there was a continual transfer of heat from the engine to the radiator by convection currents of water, which worked moderately well for the comparatively low speeds of those days.

In the steam heating plant which we have been using as an illustration, heat passes from the hot burning fuel to the bottom of the boiler by convection of hot gases, the water churns about in the boiler due to convection, changes into steam (which is relatively light), and in this form rises into the radiators, condenses, and in this heavier form, returns to the boiler. In the room where the radiator is located, the air above the radiator continually tends to rise and thus maintains a convection current in the room. Hot air heating plants, hot water heaters, and the trade winds serve as further illustrations of convection. A generalization of the idea of convection simplifies to a mere moving of a hot body from one place to another, thus transferring the heat bodily.

19-8. Radiation. A body which has any temperature above the absolute zero is continually radiating heat into space, whether that space consists of an absolute vacuum or whether it is filled with material substances. For example, the sun is surrounded by an excellent vacuum, containing only about 16 molecules per cubic inch. One of these molecules is so small that on the average it travels nearly 300 years before striking one of its neighbors. Nevertheless the sun sends out radiant energy in all directions to such an extent that at the distance of the earth (93,000,000 miles) we continually receive, on every square yard of surface perpendicular to the sun's rays, energy at the rate of 1.5 horsepower. These rays travel at the rate of 186,000 miles per second in the vacuum between the sun and the earth's atmosphere, and at somewhat slower rates in other transparent substances. A small fraction of them is visible to the eye as light, but all of the radiations will raise the temperature of any object upon which they may fall. The hotter an object is, the greater is the rate at which it will emit radiation. We shall study radiation further under the heading of *light*, but just now we are interested chiefly in the fact that it constitutes a third method of transferring heat. While radiation is not itself a form of heat (heat involves molecular motion and there are no molecules in a perfect vacuum), it has its origin in the heat of bodies and is changed again into heat upon striking other bodies. If one stands in front of a hot fireplace, he may cut off the sensation of heat on his face practically instantly by suddenly placing his hand between the source of heat and his face.

In our illustration of the steam heating plant, the bottom of the boiler receives heat from the hot coals by radiation as well as by convection. The radiators also emit energy by radiation as well as by conduction and convection. Since radiation is not heat at all (but another form of energy), a generalization of this third method of heat transfer would be the case where heat is transformed into some other form of energy (for instance, electrical), transmitted in this form to some distant place, and then changed back again into heat.

19-9. Computation of Transfer of Heat by Radiation. The amount of heat that is changed into radiant energy depends on the elapsed time, the surface area of the radiating body, the temperature of the body as well as the temperature of the surroundings, and the nature of the surface. Some surfaces are almost perfect reflectors; for example a silver surface will reflect between 97 and 99 per cent of the infrared radiation falling upon it. On the other hand, certain substances reflect almost nothing, but absorb nearly all the radiation falling upon them; these are called black bodies. Good reflectors are poor radiators, but on the other hand, good absorbers are good radiators. We shall let B stand for the blackness of a surface. For a perfect absorber, that is, a perfectly black body, $B = 1.00$; for a perfect reflector, $B = 0.00$; for all other bodies B lies between zero and unity. If H is the heat that is converted into radiant energy, t the time in seconds, A the area of the emitting surface in square meters, T the Kelvin temperature of the body surface, and T_s the Kelvin temperature of the surroundings, then

$$H = 1.368 \times 10^{-11}\, tA(T^4 - T_s^4)B$$

19-10. Illustrative Problem. Assuming that each square yard of the earth's surface receives 1.5 horsepower from the sun, compute the temperature of the sun's surface. The radius of the sun is 6.97×10^8 meters.

If we imagine a huge hollow sphere of radius 93,000,000 miles, which is the distance between the sun and the earth, surrounding the sun, with each square yard receiving 1.5 horsepower, this would give a total of $4\pi(93{,}000{,}000)^2\,(1{,}760)^2\,(1.5)\,746$ watts or 3.77×10^{26} joules per second of radiant energy leaving the sun, only a small portion of which strikes the earth. This is 9.01×10^{22} Calories every second. Thus we may substitute into the equation of the previous section the values $H = 9.01 \times 10^{22}$ Cal., $t = 1$ sec., $A = 4\pi(r_{sun})^2$, where $r_{sun} = 6.97 \times 10^8$ meters; T_s is so much less than T that we may neglect it in this problem, and B may be taken as 1.00 since the sun is so nearly a black body. Hence we have

$$9.01 \times 10^{22} = (1.368)\,(10^{-11})\,(1)\,(4\pi)\,(6.97)^2\,(10^{16})\,T^4\,(1)$$

Solving, $\quad T^4 = 0.1079 \times 10^{16}$

and $\quad T = 5{,}730°\text{K}$

The value usually given for the temperature of the sun's surface is slightly under 6,000°Kelvin. The temperature of the sun's interior is, however, much greater than this. The region near the sun's center may reach as high a figure as 20,000,000°Kelvin. The source of the sun's heat is doubtless subatomic; that is, the sun's mass is gradually being converted into heat energy.

19-11. An Illustration of Heat Insulation. The problem of preventing heat transfer also arises frequently; the thermos bottle provides an interesting illustration of this. The important feature of the thermos bottle is the double layer of glass of which it is constructed, together with the vacuum between the two glass layers. This prevents the *conduction* of heat because heat can be conducted only by molecules of material substances, and there are comparatively few molecules in a vacuum. In other words, a vacuum is the worst possible conductor of heat. *Convection* cannot take place because convection takes place only in fluids and not at all in a vacuum. But radiation takes place in a vacuum better than anywhere else. *Radiation* is prevented in a thermos bottle however by silvering the sides of the glass that are next to the vacuum and reducing B of section 19-9 to a value as nearly zero as possible. This results in reflecting the escaping radiation back in the direction from which it came. Thus, the only way in which heat can escape easily from a thermos bottle is through the cork.

19-12. Perfect Reflectors and Perfect Absorbers. There is no such thing as either a perfect reflector or a perfect absorber, but since in both cases we can make good approximations, it is possible to describe their properties. A perfect reflector would not allow any of its heat energy to escape in the form of radiant energy. The radiation would be reflected internally, back into the body, just as perfectly as radiation would be reflected externally. On the other hand, a perfect absorber would not reflect any radiation, but would convert it all into heat as fast as it arrived. A perfect absorber is also the best radiator. Furthermore, when it is considered that we are enabled to see the objects about us mainly by reflected light, we realize that a perfect absorber would also be perfectly invisible (perfectly black) unless it happened to be hot enough (above 500°C.) to emit *visible* radiation of its own. Not only are the substances that emit radiation best the best absorbers, but the worst emitters are the best reflectors.

19-13. Thermal Equilibrium. After things have been left to themselves for a sufficiently long time, a state of *equilibrium* results, after which the temperatures no longer change. When this condition exists, an object is receiving heat by all the methods at the same rate at which it is giving off heat. If, after equilibrium has been established, any of the conditions are changed, we say that the equilibrium has been disturbed. For example, if an object exposed to the sun's radiation has reached a state of equilibrium and is then cut off from the sun's rays, its temperature will drop to a new state of equilibrium where it is again receiving energy as fast as it is losing it.

For another example, see problem 19-12. Thermal equilibrium is not a static affair, but involves a lively set of interchanges of energy, yet in such a way that the rates are completely balanced.

SUMMARY OF CHAPTER 19

Technical Terms Defined

Conduction. A method of transfer of heat by handing the energy along from molecule to molecule through the body.

Convection. A method of transfer of heat in fluids which takes advantage of the difference in density of hot and cold fluids.

Radiation. Radiation is not heat, but is a form of energy which passes readily through a vacuum at the characteristic speed of 186,000 miles per second. Heat may be converted readily into radiation in accordance with the equation in section 19-9, and radiation may be converted back into heat to an extent proportional to the value of B. Thus heat may be transferred indirectly by means of radiation.

Thermal Equilibrium. A condition in which a body receives heat by all methods at the same rate that it loses heat.

PROBLEMS

19-1. Discuss how it would be possible so to shape a piece of silver and a piece of asbestos that heat would be conducted through both at the same rate with a given temperature difference.

19-2. Why does a piece of cold iron which is at the same temperature as a piece of woolen cloth feel so much colder than the cloth?

19-3. The water under a layer of ice a foot thick in a pond is at 32°F. How many B.t.u. of heat will pass through a square mile of this ice in an hour if the temperature of the air above the ice is 0°F?

19-4. Assume a house wall to consist of 8 inches of brick in contact with one inch of wood. Let the outer surface of the brick be at 10°F. and the inner surface of the wood at 70°F. Find the heat that will flow through 200 square feet of this house in 24 hours, also find the temperature at the junction of the brick and wood.

19-5. Can an iceman be considered as being in the business of transferring heat? If so, of which of the three methods of heat transfer is the process a generalization?

19-6. Suppose two sheets of metal to be thermally insulated from each other by a layer of air. Assume three cases: (1) when both surfaces are horizontal with the hot surface above, (2) when the cold surface is above the hot surface, both being horizontal, and (3) when both surfaces are vertical. Discuss the transfer of heat by all three methods in all three cases.

19-7. Draw a diagram of an automobile engine and radiator and show how, with no water pump, the water will flow while the engine is running.

19-8. If one horsepower is the same as 746 watts, and if 2.54 centimeters is equal to an inch, find the number of watts per square meter received from solar radiation.

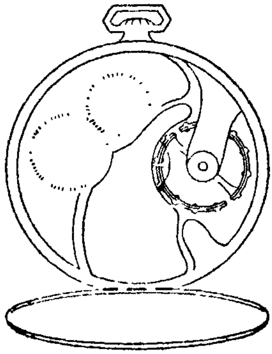
19-9. Why should a teakettle preferably have a black bottom and a polished upper surface?

19-10. Detective stories have been based upon the idea of the discovery of a perfectly black paint enabling the detective to cover himself with it and move about invisibly. What is the flaw in the idea?

19-11. The inner silvered coating of a thermos bottle has an area of 120 square inches. The hot coffee inside (100°C) is losing a Calorie per hour when the outer shell of the bottle is at 25°C. Compute the value of B.

19-12. A cake of ice and a thermometer stand near each other long enough for the thermometer to come to a constant reading. Without moving either, it is possible by means of a large reading lens to do some focusing that will result in lowering the reading of the thermometer. Does "cold" travel like light so that it can be focused on the thermometer? Explain.

CHAPTER 20



Expansion

20-1. Linear Expansion of Solids. Since the changes that substances undergo in their dimensions give us the simplest means of measuring temperatures, it is important to consider the relation between expansion and temperature change. We can talk about the lengths of solids, but not of liquids or gases. The increase in length of a solid during a rise in temperature is called a *linear expansion.* The linear expansion is proportional to three things: (1) the original length, (2) the temperature rise, and (3) a constant depending on what material is under consideration. This constant is called the *coefficient of linear expansion.* When e represents the elongation, l the original length, t_1 the lower temperature, t_2 the upper temperature, and therefore $t_2 - t_1$ the temperature rise, and k the coefficient of linear expansion, the relation just stated may be expressed by the following equation

$$e = l(t_2 - t_1)k$$

20-2. Coefficients of Linear Expansion. In the following table are a few coefficients of linear expansion, referred to the centigrade scale.

Aluminum	0 0000236	Iron	0.0000110
Brass	0.0000186	Lead	0.0000282
Copper	0.0000173	Oak, ∥ grain	0.0000049
Ebonite	0 000078	Oak, ⊥ grain	0.0000544
Fused quartz	0.00000040	Platinum	0.0000088
Glass	0.0000088	Silver	0.0000190
Gold	0.0000139	Steel	0.0000111
Ice	0.0000507	Tin	0.0000217
Invar	0.00000088	Zinc	0.0000285

These values are approximate; the exact value of a coefficient of expansion varies with the temperature as well as with the degree of purity of the specimen. It will be noted that although water expands when it freezes, the ice once formed contracts if the temperature continues to drop. A study of the table will make it clear why hot fused quartz may be plunged into cold water without cracking, whereas the same treatment will completely shatter a piece of glass. The coefficients of expansion of platinum and glass are so nearly alike that glass fused around platinum wire will cool without cracking; this is not true of copper wire. It is only at ordinary temperatures that invar (36 per cent nickel and 64 per cent iron) has a low coefficient of expansion. Above 100°C. the value rapidly approaches that of iron.

20-3. Numerical Illustration of Linear Expansion. As an illustration of the use of the equation given above, let us find the allowance that should be made for the expansion of a steel rail thirty feet long if it is to be subjected to fluctuations of temperature between 5°F. and 95°F.

Since the coefficients given in the preceding paragraph hold only for centigrade degrees, it will be necessary either to multiply these coefficients by $\frac{5}{9}$ so that they will apply to Fahrenheit degrees, or to change the Fahrenheit temperatures to centigrade readings. We shall do the former because it is easier. Since the centigrade coefficient of linear expansion of steel is 0.0000111, the Fahrenheit coefficient will be $\frac{5}{9}$ (0.0000111) or 0.00000617; this we shall call k. We set l equal to thirty feet, $t_2 = 95°$, and $t_1 = 5°$; therefore $t_2 - t_1 = 90°$. This gives us

$$e = (30)\,(90)\,(0.00000617)$$

or $e = 0.01666$ foot. This corresponds to a fifth of an inch.

20-4. Balance Wheel on a Watch. If a straight strip of brass and a straight strip of steel are welded together to form a single rod, the combination will be straight at some one temperature. Above this temperature it will be bent with the brass on the outside, and at reduced temperature it will be bent with the steel on the outside, because of the difference in the two coefficients of expansion. If no care were exercised in the construction of the balance wheel of a watch, it would run slowly on hot days and fast on cool days for two reasons: (1) the spring is weaker when hot, and (2) an expanded wheel moves more slowly. By making the balance wheel part steel and part brass (see sketch at the head of this chapter) with the brass on the outside, the diameter of the wheel becomes smaller when the temperature rises, and thus compensates for both effects when correctly adjusted.

20-5. Volume Expansion of Solids and Liquids. The change in volume v, of a solid or of a liquid, with rise of temperature, is proportional to three factors: (1) the original volume V,

(2) the temperature rise $(t_2 - t_1)$, and (3) a constant K, depending upon the material under consideration. This constant is called the *coefficient of cubical expansion.* The equation therefore becomes

$$v = V(t_2 - t_1)K$$

It can be shown that in the case of solids, the value of K is very close to three times the value of k; that is, the coefficient of cubical expansion for a given substance is practically three times the coefficient of linear expansion for the same substance. This makes unnecessary a table of coefficients of cubical expansion of solids. On the centigrade scale, the coefficient of cubical expansion of mercury is 0.000182, and that of alcohol is 0.00110. Water is a bit peculiar. A cubic centimeter of water at 0.0°C. will shrink to 0.999868 cubic centimeter at 3.98°C. At a little over 8°C., its volume is back again to 1.000000 cubic centimeter, and at 15°C. it has a volume of 1.000742 cubic centimeters and is behaving normally enough so that one can say that its coefficient of cubical expansion from that temperature on is 0.000372. If it were not for this peculiarity of water, there would probably be no life on this planet. Life is supposed to have originated in the sea. But if ice did not float on water, and if water at the freezing temperature were not lighter than water slightly above the freezing temperature, then any body of water that ever freezes at all would freeze from the bottom up, that is, would freeze solid. And since water is a poor conductor of heat, the greater part of this ice would remain frozen the year around. thus providing no opportunity for the development of life.

20-6. Numerical Illustration of Volume Expansion. A glass vessel has a volume of 100 cubic centimeters at 0°C.; find the increase in volume when the temperature is raised to 60°C. How much mercury will spill out at 68°C. if the glass vessel is just full of mercury at 0°C?

We shall use the formula in section 20-5. V is 100 cubic centimeters, $t_2 - t_1$ is 60 centigrade degrees, and K is three times the coefficient of linear expansion of glass. (3) (0.0000088) = (0.0000264) = K. The increase in volume of the glass vessel v is therefore

$$v = (100)\ (60)\ (0.0000264)$$

or 0.1584 cubic centimeter. In other words, the space inside of the glass vessel expands in exactly the same way as a solid piece of glass of the same volume. We can find the expansion of 100 cubic centimeters of mercury in a similar way. The equation will be

$$v = (100)\ (60)\ (0.000182)$$

or v = 1.092 cubic centimeters. The difference between 1.092 and 0.158, or 0.934 cubic centimeter, is the quantity of mercury that will overflow.

The fact that the coefficients of cubical expansion of liquids are greater than those of solids is the underlying principle of the ordinary thermometer.

20-7. Volume Expansion of Gases. A small change in pressure has little effect on the volume of either a solid or liquid, but in the case of a gas, pressure is important. It is therefore necessary to take both temperature and pressure into account in dealing with the volume expansion of gases. Furthermore, even if the pressure were held constant, the value of the coefficient of cubical expansion is different for every initial temperature. At constant temperatures, the pressure on a gas, multiplied by the volume of the gas, is a constant. (Boyle's law, see section 5-1.) The word *pressure* as used in this section is not simply the "gage pressure" as registered by a steam pressure gage or automobile tire gage; it is 14.7 pounds per square inch more than the "gage pressure." When a tire gage or a steam gage registers zero pounds per square inch, there is not a complete lack of pressure (perfect vacuum), but simply the same pressure inside as outside (atmospheric pressure, which is 14.7 pounds per square inch). It will now be apparent why "gage pressure" must be increased by 14.7 pounds per square inch to get the total pressure. If the temperature changes as well as the pressure, we can express the situation mathematically by saying that PV/T is constant, where P, V, and T are respectively the total pressure, the volume, and the *absolute* temperature. For the purpose of solving problems, it is convenient to introduce three more quantities: the new total pressure P', the new volume V', and the new absolute temperature T', of the same mass of gas, so that the equation becomes

$$\frac{PV}{T} = \frac{P'V'}{T'}$$

20-8. Numerical Illustration of the Gas Law. Let us consider an automobile tire, the volume of which is 1,349 cubic inches when inflated to a gage pressure of 30.3 pounds per square inch at minus 9°C. Assume that the volume of the tire increases to 1,350 cubic inches when the temperature rises to 24°C. What is the gage pressure under the new conditions?

It will be necessary to use the equation of section 20-7. $V = 1{,}349$, $V' = 1{,}350$. The gage pressure, 30.3 pounds per square inch, must be increased by 14.7 pounds per square inch to give the total pressure required in the formula. Therefore $P = 45.0$ pounds per square inch, and P' is the unknown. The temperatures, as stated, are on the ordinary centigrade scale and must be changed to the absolute scale before they will fit into the equation. $T = -9 + 273 = 264$, and $T' = 24 + 273 = 297$. The complete equation then becomes

$$\frac{(45.0)\,(1{,}349)}{264} = \frac{(P')\,(1{,}350)}{297}$$

Solving, we find that P' is 50.6 pounds per square inch total pressure, which corresponds to a gage pressure of 35.9 pounds per square inch, the required answer.

SUMMARY OF CHAPTER 20

Technical Terms Defined

Coefficient of Linear Expansion. A quantity characteristic of a particular substance in a particular condition found by dividing the increase in length of the specimen by its original length and the change in temperature which caused the elongation.

Coefficient of Cubical Expansion. The quotient of the increase in volume of a particular specimen by its original volume, and by the corresponding change in temperature. In the case of solid materials, the coefficient of cubical expansion is three times the coefficient of linear expansion.

Gas Law

The product of the total pressure of a gas by its volume divided by its absolute temperature is a constant for a given mass of gas.

PROBLEMS

20-1. A distance of 1,000 feet (correct value) is measured with a steel tape correct at 15°C. on a day when the temperature is 25°C. What is the reading of the tape?

20-2. A distance is measured with a steel tape which is correct at 15°C. when the temperature is 25°C., and the uncorrected value is found to be 2,000 feet. What is the corrected value?

20-3. Imagine two concentric circles drawn upon a sheet of copper and the material inside the smaller and outside the larger cut away. If the piece of copper is now heated 50 centigrade degrees, will the inside circle grow larger or smaller? If the two diameters are ten inches and twelve inches at the lower temperature, find the diameters at the higher temperature.

20-4. In section 20-3, what would be the two centigrade temperatures corresponding to 5°F. and 95°F.? Find the expansion of the thirty-foot steel rail between these temperatures, using the centigrade coefficient of expansion.

20-5. Referring to the data in section 20-5, describe numerically just what will happen to exactly one cubic *inch* of water as the temperature rises from zero degrees centigrade.

20-6. How much will a steel rail shorten when the temperature drops 50 centigrade degrees, if the original length is 30 feet? If Young's modulus for steel is 28,000,000 lb./in.^{2} and the rail has a cross-sectional area of 12 square inches, find the force necessary to restore the steel rail to its original length.

20-7. A steel ball one centimeter in diameter is too large to go through a hole in an aluminum plate at 0°C., but will just go through when both are heated to 80°C. Find the diameter of the hole at 0°C.

20-8. A cube of iron is 10 centimeters on an edge at 0°C. Find the change in length of one edge when the temperature rises to 100°C. Find the change in volume. Substitute in the formula in section 20-5, and compute the coefficient of cubical expansion.

20-9. The equation in section 20-1 may be supplied with units as follows: e feet = (l feet) ($t_2 - t_1$ degrees centigrade) (k/°C.) For example, k might be numerically 0.0000236 per degree centigrade, which is also sometimes read 0.0000236 reciprocal degrees centigrade. It will be noticed that the product of the three units on the right-hand side of the equation is "feet," the unit on the left-hand side; that is (feet) (°C./°C.) = feet. In a similar way, determine the unit belonging to K in section 20-5.

20-10. A block of ice at − 20°C. contains a cavity just one cubic centimeter in volume. At what temperature will the volume of the cavity be three-tenths of a per cent larger? What will the volume of the cavity be at 0°C?

20-11. If a certain mass of air occupies just one liter at 0°C. and at an absolute pressure of one atmosphere, find the volume at one atmosphere and 100°C.; at 200°C. Answers: 1.366 liters; 1.733 liters.

20-12. Using the data of the previous problem, as well as the answers, find the coefficient of cubical expansion of air when the initial temperature is 0°C.; when the initial temperature is 100°C. Does the result of this problem check the statement in section 20-7?

20-13. In the illustrative problem in section 20-8, recompute the gage pressure, assuming that the volume of the tire remains constant.

CHAPTER 21

Calorimetry

21-1. Measurement of Heat. *Temperature* represents the average energy of translatory motion of a single molecule, whereas *heat* represents the combined potential, translatory kinetic, and rotatory kinetic energies of all the molecules in the object under consideration. The quantity of heat in a body corresponding to the total energy of all its molecules may be considered equal to the product of three factors: one equal to the total energy of one molecule; a second equal to the number of molecules in one unit of mass (for example, the number of molecules per unit mass); and the third factor equal to the number of units of mass in the body. The absolute temperature is proportional to the translatory kinetic energy of one molecule, and roughly proportional to the total energy of one molecule. A physical quantity known as the *heat capacity per unit mass* is the number of heat units necessary to raise the temperature of unit mass one degree, and is roughly proportional to the number of molecules in a unit of mass. This statement of proportionality is known as *Dulong and Petit's law.* The ratio of the heat capacity per unit mass of a given substance to the heat capacity per unit mass of *water* is called the *specific heat* of that substance, and by choosing the unit of heat in such a way that the heat capacity per unit mass of water shall be unity (1.000), we can make specific heat numerically equal to heat capacities per unit mass. This reminds us of the fact that specific gravity and density are numerically alike in the centimeter-gram system (section 5-2). The *British thermal unit* (B.t.u.) is the quantity of heat necessary to raise the temperature of one pound of water one degree Fahrenheit, and the large Calorie (or in this book, simply Calorie with a capital *C*) is the quantity of heat necessary to raise the temperature of one kilogram of water one degree centigrade. Since in this book it is understood that we work only to slide-rule accuracy, which is sufficient for engineering purposes, it is not necessary to specify *which* degree the water has been raised through, although it does make a slight difference. The *small calorie*, written with a small *c*, is equal to 1/1,000 of a Calorie. 3.97 B.t.u. equal 1 Calorie, 3,410 B.t.u. equal one kilowatt-hour, and 858 Calories equal a kilowatt-hour. This is the Calorie we hear so much about in dietetics.

When we raise the temperature of a body, then, the increase of heat is the product of the increase of temperature, the heat capacity per unit mass (which is numerically equal to the specific heat), and the mass. If we represent the increase of heat by H, the heat capacity per unit mass by c, and the mass by m, the relation may be represented by the following equation

$$H = (t_2 - t_1)\,(c)\,(m)$$

21-2. Definition of Specific Heat. It is possible to define specific heat without the use of the term heat capacity just as it was possible to define specific gravity without the use of the term density, but such a definition is somewhat clumsy. The specific heat of a substance is the ratio of the quantity of heat necessary to raise the temperature of a given mass of the substance a certain number of degrees to the quantity of heat necessary to raise the same mass of water the same number of degrees. From this definition it is clear why the specific heat of a substance has no units, and why it has the same value regardless of whether centigrade degrees or Fahrenheit degrees are used and whether pounds or kilograms are employed. Since the heat capacity of a body is the quantity of heat necessary to raise the temperature of the body one degree, the introduction of the term heat capacity into the definition of specific heat tends to brevity.

21-3. Numerical Illustrations of Calorimetry. The measurement of heat is known as *calorimetry*; two problems will be worked as illustrations.

(1) How many pounds of boiling hot water (212°F.) must be added to 610 pounds of water at 40°F. in order to get a final mixture at 90°F.? The procedure will be to equate the heat gained by the cold water to the heat lost by the hot water, thus utilizing the law of conservation of energy. For this purpose we use the equation in section 21-1. Since the specific heat of water is unity, the heat gained by the cold water is (90 − 40) (1) (610) or 30,500 B.t.u. Similarly the heat lost by the hot water is equal to

(212 − 90) (1) (*m*) where *m* represents the number of pounds of hot water necessary. Equating these two quantities of heat and solving for *m*, we obtain 250 pounds for our answer. The student will notice that while the mechanical engineer uses the pound as a unit of weight, the heat engineer uses it as a unit of mass. (See section 11-9.)

(2) Let us suppose that an experiment is conducted for the purpose of determining the specific heat of lead (which is 0.03). Let us assume that 0.05 kilogram of lead shot have been heated to a temperature of 100°C. and then dropped into 0.120 kilogram of water at 19°C. Let us assume further that the resulting mixture has a temperature of 20°C. The equation expressing the fact that the heat lost by the lead is gained by the water is

$$(100 - 20)\,(c)\,(0.050) = (20 - 19)\,(1)\,(0.120).$$

Solving, we find that $c = 0.120/4$ or 0.030.

21-4. States of Matter. Matter is usually said to exist in three states: solid, liquid, and gaseous. In the solid state, the word *molecule* loses its significance; the solid is practically one large molecule in which the atoms are very closely packed and do not on the average leave their positions, but merely oscillate about a mean position, the rate of oscillation depending upon the temperature. In a liquid the word *molecule* regains its usual meaning, although the particles are nearly as close to each other as in a solid, but, due to a higher rate of motion, they now zigzag about among their neighbors. This results in the fact that a liquid has no fixed shape of its own, but merely a fixed volume (at a given temperature). In the case of a gas, the molecules actually move fast enough to result in a complete separation from the neighboring molecules. Thus, a gas has neither a fixed shape nor a fixed volume, but expands to fill the space available. When one compresses a gas, he is really compressing the spaces between the molecules. Molecules attract each other; this is why the parts of a solid stick together so securely and why liquids tend to cling together in drops. In gases, however, the molecules are sufficiently far apart to exert very little attraction on each other.

21-5. Energy is Required to Separate Molecules. The attractive forces between molecules are labeled *cohesion* when the molecules involved are alike and *adhesion* when the molecules are different. Whenever two molecules have been separated, work has been done and we say that energy has been expended. We recognize the fact that the potential energy of the molecules has been increased. If the space under the plunger of an air pump is increased, the motion of the molecules drives them farther apart and utilizes the extra volume. But the energy necessary to do this is at the expense of the kinetic energy of the molecules, and as a result, their average speed has been decreased. Since the temperature is proportional to the average energy of translatory motion of the molecules, the temperature is lowered by a sudden expansion. On the other hand, a sudden compression of air, as in a bicycle pump, will raise its temperature. A very striking case of increase of molecular potential energy is when a liquid changes into a vapor. We call this process *evaporation* and observe that the remaining liquid tends to be cooled by the process. If, however, we supply the necessary energy in the form of heat from some external source, it is observed that a definite quantity of heat is necessary to vaporize a definite mass of the substance.

21-6. The Triple Point Diagram. The relation between pressure, temperature, and changes of state are best shown by the so-called triple point diagram (figure 21-1). Suppose we start with

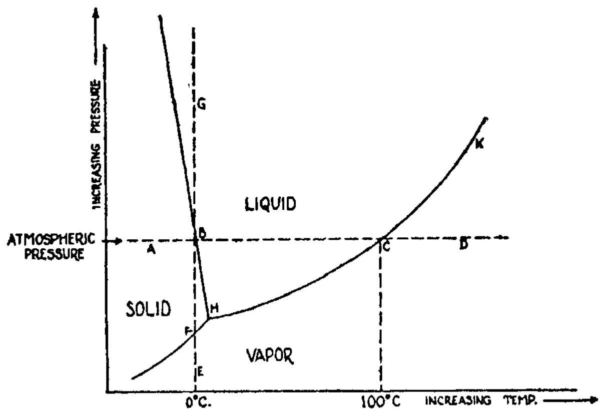
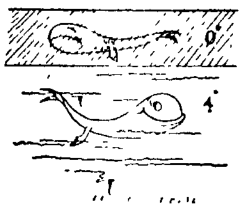


Figure 21-1.

a sample of ice (solid) at atmospheric pressure, and at a temperature below 0°C., point A on the diagram. Keeping the pressure constant and raising the temperature, we presently reach point B. The temperature remains at this value (0°C.) until all the ice is melted. A further rise in temperature brings us to point C, and before the temperature can rise any further, all the water must vaporize. Point D therefore represents water vapor. It is possible, however, to start with a low pressure and temperature (point E) with the vapor phase. Keeping the temperature constant and increasing the pressure, the vapor will condense to ice at point F. So far the pressure has been less than one atmosphere. When the pressure reaches one atmosphere, the ice will melt, and at higher pressures (at 0.°C.) will remain liquid water. An inspection of the diagram will reveal the fact that at pressures less than one atmosphere, the melting point of ice is above 0°C., and the boiling point below 100°C. Ice at low pressures will "sublime," that is, change directly from solid to vapor with rise of temperature.

The boundary line between the solid and liquid region slopes upward to the left in the case of water and a few other substances. But in the large majority of cases this line slopes upward to the right. There is a direct connection between the slope of this line and the fact that water expands when it freezes. When a substance expands as it freezes, an increase of pressure tends to put it into the state in which it has the smaller volume, namely, the liquid state. On the other hand, a substance like paraffin or aluminum, which contracts as it freezes, in tending to go into the state with the smaller volume, solidifies. This means that the freezing point of water *decreases* with increase of pressure, while the freezing points of paraffin and aluminum *increase* with increase of pressure. Skating is much easier when the temperature is close to 0°C. than when it is very cold, because in the former instance the skater is actually skating upon water. This is because the pressure is so high directly under the skate that the ice finds itself above its freezing point and liquefies, although it immediately freezes again after the skater has passed on.



The point H is called the triple point. Imagine a vessel containing ice, liquid water, and water vapor sealed off and maintained for an indefinite time at the temperature and pressure represented by H. The proportions of the three phases will not change. This does not mean that if we started with a perfect cube of ice floating upon some water with water vapor in contact with each, that we could come back after a week and find our ice still in the form of a perfect cube. There would still be the same volume of ice present but its shape would be different. This is because in the equilibrium under consideration, six things are going on at once, but at balancing rates: ice is both melting and subliming, water is both evaporating and freezing, and water vapor is condensing to both the solid and liquid forms, all simultaneously.

The curve HCK comes to a definite end at the so-called *critical* *point.* The critical temperature in the case of water is 365°C.; above this temperature water exists only in the gaseous state. Oxygen, nitrogen, and hydrogen, as well as several minor constituents of air, are gases which at ordinary temperatures are well above their critical temperatures. Before the relations depicted in the triple point diagram were understood by scientists, time and money were wasted in the attempt to liquefy these gases by putting them under pressure. It is now known that their temperatures must be lowered at least to the critical point before there is any hope of liquefying them.

21-7. Artificial Refrigeration. Artificial refrigeration depends on the principle that evaporation is a cooling process. A working substance is compressed or condensed, and the resulting heat removed. Then the reverse process is allowed to transpire in the place where the refrigeration is desired, with the result that heat is withdrawn from the objects in the refrigerator in order to supply the necessary energy for expansion, or evaporation.

21-8. Heat of Vaporization. When heat is added to a liquid at its boiling point, the temperature does not rise while the boiling is taking place, but the newly formed vapor has the same temperature as the liquid, and during this process, a definite amount of heat is necessary to vaporize each unit of mass. For example, it requires 540 Calories to vaporize each kilogram of water. Letting L stand for the heat of vaporization, we can say that when m grams of liquid change to m grams of vapor, the amount of heat necessary to produce the change H, is given by the equation

$$H = (L)\,(m)$$

With a different value for L, the same equation can be used when the units are B.t.u. and pounds. The equation in section 21-1 holds when there is a change of temperature, but no change of state, and the equation just stated above holds when there is a change of state and no change of temperature.

21-9. Illustrative Problem. By the use of these two equations, we can solve a problem in which we have both a change of state and a change of temperature. How much heat is necessary to change 10 kilograms of water at 90°C. into steam at 115°C.? The specific heat of steam is 0.48, considerably different from that of liquid water.

The problem must be separated into three parts: first find the heat necessary to raise the liquid water to the boiling point, next find the heat necessary to vaporize the water at 100°C., and finally find the heat necessary to raise the steam from 100°C. to 115°C. This means applying the equation in section 21-1 twice, and the equation of section 21-8 once.

Putting in all the numerical values, we have $H = (100 - 90)\,(1)\,(10) + (540)\,(10) + (115 - 100)\,(0.48)\,(10)$, or H equals 5,570 Calories, rounding the answer off, as is our custom, to the first three significant figures.

21-10. Heat of Fusion. To a less degree, a similar situation holds during the transition from the solid to the liquid state. It is necessary to add a given quantity of heat to change a given mass of solid to the liquid form, and the mixture remains at the temperature of melting as long as there is both solid and liquid present, efficient stirring being assumed. It requires 80 Calories to melt one kilogram of ice; this quantity is called the heat of fusion. Eighty Calories per kilogram is equivalent to 144 B.t.u. per pound. The process is reversible; by extracting 80 Calories of heat from each kilogram of water at 0°C., it is possible to freeze water. The equation in section 21-8 may be used for fusion if L be interpreted as heat of fusion instead of heat of vaporization.

21-11. Illustrative Problem. Imagine a mixture of 20 kilograms of ice at 0°C. and 100 kilograms of water also at 0°C. contained in a copper tank weighing 25 kilograms. The specific heat of copper is 0.093. How much steam at 110°C. must be passed into the mixture to bring the temperature up to 20°C.?

The steam will lose a certain quantity of heat, and the ice, water, and copper vessel will gain this same heat, therefore the procedure will be to put on one side of an equation the heat lost and on the other side of the equation the heat gained. The equation thus becomes

$$\begin{aligned}(110 - 100)\,(0.48)\,(m) + (540)\,(m) + (100 - 20)\,(1)\,(m)\\ = (80)\,(20) + (20 - 0)\,(1)\,(100 + 20) + (20 - 0)\,(0.093)\,(25)\end{aligned}$$

m is therefore equal to $(1{,}600 + 2{,}400 + 46.5)/(4.8 + 540 + 80)$, or 6.48 kilograms of steam.

SUMMARY OF CHAPTER 21

Technical Terms Defined

Heat Capacity. The heat capacity of a body is the number of heat units necessary to raise its temperature one unit. Units are Calorie per degree centigrade or B.t.u. per degree Fahrenheit.

Specific Heat. The ratio of the heat capacity of the given body to the heat capacity of the same mass of water. It is therefore a pure number. Numerically it has the same magnitude as the heat capacity per unit mass, but the latter has units (either B.t.u. per pound per degree Fahrenheit, or Calories per kilogram per degree centigrade).

Triple Point. The point on a temperature-pressure diagram at which solid, liquid, and vapor are in equilibrium.

Sublimation. A direct change from the solid state to the vapor state without passing through the liquid state.

Critical Temperature. The highest temperature at which it is possible to liquefy a vapor by increasing the pressure sufficiently.

Critical Pressure. The pressure necessary to liquefy a vapor at the critical temperature.

Heat Of Vaporization. The number of heat units necessary to vaporize unit mass of liquid at normal atmospheric pressure. The process is called *boiling.*

Heat of Fusion. The number of heat units necessary to melt unit mass of solid at normal atmospheric pressure.

PROBLEMS

21-1. Iodine passes directly from the solid to the vapor state (sublimes) under ordinary conditions. What could be done to obtain iodine in the liquid state?

21-2. If figure 21-1 were to be replaced by the incorrect figure 21-2, in what state would the substance be inside the small triangle *PQR*? How would you proceed to prove that these three lines would have to meet at one point *H* as in figure 21-1?

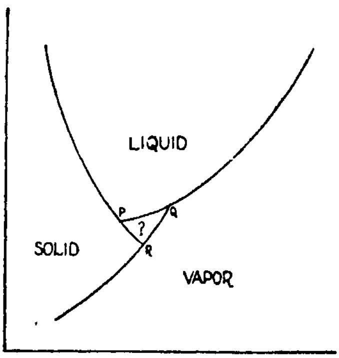


Figure 21-2.

21-3. Imagine compressing a quantity of gaseous water at 380° C. until all the molecules are in contact with each other. What property of a solid will the mass still fail to have? What property of a liquid will it fail to have?

21-4. Why is the boiling point of water only 86°C. on Pikes Peak, Colorado? At which place will an egg cook sooner by boiling, Pikes Peak or Boston?

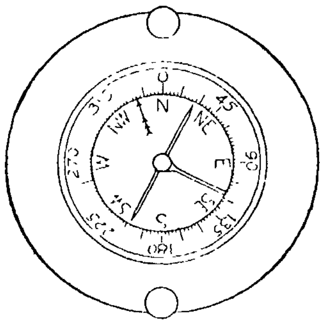
21-5. How much heat is necessary to change 2 kilograms of ice at −10°C. to water at 90°C., if the specific heat of ice is 0.5?

21-6. What is the resulting temperature when 50 grams of lead shot (specific heat = 0.03) at 90°C. are poured into 1 kilogram of water contained in a brass calorimeter weighing 200 grams, both the water and container being originally at 20°C.? The specific heat of brass is 0.09.

21-7. How many pounds of steam at 212°F. let into a swimming pool containing 10,000 cubic feet of water will be necessary to raise its temperature from 62° to 70°F.? One cubic foot of water weighs 62.4 pounds.

21-8. How many pounds of coal, the heat value of which is 14,400 B.t.u. per pound, will be needed in a boiler the efficiency of which is 100 per cent, to convert 100 pounds of water at 62°F. into steam at 212°F.? (No boiler can have an efficiency of 100 per cent.) How many pounds of coal will it take if the efficiency is 50 per cent?

CHAPTER 22



Magnetism

22-1. Elementary Facts of Magnetism. Every boy knows that a horseshoe magnet will *attract* a steel needle. Moreover, if the needle is first stroked several times in the same direction by one of the ends of the magnet, it will be found possible to hold the horseshoe magnet at a suitable distance from the now magnetized needle such that *repulsion* will take place if the relative positions are correct. If the large magnet is brought too near the needle, the magnet is likely to reverse the newly acquired magnetization of the needle and produce attraction. Or, if the ends of the needle are just reversed in position, attraction will result. If a knitting needle is magnetized in the manner described above and suspended by a single thread, it will set itself in a north-and-south position, thus constituting a magnetic compass. This provides us with a basis for naming the two ends. The end that points north will henceforth be called the *north pole*, and the other end will be called the *south pole*. If two knitting needles are magnetized and similarly suspended near each other, it will be found that (1) their north poles repel each other, (2) their south poles repel each other, but (3) a north pole attracts a south pole. If a magnetized knitting needle is thrown violently on the floor several times in random directions, it will lose most of its magnetism; or if it is heated red hot, it will cease to be a magnet. But if an unmagnetized knitting needle is held in a north-and-south direction and hammered a number of times, it will acquire a slight amount of magnetism. Finally, if a magnetized knitting needle is cut in two, both halves will now be found to be complete magnets; this process may be continued indefinitely.

22-2. The Underlying Theory. Everything that has been said in the previous paragraph may be explained if we think of steel (or iron) as made up of a very large number of little magnets, each of which is more or less free to change its orientation. An unmagnetized needle is merely an aggregation of these little magnets, all in complete disarray, so that the north poles of all the little magnets point in thoroughly haphazard directions; therefore the needle as a whole exhibits no evidences of magnetism. Stroking the needle systematically from one end to the other (but not in both directions) with the north pole of a strong magnet, swings most of the south poles of the little magnets into similar positions, and when they are thus lined up, the needle as a whole behaves like another magnet. The elementary magnets swing more freely in soft iron than they do in hardened steel, and for this reason it is easier to magnetize soft iron than it is steel, but likewise it is easier for soft iron to lose its magnetism; in fact, soft iron loses its magnetism almost immediately after the magnetizing process stops. In the case of steel, where the elementary magnets swing with more difficulty, another effect has a chance to show itself; once the little magnets are lined up, each one tends to be held in position by its neighbors, since north poles attract south poles. Since the earth is itself a huge magnet, hammering a knitting needle held in a north-and-south position (especially with the north end held lower than the south end) tends to make the elementary magnets behave like little compasses and line up in parallel directions. This is why the hulls of steel ships are found to be magnetized after the riveting process is completed. Heating the magnet to a red heat agitates the individual molecules violently in a haphazard way, breaks up the formation of the elementary magnets, and thus produces demagnetization. Similarly, throwing the needle on the floor tends to demagnetize it by disarranging the little magnets. Furthermore we can now see why a magnet cut in two yields two complete magnets, because, if the cutting were continued until we had the individual elementary magnets all separated, each would still have its own north and south poles.

22-3. The Earth as a Magnet. We have seen that the north pole of a magnet is so called because it points nearly north when the magnet is mounted so that it is free to swing, that is, when functioning as a compass. But we have also seen that north poles repel each other and that north and south poles attract. Therefore the polarity of the geographically northern end of the earth must be magnetically south! In the United States, the north pole of a perfectly balanced magnet, in this case called a *dipping needle*, tends to pull down as well as to point north. This action is called dipping, and is due to the fact that we are nearer to the north pole than we are to the south pole. If we were to stand directly over a magnetic pole of the earth, the magnet would orient itself vertically. The earth is a magnet because of its rotation relative to an excess of positive ions in the upper atmosphere. The magnetic poles do not exactly coincide with the geographic poles because of the irregular distribution of iron in the earth together with the fact that the earth is constantly being bombarded with electrons from the solar sunspots in an irregular fashion. We shall see later that motion of electric charges (which includes electrons and positive ions) produces magnetic effects.

22-4. Magnetic Lines of Force. It becomes rather easier to discuss both electricity and magnetism if we introduce the idea of "magnetic lines of force." We visualize these lines as coming out of the north end of a magnet and going in at the south end. Their direction in space at any point is that which a small compass needle would take at the point in question. Thus the north end of a small compass needle would point toward the south end of a large magnet. If a sheet of paper, or a glass plate, or some other thin nonmagnetic substance is placed upon a strong magnet, and iron filings sprinkled upon it, each filing will constitute a tiny compass and will set itself parallel to a line of force at that point. Thus we can determine experimentally the direction of the lines of force. A line of force is considered to be a closed curve, passing through the magnet as well as through the region outside of the magnet. By definition, the lines pass from the south end of the magnet to the north end *inside* of the magnet, and from the north end to the south end *outside* of the magnet. (See figure 22-1). In terms of lines of force, a north pole is a region where lines of force emerge from a piece of iron, and a south pole is a region where lines of force enter a piece of iron. Lines of force prefer to pass through iron rather than air. Lines of force tend to shorten, and two lines of force adjacent to each other repel each other. The works of a watch may be considerably shielded from magnetic effects by enclosing them in an iron case. Then any lines of force in the region of the watch will pass through the iron of the case rather than through the air within the case, and very little magnetic effect will be experienced inside the case. A *magnetic field* is a region containing magnetic lines of force.

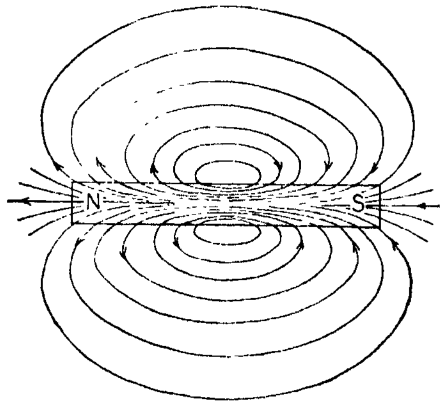


Figure 22-1.

22-5. Quantitative Aspects of Magnetism. Two magnetic poles repel or attract each other more strongly when near each other than when far apart. If we double the distance between them, the effect becomes only one fourth as great, and if we treble the distance, the effect drops to one ninth. If we let the strength of the two poles be represented by p_1 and p_2 respectively, the distance between them by d, and the force of repulsion by F, the relation* is

$$F = \frac{p_1 p_2}{\mu d^2} k_m$$

We shall assume in this equation that d is in meters, F in newtons, and p_1 and p_2 in the corresponding unit of the so-called practical system of electrical units which ties in with the meter-kilogram-second system. This unit of pole has no name, so we shall refer to it as a "pole unit." The value of k_m is 10,000,000 newton-meters^2 per pole unit squared. We shall find it more convenient to express this number as 10^7. μ represents a pure number which is called the *permeability* of the medium. Its value for a vacuum is exactly unity. For air it is slightly more, 1.00026, and for a few materials considerably more, but in most cases we may safely ignore it as a factor. Since a given magnet pole is always attached to another of equal strength and opposite polarity, the poles which are not involved in the equation just mentioned must be considered to be a great distance away. That is, the magnets concerned are very long.

It is often convenient to compute the force which would exist at a given point on a north pole of unit strength. In order to visualize this, we must imagine the unit magnet again to be very long so that the south pole which inevitably accompanies a north

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* This relation was discovered by the French physicist, Charles A. Coulomb, 1736-1806, and is often labeled with his name.

pole is far enough away to have a negligible effect on the situation. The force per unit north pole at the given point is called the *field intensity* at that point, and is generally represented by the letter H. Knowing the value of the field intensity, it is merely necessary to multiply it by the strength of an actual pole placed at the given point to find the force on the actual pole. That is

$$F = pH$$

where F is in newtons, p in "pole units," and H in a unit which is one thousandth of a certain unit in the c.g.s. system called the *oersted*; we may therefore speak of H as measured in millioersteds. A uniform field of H millioersteds is considered to consist of lines of force just far enough apart so that H of them would pass through a square meter placed at right angles to the field.

A convenient method of measuring the field intensity at a given point is to set up a small compass needle there, displace it slightly from its equilibrium position, and let it oscillate. Then perform the same experiment with the same compass needle at a place where the magnetic field is known. If the two fields are represented respectively by H_1 and H_2 and the two frequencies of oscillation respectively by f_1 and f_2, the relation between the four quantities will be

$$\frac{H_1}{H_2} = \frac{f_1^2}{f_2^2}$$

The student will be able to convince himself that this equation is correct by recalling the relations for a compound pendulum in a gravitational field (section 15-11) where the period T was shown to be inversely proportional to the square root of the gravitational field, g. Therefore the frequency is directly proportional to the square root of the gravitational field, or in our present problem, to the square root of the magnetic field.*

22-6. Illustrative Problems. (1) Given two long magnets one with poles of 5 microunits and the other of 6 microunits. (*Micro* means *one millionth* in scientific work). If the north pole of the first magnet is placed within 0.1 meter of the north pole of the second magnet in air (with the two south poles as far apart as possible), find the repulsive force exerted by one north pole on the other.

Solution: Let $p_1 = 0.000005$, $p_2 = 0.000006$, $k_m = 10{,}000{,}000$, and $d = 0.1$. Substituting these values into the equation $F = k_m p_1 p_2/d^2$ gives

* The period T, of the oscillating magnet in a horizontal magnetic field H, is given by

$$T = 2\pi\sqrt{I/Hlp}$$

where I is the moment of inertia of the magnet, l its length, and p its pole strength.

$$F = \frac{(10{,}000{,}000)\ (0.000005)\ (0.000006)}{(0.1)^2} = 0.03 \quad \text{newton}$$

A newton is a little less than a quarter of a pound (4.45 newtons = 1 pound) so it is seen that this force is not very large, although the magnets of this problem represent fairly strong "permanent magnets." Electromagnets can be made much stronger.

In handling large or small numbers the so-called index notation is convenient. Thus in this problem it would have been easier to state the equation

$$F = \frac{(10^7)\ (5)\ (10^{-6})\ (6)\ (10^{-6})}{(10^{-1})^2} = 3(10^{-2}) = 0.03 \text{ newton}$$

(2) (a) Find the field at the vertex of an equilateral triangle 0.2 meter on a side if at the other two vertices we have respectively the north and south poles of a cobalt steel magnet of pole strength 4 micropole units. (b) How many lines per square meter would we have in a uniform magnetic field of this strength? (c) What force will act on a magnet pole of 0.000001 pole unit if placed at the point p?

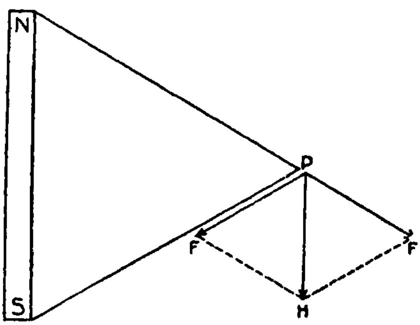


Figure 22-2.

Solution: (a) In order to find the field at the point p of figure 22-2, we first imagine a unit north pole at the point p, then compute the force F_1 due to the repulsion arising from N, then find the force F_2 due to the attraction of S, and lastly get the resultant H.

$$F_1 = \frac{10^7(4)10^{-6}(1)}{(0.2)^2} = 1{,}000 \text{ millioersteds}$$

Since exactly the same numerical values enter into the computation of F_2, this is also 1,000 millioersteds. And since the parallelogram contains two equal forces 120 degrees apart, the resultant H is also 1,000 millioersteds, parallel to the magnet.

(b) The number of lines per square meter will be numerically equal to the field strength in millioersteds. Therefore in this case the field corresponds to 1,000 lines per square meter.

(c) If a pole of 10^{-6} units is placed at the point p, the force acting on it will be given by the relation $F = pH$. In this case it will be

$$F = (10^{-6})\ (10^3) = 10^{-3} \text{ newton}$$

This is a small force. A newton is about 3.6 ounces, therefore this force is about 1/278 of an ounce.

(3) A certain compass needle takes 50 seconds to make one complete oscillation in the earth's magnetic field, the horizontal component of which

is 166 millioersteds. When placed near the poles of a certain electromagnet it oscillates five times per second. Find the field at this point.

Solution: Let f_1 be 0.02 vibration per second, the reciprocal of a period of 50 seconds per vibration. Let $f_2 = 5$ vib./sec. Let H_1 be 166 millioersteds. H_2 may be found by substituting in the equation $H_2/H_1 = f_2^2/f_1^2$

$$\frac{H_2}{166} = \frac{5^2}{(0.02)^2}$$

Solving, we obtain $H_2 = 10{,}380{,}000$ millioersteds. Fields more than ten times this figure may be obtained by well designed electromagnets.

22-7. Demagnetization. A piece of soft iron will become temporarily magnetized very easily when brought near a strong magnet, but it will immediately lose its magnetism, that is, become demagnetized, when it is taken away again. Soft iron is therefore used for the cores of electromagnets. On the other hand, it is more difficult to magnetize a piece of hardened steel, but once magnetized it tends to retain its magnetism and becomes a *permanent magnet.* By shaping the permanent magnet like a horseshoe, its two poles are brought near to each other, thus resulting in a more concentrated magnetic field. It is sometimes desired to demagnetize a piece of magnetized steel. Heating is a possible method, for heat represents a random molecular motion and therefore tends to disarrange the little magnets that constitute a permanent magnet. Raising to a red heat will spoil any permanent magnet. The hairspring of a watch cannot be demagnetized by this method, however. The magnetization in this case can be removed by placing the watch in the magnetic field produced by an alternating electric current and then slowly removing it. All electric currents are surrounded by magnetic fields, and an alternating current is surrounded by an alternating field that reverses its direction many times a second. Thus the elementary magnets in the hairspring attempt to wheel about every time that the field reverses its direction. As the watch is gradually removed from the field, a decreasing number of the little magnets within the hairspring is able to obey the alternating field until finally the little magnets are completely disarranged. The same effect may be produced by means of a permanent magnetic field and a rotating watch. In this case the watch is merely suspended from a twisted string in a strong horizontal magnetic field and gradually removed as the string untwists.

22-8. Additional Evidence of the Identification of Magnetism with Arrangement of Elementary Magnets. Anyone who has ever been near a large electric transformer operating on a 60-cycle alternating current has noticed the so-called "60-cycle hum." In fact, if one listens carefully to a small transformer or even an electromagnet with alternating current flowing through it, the same 60-cycle tone will be heard. This is due to the fact that the little magnets in the iron of the transformer are obliged to turn over with great regularity every time the current reverses direction. A regular vibration in the region which the ear can detect constitutes a musical tone. The 60-cycle tone is a fairly low pitch; on the other hand, a 500-cycle transformer emits a rather high-pitched squeal.

22-9. Magnetism Not Confined to Iron. Every substance is affected to some slight degree by a magnetic field, but it requires rather strong magnetic fields to demonstrate it except in the case of iron. For this reason it is usual to consider iron the only magnetic material, in spite of the fact that magnetism is actually a universal property of matter. Some substances make a feeble attempt to get out of a strong magnetic field; they are called *diamagnetic.* Other substances which make a feeble effort to get into a strong magnetic field are called *paramagnetic.* Iron, which is in a class almost by itself, is called *ferromagnetic.*

SUMMARY OF CHAPTER 22

Technical Terms Defined

Magnetism. A property possessed feebly by all substances but to a pronounced extent by iron; it involves a rearrangement of the constituent particles of the material and results in attractions and repulsions between the magnetized objects. A magnetized piece of iron or steel is called a *magnet.*

Magnetic Pole. One of the ends of a magnet or other portion where the forces are greatest.

Compass. A magnet in the shape of a bar or needle mounted so that it is free to rotate in a horizontal plane. The end that points north is labeled the north pole and the other end the south pole.

Unit Pole. Two poles, just alike, in vacuo, which will repel each other with a force of 10,000,000 newtons when separated by a distance of one meter, are said to be unit poles in the practical system of electrical units.

Magnetic Field Strength. The strength of a magnetic field at a given point is the magnetic force in newtons which will be exerted on a unit north pole placed at that point.

Magnetic Lines of Force. A magnetic field is described by "lines of force." These are thought of as emerging from north poles and re-entering the magnets at the south poles, forming closed curves. A small compass needle or iron filing will set itself parallel to a line of force. It is considered that there is the same number of lines per square meter as the numerical value of the field strength in newtons per unit pole.

Diamagnetic Substances. Substances which tend feebly to remove themselves from a magnetic field. Usually the best they can do is to set themselves crosswise in a field.

Paramagnetic Substances. Substances that feebly set themselves parallel to a magnetic field.

Laws of Magnetism

Like poles repel each other; unlike poles attract.

A given magnet always possesses the same strengths of north and south polarity.

It is impossible to separate a north pole from a south pole.

Coulomb's Law. A magnetic pole will exert a force on another pole which is proportional to the product of the two pole strengths and inversely proportional to the square of the distance between them.

PROBLEMS AND EXERCISES

22-1. Why is it impossible to have a north pole unaccompanied by a south pole?

22-2. Assume two bars, one of soft iron and the other of steel, the latter magnetized; state how you could distinguish the magnetized bar from the other, with no equipment other than the two bars and your two hands.

22-3. Give an opinion as to whether a compass floated on the ocean will start a journey toward the nearer of the earth's magnetic poles.

22-4. Discuss the permanency of a bar magnet made by filling a soda-fountain straw with iron filings, and plugging the ends.

22-5. An imitation of Mohammed's coffin, which was said to float in mid-air, is sometimes accomplished by means of two strong bar magnets. Make specifications for a piece of lecture demonstration apparatus which will illustrate this.

22-6. Specify the best arrangement of two bar magnets when stored side by side in the same box.

22-7. Find the attraction between a north and south pole of strengths 1 and 2 micropole units respectively when placed 0.05 meter apart in air.

22-8. Two equal poles 0.04 meter apart in air repel each other with a force of 0.00016 newton. Find the pole strength of each.

22-9. Two unlike poles attract each other with a force of 0.004 newton at a distance of 0.06 meter. If the south pole has a strength of 0.6 micropole unit, what is the strength of the north pole?

22-10. Two like poles, each with a strength of 10^{-6} unit, repel each other with a force of 0.004 newton. What is the distance between the poles?

22-11. Find the field strength at a point distant 0.05 meter from a pole of strength one micropole unit. How are the lines of force spaced at this point?

22-12. Calculate the strength of the magnetic field at a point 0.08 meter from the north pole of a bar magnet and 0.1 meter from the south pole. The distance between the poles of the magnet is 0.06 meter, and each pole has a strength of 96×10^{-8} unit.

22-13. Find how many complete oscillations a compass needle would make per minute at a place where the horizontal field strength is 180 millioersteds, if its period is 6 seconds where the field is 185 lines per square meter.

22-14. Two like magnets are placed in line with their north poles 0.1 meter apart and their south poles half a meter apart in air. (1) What is the repulsion between these magnets if each pole is 2 microunits? (2) What is the magnetic field strength midway between the north poles of these magnets?

CHAPTER 23

Static Electricity

23-1. How Atoms Are Put Together. In order to understand the basic facts of electricity, it is helpful to get a mental picture of the internal structure of atoms. They are no longer considered to be hard, spherical, uncuttable entities, but we speak quite confidently of three of their constituents (and speculate on the possibility of a fourth). Atoms are constructed out of three kinds of particles called electrons, protons, and neutrons. Protons and neutrons have about the same mass (1.671×10^{-28} kilogram) while electrons are lighter (1/1,834 as much, or 9.11×10^{-31} kilogram). Protons repel one another (except when *very* close together, when they attract one another). Electrons also repel each other, but protons and electrons attract each other at all distances. Neutrons exert no forces on one another or on other particles except at *very* close range, when the force is one of attraction. 6.24×10^{18} electrons constitute a coulomb of negative electricity, likewise the same number of protons constitute one coulomb of positive electricity.

In the structure of the typical atom, the protons and neutrons are concentrated in a comparatively small region called the *nucleus* at the center of the atom. In most atoms, there are at least as many neutrons as protons and usually more; the total number of both together is called the *atomic weight.*

When in the normal state, an atom contains a series of concentric shells which are named (starting from the inner one) respectively, the *K-shell*, *L-shell*, and so on. These shells are not to be thought of as made of anything; they are rather the average location of groups of electrons, two in the K-shell, 8 in the L-shell, 18 in the M-shell, and so on, until there are as many electrons present as there are protons in the nucleus. It will be noticed that the numbers 2, 8, 18, 32, and so on, are twice the perfect squares, 1, 4, 9, 16, and so on. Since the recent discovery of neptunium and plutonium, there are 94 different kinds of known atoms (with some doubt about two of them) and probably numbers 95 and 96 will be added to the list before long. These make a continuous series all the way from hydrogen with one proton in the nucleus and one electron in the K-shell, up to plutonium with 94 protons and 145 neutrons in the nucleus and 94 electrons distributed among several shells.

23-2. Conductors and Insulators. The nearly one hundred varieties of atoms may be divided into two classes, called by the electrician *conductors* and *insulators*, and by the chemist *metals* and *nonmetals*. Metallic atoms all have one, two, three, or in some cases four, electrons held loosely enough so that it is perfectly possible for the atom temporarily to lose them entirely. Since the nucleus is positive, and since under normal conditions there are just enough electrons to balance exactly this positive charge, it will be clear that if one or more electrons are lost from the atom, what is left will have an excess positive charge. The technical term for this positive aggregation is *positive ion*. A nonmetallic atom, on the contrary, is so constituted that it has an attraction for more electrons than are necessary to balance the positive charge of the nucleus. An aggregation of this sort, containing an excess of electrons, is called a *negative ion*. In an insulator, the nonmetallic atoms are predominant; no free electrons to speak of are present. But in a group of metallic atoms, there will always be plenty of free electrons roaming about between the atoms. Given a metallic wire, it is possible to force in extra electrons at one end and remove electrons from the other end, while, in between, the electrons drift along from atom to atom. Such a wire is said to conduct electricity.

23-3. Static Electricity. The tendency of a substance to gain or lose electrons varies from substance to substance. Therefore if any two dissimilar materials be placed in very close contact, electrons will tend to desert one substance and cling to the other. The former material thus becomes positively charged and the latter negatively charged. Examples are: leather belts running on steel pulleys, fountain pens in contact with cloth, rubber tires in contact with the road, combs pulled through hair, shoes scuffed on rugs, and so on. If one strokes a cat on an exceptionally dry day and then touches the cat's nose, a small spark will jump. When a glass rod is rubbed with a silk cloth, the glass loses electrons and the silk gains them, that is, the glass becomes positively charged and the silk negatively. It is customary to refer to this phenomenon as *frictional electricity* or *static electricity*; it is however the contact rather than the friction which produces the effect. If two very light objects, such as small balls made of pith covered with metallic foil, and suspended by means of nonconducting threads, are now given, one some of the charge that is on the glass rod, and the other some of the charge on the silk cloth, it will be found that the two pith balls attract each other. If, however, both balls are charged from the same object, say the silk cloth, they will repel each other. Furthermore, if an insulator be given a negative charge by the method just mentioned and then rotated rapidly in a clockwise direction as viewed from above, the upper side will be found to be (during the rotation) a feeble north magnetic pole, and the under side a south pole of equal strength. If on the other hand the insulator is given a positive charge, it will have to be whirled in a counter-clockwise direction as seen from above to make the upper side a north pole.

23-4. Coulomb's Electrostatic Law. The electrician's unit of electric charge is called the *coulomb*. The number of electrons necessary to constitute a coulomb is very large; a microcoulomb is one millionth of a coulomb and is still large as electrostatic charges go. The equation giving the relationship between the electrical charges q_1 and q_2, the distance d, between them and the force F, that each exerts on the other is

$$F = k_e \frac{q_1 q_2}{\epsilon d^2}$$

In this case F is in newtons, d in meters, q_1 and q_2 in coulombs, and k_e is 9×10^9 newton-meters^2 per coulomb^2. If everything takes place in a vacuum ϵ is exactly unity. If the medium is air, ϵ is slightly more, 1.00059. ϵ is known as the *dielectric constant* for the medium between the charges. Typical values are 16.5 for diamond, 9.9 for heavy flint glass, 5.8 for mica, 2.1 for both paraffin and kerosene.

23-5. Illustrative Problem. Two pith balls weighing 90 milligrams each are suspended in air by nonconducting silk threads, each 0.13 meter long (the weight of which may be neglected) from a single point, and given equal positive charges. As a result the centers of the pith balls stand 0.10 meter apart. See figure 23-1. Find the charge on each, in coulombs and in microcoulombs.

Solution: Since both pith balls are in equilibrium, the tension in one of the threads, the electrostatic repulsion, and the pull of gravity represent three forces on one pith ball which are all in equilibrium. If we resolve the tension into a vertical and a horizontal component (figure 23-2), we discover that the diagram contains 5, 12, 13 triangles. The vertical component balances mg. In this case $m = 0.000090$ kilogram and $g = 9.80$ newtons/kilogram, making $mg = 0.000882$ newton. Since this represents the 12 side of the 5, 12, 13 triangle, and the force F, which we need, balances the 5 side, F is 5/12 of 0.000882 newton or 0.000368 newton. We shall substitute then into the equation of section 23-4 this value together with $k_e = 9 \times 10^9$ newton-meters²/coulomb², $d = 0.10$ meter, and $\epsilon = 1.001$. This gives

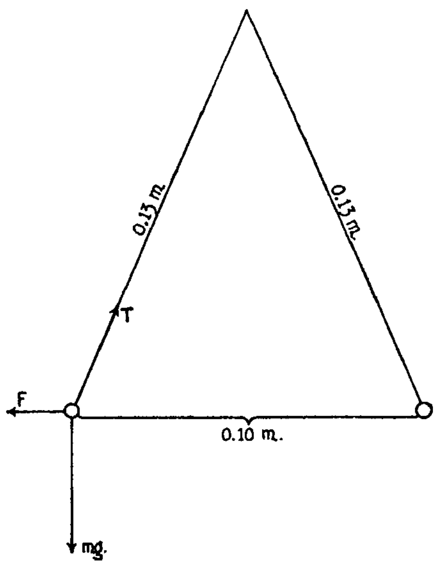


Figure 23-1.

$$0.000368 = \frac{9 \times 10^9 q^2}{(1.001)\ (0.10)^2}$$

Solving for q, we obtain $q = 0.0000000202$ coulomb, or better 2.02×10^{-8} coulomb. This can also be expressed as 0.0202 microcoulomb or 2.02×10^{-2} microcoulomb.

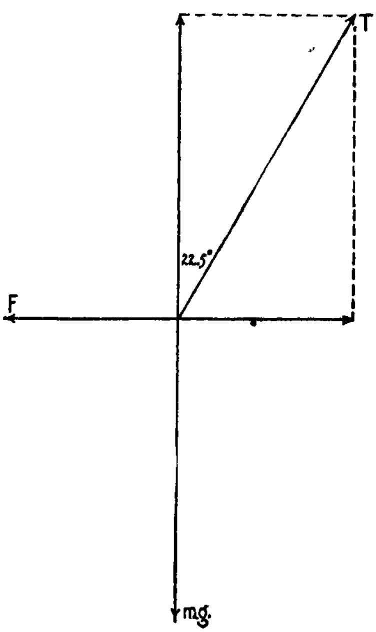


Figure 23-2.

23-6. Condensers; Capacitance.

A very useful piece of electrical apparatus consists of two sheets of conducting material, such as tin foil, separated by a nonconductor, such as glass or mica or paraffined paper. This is called a *condenser.* If one of the conductors is given an electric charge and the other conductor is connected to the ground, say by joining it by means of a copper wire to a water pipe, the grounded side of the condenser will be found to have a charge of opposite sign but equal in magnitude to the original charge. The two opposite charges hold each other in place by their electrostatic attractions. The charged condenser may be "discharged" by making a metallic connection between the two plates.

A physical quantity known as the *capacitance* of the condenser, represented by the letter C, is proportional to the overlapping area of the two conductors A, in square meters, inversely proportional to the thickness d, in meters, of the nonconductor (which is often called a *dielectric*) also proportional to the *dielectric constant* ϵ, of the nonconductor. The equation expressing these relations is

$$C = \frac{\epsilon A}{4\pi k_e d}$$

k_e is as before 9×10^9 newton-meters^2/coulomb^2. We should expect to express the capacitance C, in coulomb^2 per joule, indeed this would be an entirely correct unit, but it is customary instead to express the capacitance in farads. Like the coulomb, this is a very large unit, so that we find the microfarad, which is a millionth of a farad, much more convenient for ordinary use.

23-7. Voltage. The ratio of the positive charge on one side of a condenser to the capacitance of the condenser is called the *voltage* across the condenser. Sometimes we use the expression *potential difference* instead of *voltage*. We should expect the unit of this ratio to be either coulombs per farad or joules per coulomb. While both of these are correct, still another unit is used, the *volt*. The best definition of the volt is the energy in joules necessary to transfer a coulomb of electricity from one side of the condenser to the other. At the same time, it is convenient to think of voltage as the degree of abnormality of distribution of electrons. This results in a tendency to force the electrons back into the atoms where they normally belong. Thus if we desired a large flow of electrons in a wire the thing to do would be to create a large potential difference or voltage across the ends of the wire.

23-8. Illustrative Problems. (1) Find the capacitance of a condenser which consists of a cylindrical glass jar 11.2 centimeters in diameter, thickness of glass 3.5 millimeters, with 700 square centimeters of its inner area coated with tin foil opposite an equal area of tin foil on the outer surface. Consider the dielectric constant of the glass to be 8.

Solution: It is necessary to substitute into the equation of section 23-6 the values $\epsilon = 8$, $A = 0.0700$ meter^2, $k_e = 9 \times 10^9$ newton-meters^2/coulomb^2, and $d = 0.0035$ meter. Thus

$$C = \frac{(8)\,(0.0700)}{4\pi 9(10^9)\,(0.0035)}$$

or $C = 1.415 \times 10^{-9}$ farad. It is more customary to express this as 1.415×10^{-3} microfarad or even 1,415 micromicrofarad. The piece of apparatus described in this problem is called a *Leyden jar* and has more historical than practical interest. Much greater capacitances than this may be obtained by using alternate layers of tin foil and waxed paper with every other piece of tin foil connected electrically. The whole can then be rolled up into a compact cylinder.

(2) If the two metallic surfaces of the Leyden jar of the previous problem are connected respectively to the positive and negative terminals of an electrostatic "influence machine" which furnishes a potential difference of 50,000 volts, find the quantity of electricity that can be removed from one surface of the Leyden jar and placed on the other.

Solution: Substitute into the equation

$$V = \frac{q}{C}$$

implied in the first sentence of section 23-7 the values $V = 50{,}000$ volts and $C = 1.415 \times 10^{-9}$ farad, and solve for q. This gives $q = 7.08 \times 10^{-5}$ microcoulomb.

(3) How much energy goes into the electric spark when this Leyden jar is discharged?

Solution: A volt may also be called a *joule per coulomb.* While the Leyden jar is being discharged, we may think of its voltage as dropping from 50,000 volts to zero with an average value of 25,000 volts, or 25,000 joules per coulomb. This multiplied by the 7.08×10^{-5} coulombs, which is to be transferred from one metallic surface of the Leyden jar to the other, gives 1.77 joules of energy. This energy is not annihilated but appears in the form of heat, light, and sound in the electric spark. 1.77 joules is equivalent to 1.31 foot-pounds, which represents the mechanical work done by the man that turned the crank of the "influence machine" in the first place.

23-9. Comparison of Magnetic and Electrostatic Effects.

Magnetism	Electrostatics
Occurs to a noticeable extent only in iron and its alloys.	Is noticeable in all substances, especially in nonconductors.
Magnetism in a piece of steel is produced by lining up the elementary magnets of which the sample is made.	An electric charge in a nonconductor is produced by giving it extra electrons, or taking some away.
Magnet poles always occur in pairs which cannot be separated.	Unlike electric charges may be separated to any extent.
$F = \frac{k\ p_1 p_2}{\mu r^2}$	$F = \frac{k_e q_1 q_2}{\epsilon r^2}$
Magnet poles are designated by the terms *north* and *south*.	**Electric charges are designated by the terms *negative* and *positive*.**

A magnetic pole at rest has no effect on an electric charge at rest.

SUMMARY OF CHAPTER 23

Technical Terms Defined

Electron. A minute particle having a mass of 9.11×10^{-31} kilogram and a negative electric charge of 1.63×10^{-19} coulomb. It represents the smallest quantity of negative electricity discovered; all other charges are integral multiples of this.

Proton. A particle having a mass of 1.67×10^{-28} kilogram and a positive charge of 1.63×10^{-19} coulomb. It is one of the constituents of the nucleus of an atom and can be removed from an atom only by very extreme measures.

Neutron. A particle with approximately the same mass as the proton and with no electrical charge. It is also one of the constituents of atomic nuclei.

Atom. The chemical unit of matter. So far, nearly 100 kinds of atoms are known. An atom represents an aggregation of protons in its nucleus, an equal number of electrons in concentric shells surrounding the nucleus, also, with one exception, neutrons in the nucleus.

Conductor. A metallic substance in which there are free electrons moving between the atoms and belonging to these atoms. This motion of electrons in a conductor is called an *electric current.*

Insulator. A nonmetallic substance containing practically no free electrons. A nonconductor of electricity.

Dielectric Constant. A property of a nonconductor. The greater the dielectric constant of an insulator, the smaller is the electrostatic force that exists between two charges embedded in the insulator.

Condenser. A piece of apparatus consisting essentially of two sheets of conducting material with a sheet of insulating material between.

Voltage. The energy in joules necessary to remove one coulomb of electricity from one side of a condenser and place it on the other side. One joule per coulomb is called *one volt.*

Capacitance. The ratio between the quantity of electricity on one side of a charged condenser and the voltage across the condenser. Its unit is the farad. One microfarad is a millionth of a farad. The capacitance of a condenser is directly proportional to the dielectric constant of the insulating sheet, directly proportional to the overlapping area of the conducting sheet, and inversely proportional to the thickness of the insulator.

Coulomb's Law. The force with which one electrostatic charge q_1, repels or attracts another charge q_2, varies directly with the charges, inversely with the square of their separation, and inversely with the dielectric constant of the intervening medium. Like charges repel and unlike charges attract.

PROBLEMS

23-1. Given a mounted compass needle not enclosed in a case. What would be the effect of bringing near the north pole of the needle each end of (1) a magnet, (2) a bar of unmagnetized soft iron, (3) a stick of wood, and (4) an electrified glass rod? In what respects are effects (1) and (2) similar?

23-2. Two pith balls in air are charged with three microcoulombs each and their centers are two centimeters apart. What force does each exert on the other?

23-3. A positive charge of 10 microcoulombs and a negative charge of 5 microcoulombs are separated 0.2 meter by kerosene of dielectric constant 3. What force does each charge exert on the other?

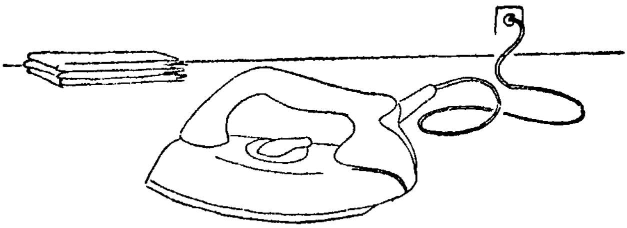
23-4. Find the capacitance (in both farads and microfarads) of a condenser consisting of two sheets of lead foil each 20 cm.^{2} area and one sheet of paraffined paper 0.05 mm. thick, of dielectric constant 2.

23-5. If a condenser of one farad capacitance were to be constructed of lead foil and paraffined paper 0.05 mm. thick of dielectric constant 2, find the necessary area of the "plates." From the result of this problem, state the reason for the unpopularity of the farad as a unit.

23-6. If a condenser of 2 microfarads capacitance is charged to a potential difference of 500 volts, find the charge on each side.

23-7. What energy was expended in charging the condenser of the previous problem? Express this in joules, foot-pounds, and British thermal units. What ultimately becomes of this energy?

CHAPTER 24



Electricity In Motion; Heating Effect

24-1. Electric Currents. A negatively charged object is one with more electrons than normal, while a positively charged object has less electrons than normal. There is no easy method of moving protons about; they remain fixed quite permanently in the nuclei of the atoms present, but the electrons are free to move. If, then, by some means it is possible to arrange two regions, one of which maintains more electrons than enough to balance the protons in the nuclei of the atoms, and the other less electrons than protons, and if these two regions are connected by means of a conducting wire which contains a great many loose electrons, but an equal number of positively charged atoms (so that the wire as a whole is electrically neutral), the result will be that extra electrons will flow into the wire at one end and electrons will flow out at the other end at an equal rate, and the wire as a whole will still remain neutral. This migration of electrons through the wire is known as an *electron current.* (Electricians are coming to use the idea of electron current more and more in preference to the old "conventional current" which was said to flow in the other direction and which dates back to the time when no one knew just what was going on.) In this book, from here on, the word *current* will be understood to mean *electron current*, or *current of negative charges.* It will flow from an excess of electrons to a deficiency of electrons, which is equivalent to saying that it will flow from a low potential to a high potential. If it were possible to count the electrons that went by a point of the wire in one second we could visualize one ampere as a flow of 6.24×10^{18} electrons per second, which is one coulomb per second.

24-2. Drift Speed of the Electrons Versus Signal Speed. The actual progress of an electron along a wire is in a very zigzag fashion. It is constantly colliding with the other particles of the wire, so that a current of one ampere flowing in a copper wire a square millimeter in cross section would correspond to an actual drift speed of about a foot per hour. How is it then that we can carry on a telephone conversation with someone 3,000 miles away and notice no delay whatever in the return of the other person's answer? The answer is that the *drift* of the electrons and the *signal* speed are two utterly unrelated quantities. As an illustration of how this can be, consider a pipe full of water with a tightly fitting plunger inserted at each end. If now one plunger is moved forward slowly, say at the rate of one inch per second, all the water in the pipe will also move at that rate, and so will the plunger at the other end. The *drift speed* is therefore one inch per second. The question now arises as to the length of time intervening between the motion of the first plunger and that of the second plunger. If both the water and the pipe were incompressible, the process would be instantaneous, although no drop of water actually travels faster than one inch per second. Thus a series of signals can be transmitted with this mechanism much more rapidly than the actual motion of the particles because all the particles start moving at the same time. The strong repulsions that the free electrons in a wire exert upon each other make them behave like a nearly incompressible fluid in a pipe, so that the signal speed is practically the speed of light (186,000 miles per second) in spite of the fact that the drift speed is so small.

24-3. Electromotive Force. Since the drift of electrons through a wire is so analogous to the flow of water in a pipe, it will be helpful to use the illustration further. Consider a "centrifugal" water pump with the outlet connected to the inlet by means of a long pipe. (See figure 24-1.) The pipe will be somewhat more analogous to an electric wire if we imagine it filled with pebbles to correspond to the atoms and ions all through the cross section of the wire. If both the pump and the pipe are full of water to begin with, to correspond to the free electrons which are always present in a conductor, it will be clear that there will be no circulation of water in the system until the pump begins to act. Let us however imagine the pipe to be equipped with a valve which is closed at the beginning of our discussion; this corresponds to a break in the electric circuit, that is, a nonconductor. If the centrifugal pump is started while the valve is still closed, the only result from the action of the pump will be to produce a difference in pressure on the two sides of the closed valve, which may be determined by reading

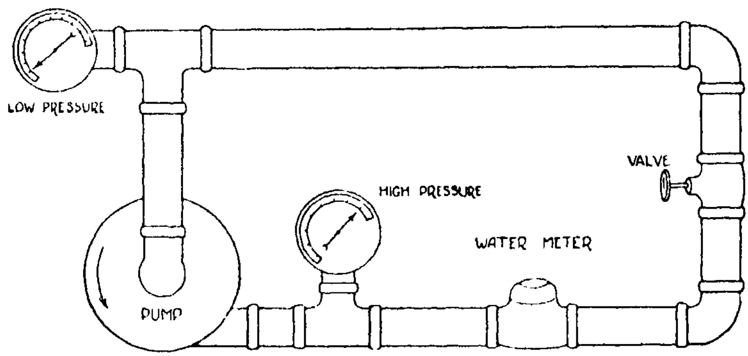


Figure 24-1.

the two gages. Pressure in mechanics is equivalent to energy per unit volume (see section 5-1); voltage in electricity is equivalent to energy per unit charge; thus if we consider volume of water in this analogy to correspond to quantity of electricity, this pressure difference on the two sides of the closed valve corresponds to a potential difference in the electrical case. There exist devices like battery cells (often called *voltaic cells*) which are capable of maintaining a difference in potential across a break in an electric circuit, and we call this voltage the *electromotive force* of the voltaic cell.

Now if we open the valve and allow the water to flow, the pressure gages will change their readings so that the difference will be less. It will still require a certain amount of energy to force unit volume of water all the way around the circuit once, through the pebbles, but this energy can no longer be determined by reading the gages; the gages will give only that part of the energy utilized between the high pressure and the low pressure gages, and not that used up in the pump. This process is equivalent to closing the switch in figure 24-2. The energy in joules necessary to make one coulomb of electrons go around the circuit once is again the electromotive force of the cell in volts, but while it could be read directly from the voltmeter when the switch was open as in the diagram, the electromotive force must be computed and not measured when the switch is closed.

The volume of water which passes any given cross section of the pipe per second, that is, the current, is directly proportional to the pump action and inversely proportional to the resistance offered by the pebbles. Likewise the electron current in amperes is directly proportional to the electromotive force of the cell and inversely proportional to the resistance offered by the electric circuit.

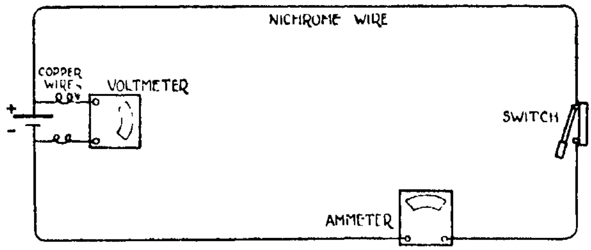


Figure 24-2.

24-4. Ohm's Law. The proportionality between the electromotive force in an electric circuit and the current that flows is known as *Ohm's law* and the proportionality constant is technically known as the *resistance* of the circuit. Thus, if E stands for the electromotive force of the circuit, I for the current flowing through the circuit, and R for the resistance of the circuit, Ohm's law is expressed by the equation

$$E = IR$$

This is equivalent to defining the resistance of a circuit as the ratio between the electromotive force in volts and the current in amperes. But instead of expressing the resistance in volts per ampere we express it in ohms.

We also speak of the voltage V, across a portion of a circuit; this is equal to the current flowing through this part of the circuit multiplied by the resistance of the same portion of the circuit

$$V = IR$$

24-5. Distinction Between Electromotive Force and Voltage. In order to make electrons flow around an electric circuit, energy must be supplied somewhere in the circuit. This usually happens in a comparatively small portion of the circuit; the source may be chemical energy, heat energy, or mechanical energy, but however it is done, the result is a more or less sudden jump in electrical pressure called an *electromotive force.* There will then be a gradual drop in electrical pressure all through the rest of the circuit so that the sudden rise (the electromotive force) is equal to the sum of the gradual drops. These gradual drops are sometimes called

IR drops, sometimes *potential drops*, and sometimes merely the "voltage across such and such a resistance."

Electromotive forces may be negative as well as positive. An example of this is a battery put into a circuit backward, as is done in charging a storage battery. In this case there is a sudden drop in electrical pressure and electrical energy is converted into chemical or some other form of energy.

24-6. Illustrative Problems. (1) Assume that in figure 24-2 there is just enough No. 20 nichrome wire connected to the terminals of the dry cell to have a resistance of one ohm (1.70 feet), and that the switch is closed. Let the resistance of the cell itself be 0.05 ohm and assume that the ammeter reads 1.43 amperes. What is the electromotive force of the cell?

Solution: In a simple circuit of this type (with no branching) the current is the same throughout, namely, 1.43 amperes. The total resistance of the circuit is the sum of 0.05 ohm in the cell and 1.00 ohm in the rest of the circuit, or 1.05 ohms. This assumes that the resistance of the ammeter is negligibly small and the resistance of the voltmeter so large that practically no current goes through it. Substituting into the equation $E = IR$ we obtain

$$E = (1.43)\ (1.05)$$

or $E = 1.50$ volts.

(2) Find the reading of the voltmeter in part (1).

Solution: This may be done by two methods. First we may use the equation $V = IR$ for the part of the circuit consisting of the nichrome wire, switch, and ammeter. In this case, $R = 1$ ohm, $I = 1.43$ amperes, and therefore $V = 1.43$ volts. The same result may be obtained by arguing that at the place where the voltmeter is connected we have almost the full effect of the electromotive force of 1.50 volts. There is only the IR drop through the battery to be subtracted. This is (1.43) (0.05), or 0.0715 volt. 1.50 volts minus 0.07 volt is 1.43 volts as before.

(3) What will the voltmeter read if the switch is opened?

Solution: When the switch is opened the current drops to zero, there are no IR drops, and therefore the voltmeter reads the full electromotive force of 1.50 volts.

24-7. Resistivity. We can predict the resistance of a piece of wire if we know its length, cross sectional area, and the material of which it is made. If R = resistance, A cross sectional area, and l length of the wire the relation is

$$R = \frac{rl}{A}$$

The proportionality constant r is called *resistivity*. This equation states that a long wire offers more resistance to the flow of electrons than a short wire, a fat wire offers less resistance than a thin wire, and some materials are better conductors than others. As in the case of heat conduction, silver and copper are the best conductors of electricity. If l is in meters and A in square meters, a few values of r are: aluminum, 2.83×10^{-8}; carbon, $3{,}500 \times 10^{-8}$; copper, 1.692×10^{-8}; German silver, 33×10^{-8}; gold, 2.44×10^{-8}; iron, 10×10^{-8}; lead, 22×10^{-8}; mercury, 95.8×10^{-8}; nichrome, 100×10^{-8}; silver, 1.65×10^{-8}.

These values hold at 20°C. At higher temperatures, the resistance of metals increases according to the same type of law as linear expansions. That is, the increase of resistance is proportional to the original resistance times the temperature coefficient times the increase in temperature. The temperature coefficients of non-metals are negative. A few centigrade temperature coefficients are: aluminum, 0.0039; carbon, − 0.0005; copper, 0.0039; German silver, 0.0004; gold, 0.0034; iron, 0.0050; lead, 0.0043; mercury, 0.00089; nichrome, 0.0004; silver, 0.0038.

24-8. Illustrative Problems. (1) How long must a piece of nichrome wire be in order to have one ohm resistance if its diameter is 0.03196 inch?

Solution: Here it is necessary to convert the resistivity into English units or convert the diameter into metric units. We shall do the latter. Since there are 39.37 inches in a meter, 0.03196 inch is 0.03196/39.37 or 0.000812 meter (= 0.812 millimeter). Thus we have $r = 100 \times 10^{-8}$ or $= 10^{-6}$ from the previous section, $R = 1$ ohm, and $A = \pi(0.000406)^2 = 5.18 \times 10^{-7} m^2$. Substituting these three values into the equation of section 24-7 gives

$$1 = \frac{(10^{-6})\, l}{5.18 \times 10^{-7}}$$

Thus, $l = 0.518$ meter. Since there are 3.28 feet in a meter, l may also be expressed as (0.518) (3.28), or 1.699 feet.

(2) Find the resistance of this piece of wire at 100°C., also at 0°C., also find its temperature coefficient at 0°C.

Solution: The increase in resistance from 20°C., at which the resistance is 1.000 ohm, up to 100°C. is the product of 1 ohm by 0.0004 per °C. by 80°. This product is 0.032 ohm. Thus the resistance at 100°C. is 1.032 ohms.

The decrease in resistance from 20°C. down to 0°C. is similarly the product of 1 ohm by 0.0004/°C. by 20°, or 0.008 ohm. Thus the resistance at 0°C. is 0.992 ohm.

If we wish a new temperature coefficient referred to 0°C., we must solve the equation

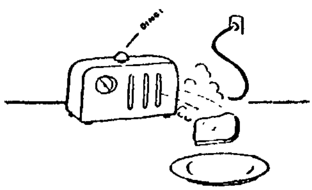
$$1.032 - 0.992 = (0.992)\,(x)\,(100)$$

This gives $x = 0.040/99.2 = 0.000403$ per degree centigrade.

24-9. Heat Produced by an Electric Current. If we remember that V volts may also be written V joules per coulomb, also that an ampere is the same as a coulomb per second, and thus that a coulomb is the product of the current in amperes by the number of seconds it flows, that is

$$q = It$$

then it will be clear that the product of V joules per coulomb and It coulombs will be the energy in joules involved in causing a current I amperes to flow t seconds through a potential drop of V volts.



$$\text{joules} = VIt$$

If there are no negative electromotive forces present, this energy will be converted into heat in the circuit. In order to avoid any qualifying "ifs," an equivalent expression may be obtained eliminating the voltage V. Since $V = IR$, we have

$$\text{heat in joules} = I^2Rt$$

That is, the current in amperes, squared, times the resistance in ohms times the time in seconds will always give the heat developed in this resistance in joules.

24-10. Illustrative Problem. Find the heat developed by a 20-ohm, 120-volt electric stove in half an hour. Express the result in joules, Calories, B.t.u., and KWH.

Solution: Using Ohm's law, $V = IR$, when V is 120 volts and R is 20 ohms, we conclude that the current is 6 amperes. Substituting $I = 6$ amperes, $R = 20$ ohms, and $t = 1{,}800$ seconds into the last equation of section 24-9 gives

$$\text{heat in joules} = (36)\ (20)\ (1{,}800)$$

or 1,296,000 joules of heat are developed in the half-hour.

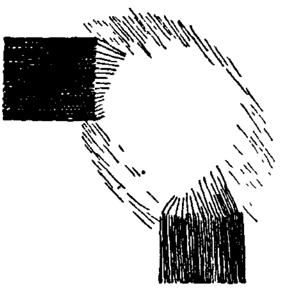
Since there are 4,190 joules in a Calorie, this is equivalent to 3,090 Calories.

Multiplying 3,090 Calories by 3.97 B.t.u./Cal. gives 12,270 British thermal units.

Dividing 1,296,000 joules by 3,600,000 joules/KWH gives 0.360 KWH.

24-11. Hot Wire Ammeters. Since an electric current always produces heat and since heating a wire changes its length, ammeters can be constructed on this principle to measure currents. It is only necessary to put the ammeter into the circuit so that the same current flows through it that flows through the rest of the circuit. The wire whose length is to change (on account of the heating effect of the current through it) is kept taut by a spring; the motion of this spring is communicated to an indicating pointer which moves over a calibrated scale.

24-12. Electric Light. If sufficient heat is produced in a wire, it will become red-hot, or even white-hot, and emit considerable light. The fraction of the energy thus converted into light is, however, small and dependent on the temperature. Incandescent lamps with tungsten filaments surrounded by an atmosphere of nitrogen emit about 11 per cent of the energy consumed in the form of light. These filaments reach temperatures of 2,800°C.



The electric arc between carbon terminals in air reaches temperatures of 3,000°C. at the negative terminal, and 3,500°C. at the positive terminal. These temperatures may be increased by enclosing the arc in an atmosphere of carbon dioxide and increasing the pressure. In this way temperatures of 6,000°C. have been reached at about 30 atmospheres pressure.

24-13. Electric Power. Since power is the rate of doing work, the quotient of energy by time gives power. Since the energy in joules is VIt we have

$$\text{power in watts} = VI$$

That is, current in amperes times voltage in volts gives power in watts. A 600-watt flatiron on 120 volts, for example, carries 5 amperes.

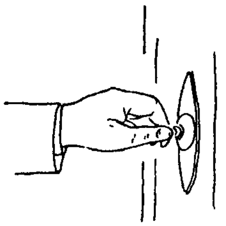
By using $V = IR$ we may eliminate V and obtain

$$\text{power in watts} = I^2R$$

For example, in the problem of section 24-10, the power was $(6)^2(20) = 720$ watts. This is equivalent to 0.720 kilowatt. We could have obtained the last answer by multiplying 0.720 kilowatt by 0.5 hour and obtaining 0.360 *KWH*.

24-14. Thermoelectricity. If two wires of different materials are connected at both ends, no current tends to flow so long as the two ends are at the same temperature. But if the two junctions are maintained at different temperatures, a small electromotive force is developed resulting in a feeble current. This current is too small for any commercial application other than for the purpose of measuring temperatures. The arrangement used in this may be called a *thermocouple*. For example, if one junction is in contact with an automobile engine and the other is at the temperature of the instrument board, a current will flow sufficient to give an indication on a *galvanometer* which may be calibrated to read the engine temperature in degrees Fahrenheit. It is also of theoretical interest that one of the three principal ways of creating an electromotive force is thermally.

24-15. Some Practical Aspects of an Electric Current. In practical work, three things are necessary for the satisfactory operation of an electrical device (assuming of course that the device itself is properly designed): (1) the proper connection of a proper source of electromotive force, (2) a complete circuit consisting of conductors, and (3) the precaution of preventing the current from flowing in undesired directions, accomplished by the use of insulation and insulators. As an illustration, consider any simple electrical apparatus, such as an electric bell or electric flatiron. The electromotive force for the bell may be supplied by a couple of dry cells or some other source of low voltage. If the cells are used, the first item consists in making sure that the carbon (center terminal) of one cell is connected to the zinc (outside terminal) of the other cell. The second item includes a check of the wiring, the condition of the bell itself, and the switch. The wiring must be arranged so that when the switch is closed, there is a complete circuit from the zinc of one cell to the carbon of the next, then from the zinc of the next cell through the bell and the switch, back to the carbon of the first cell.



SUMMARY OF CHAPTER 24

Technical Terms Defined

Electric Current. Migration of free electrons through the body of a conductor, measured in amperes, or coulombs per second.

Electromotive Force. The electromotive force of a battery or other source of electrical power is the energy in joules that it is capable of expending while pushing a coulomb of electricity completely around a closed circuit. This is a characteristic of the battery and not of the circuit and will be the same, whatever the nature of the circuit. It is measured in volts.

Potential Difference. The energy in joules expended in forcing a coulomb of electricity through a portion of a circuit. It is also measured in volts.

Resistance. The ratio between the potential difference in volts across a portion of an electric circuit and the current in amperes. It is characteristic of the conductor and is measured in ohms.

Resistivity. The resistance of a specimen of material of unit length and unit cross section. The resistivity is a characteristic of the material at a definite temperature. The change in resistivity of a conductor with temperature is directly proportional to its original resistance, the change in temperature, and its temperature coefficient.

Thermoelectricity. A small electromotive force may be produced by joining two or more dissimilar metals to form a closed circuit and maintaining different temperatures at the junctions.

Laws

Ohm's Law. This law states that the current flowing in a circuit is directly proportional to the total electromotive force present and inversely proportional to its resistance. Ohm's law may likewise be applied to a portion of a circuit. The voltage across any portion of a circuit is proportional to the current flowing and to the resistance of this portion of the circuit.

Joule's Law. The heat developed in a portion of an electrical circuit is proportional to its resistance, to the time, and to the square of the current.

The power applied to a circuit is proportional to both the voltage and the current.

PROBLEMS

24-1. If $E = 100$ volts and $I = 20$ amperes, find the resistance, R.

24-2. What current will flow in an electric lamp of 220 ohms resistance on a 110-volt circuit?

24-3. Find the drop in voltage in a five-mile trolley wire carrying a current of 20 amperes, if the resistance is 0.5 ohm per mile. If the power station supplies 550 volts, what is available for the trolley?

24-4. Figure out a wiring diagram which will make it possible to turn on or off an electric lamp either at the head or foot of a flight of stairs, regardless of how the other switch stands.

24-5. Repeat problem 24-4 with three independent switches for the one lamp.

24-6. If there are 8.5×10^{19} free electrons in a cubic millimeter of a copper wire, find the drift speed of electrons in a wire of one square millimeter cross section while carrying a current of ten amperes.

24-7. The resistance of a piece of wire varies directly as its length and inversely as the square of its diameter. If a copper wire 1 foot long and 0.001 inch in diameter (sometimes called a *mil-foot*) has a resistance of 10.4 ohms, find the resistance of 100 feet of copper wire 0.0403 inch in diameter (# 18 A.W.G.)

24-8. The change in resistance of a copper wire with temperature follows the same type of law as change of length, that is

$$\text{change of resistance} = R_o t\,(0.00426)$$

where R_o is the resistance at 0°C. and t is the centigrade temperature. A piece of copper wire has the resistance of 15 ohms at 0°C. What is its resistance at 30°C.?

24-9. A 32-volt lamp has a resistance of 10 ohms and is to be connected with a 110-volt line. Compute the resistance which must be used in series with the lamp.

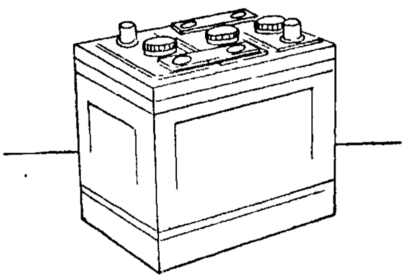
24-10. Three pieces of apparatus of 20, 24, and 36 ohms respectively are connected in series across a 115-volt line. Compute (1) the current in, and (2) the voltage across, each piece.

24-11. Two rods are welded together by driving a current of 500 amperes through the contact in which most of the resistance is concentrated. If the resistance of the contact is 0.02 ohm, find the number of Calories developed in 8 seconds.

24-12. How much current is intended to flow through a lamp marked 60 watts, 110 volts? What is its resistance when in use? How many Calories are developed in it per second? Would its resistance be more or less when cold?

24-13. A certain fuse wire has a resistance of 0.003 ohm, a mass of 0.005 gram, a specific heat of 0.04, and a temperature of 20°C. It melts at 140°C. If a current of 10 amperes is sent through it, how long will it take for the fuse to "blow"?

CHAPTER 25



Voltaic and Electrolytic Cells; Simple Circuits

25-1. Voltaic Cells. A strip of metallic zinc is made up of neutral zinc atoms together with a considerable number of zinc ions. The latter have each lost two electrons from the outermost shell; these detached electrons are also present in the strip. The zinc atoms are not soluble in water to any appreciable extent; on the other hand the positively charged ions are quite readily soluble in water. The result is that when a strip of zinc is dipped into water, it is found that the water takes on a positive charge and the metallic zinc an equal negative charge. If at the same time a rod of carbon or copper or some metal that is much less active than zinc is also placed in the same solution with the strip of zinc, the zinc will be negatively, and the other metal positively, charged. This means that the carbon will now contain too few electrons, because electrons have been attracted into the solution by the positive zinc ions, and the zinc contains an excess of electrons. Pure water is a very poor conductor of electricity, therefore before any very practical use can be made of this arrangement, the conductivity of the water must be improved by dissolving in the water some cheap electrolyte, such as sal-ammoniac.

If the negative zinc and positive carbon terminals are now connected with a wire or other electrical apparatus possessing more or less resistance, so as to form a closed circuit, there will be a flow of electrons from the zinc through the wire to the carbon. As these electrons arrive, they are also attracted through the carbon rod into the solution, where they neutralize positive zinc ions and positive hydrogen ions. Both the neutral zinc atoms and hydrogen atoms thus formed become insoluble and tend to "plate out" on the carbon. When this happens, the cell is said to be *polarized.* The polarization may be prevented by adding an oxidizing agent like manganese dioxide to the solution. When positively charged ions are neutralized by the electrons which arrive by way of the carbon, more zinc ions go into solution from the metallic zinc. Thus the cycle of events in the circuit is complete. The combination of two dissimilar metals in a conducting solution is called a *voltaic cell* after its discoverer, Alessandro Volta (1745-1827).

25-2. Dry Cells. In order to utilize the voltaic cell efficiently, it is convenient to use a zinc container as the negative electrode, put a carbon rod down through the center as positive electrode; and fill in the intervening space with a paste containing ammonium chloride, the depolarizing agent, and manganese dioxide. This paste is sealed in with pitch, a nonconductor which prevents evaporation of the paste. There is not much to get out of order in this cell; it lasts until enough zinc has gone into solution to eat a hole in the container, after which the paste evaporates and the "dry cell," which heretofore has been dry only on the outside; becomes dry inside as well, and ceases to function.

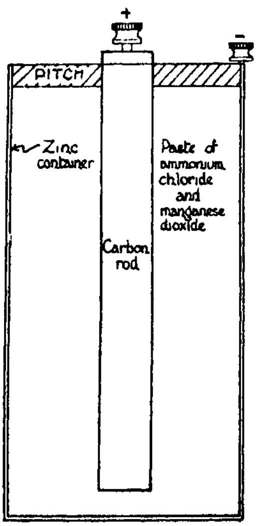


Figure 25-1.

25-3. Storage Batteries. Several voltaic cells may be used together, with the positive terminal of one connected directly to the negative terminal of the next and so on; the combination is called a battery. A type commonly used in automobiles, airplanes, trains, boats, and so on, is called a *storage battery* because its action may be reversed by forcing a current through it backward. This process is known as *charging*; after this, the battery is ready to be used all over again.

The commonest type of storage battery is the *lead accumulator.* The negative electrode is metallic lead, put into a "spongy" condition to increase its surface. The positive electrode consists of lead dioxide, and the solution is dilute sulphuric acid of specific gravity about 1.250. When the battery delivers current to a circuit, the electrons leave the lead electrode and pass around through the external circuit, arriving by the lead dioxide (positive electrode) at the solution. Here the electrons encounter positive hydrogen ions, with which they join up, forming hydrogen atoms. The latter tend to pick off oxygen atoms from molecules of lead dioxide, forming lead monoxide and water. The lead monoxide combines with sulphuric acid to form insoluble lead sulphate. Likewise at the negative electrodes, as fast as double-charged lead ions go into solution, they combine with sulphate ions to form insoluble lead sulphate. That is, the action of the battery is to make both electrodes alike and to dilute the solution with water. If this tendency goes anywhere near to completion, the battery is said to be *run down*, or *discharged.*

But the utility of the lead storage battery lies in the fact that when an electric current is put through the battery backward, all these changes take place in the reverse direction; the acid solution becomes stronger, and the lead sulphate disappears from both electrodes, leaving them spongy lead and lead dioxide respectively, that is, if the battery has not been left discharged long enough to become "sulphated."

25-4. Chemical Effect of the Electric Current. The last paragraph of the preceding section is but one illustration of a phenomenon known as *electrolysis*. Electroplating is another illustration. Electroplating always entails the use of solutions. In any inorganic solution, the substance dissolved is usually present in the form of positive or negative ions. Since the solution as a whole is neutral, there are just as many total negative charges present due to the negative ions as there are positive charges from the positive ions. Electroplating is done by passing an electric current through a solution of some compound of the metal involved. For example, silver plating may be accomplished by passing the current through a solution of silver nitrate which contains, in addition to water molecules, positive silver ions and negative nitrate ions. The negative nitrate ions, while the current is passing, move through the solution at a slow rate in the direction of the electron current. The positive silver ions move still more slowly through the solution in the other direction. In fact, it may be said that the combined motion of these two kinds of ions constitutes the whole electric current *in the solution.* The student should contrast the situation in a wire, where the current is solely due to the motion of electrons, with the situation in a solution, where the current is wholly due to the combined motions of positive and negative ions. When the positive silver ions land at the electrode where the electron current is entering the solution, the silver ions become silver atoms by absorbing one electron each and "plate out" on the electrode. The negative electrode upon which the silver plates out is called the *cathode*, and the positive electrode is called the *anode*. The number of grams of an element plated out by the current is proportional to the number of coulombs allowed to pass through the circuit, because the number of atoms plated out is proportional to the number of electrons entering the solution. One coulomb will plate out 0.00111800 gram of silver. The fact is not only taken as the *legal* definition of the coulomb in this country, but it is also the international coulomb, and the legal definition of the ampere is then the same as that given in section 24-1. It has been remarked facetiously that the number just mentioned is easy to remember since it consists of *one* decimal point, *two* zeros, *three* ones, and *four* twos (to say nothing of the extra zeros at both ends).

Electrolysis and electroplating find a very extensive use in industry, not only in extracting metals from their compounds that occur in nature, but in the purification of unrefined grades of metals. For example, ordinary copper has too great a resistance for use in the electrical industries, but by using it as the anode of an electrolytic cell, very pure "electrolytic copper" plates out at the cathode. This has a much lower resistance.

25-5. Hill Diagram. In order to visualize the potentials around an electric circuit, it is helpful to make a graph in which potentials are plotted as ordinates against position in the circuit as

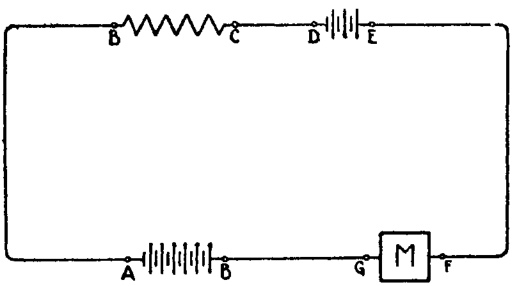


Figure 25-2.

abscissas. For this purpose consider the circuit shown in figure 25-2. This consists of six storage cells in series with an ammeter, *M*; a three-celled storage battery which is being charged; and a resistance. The circuit is slightly absurd because it is not customary to charge one storage battery at the expense of another.

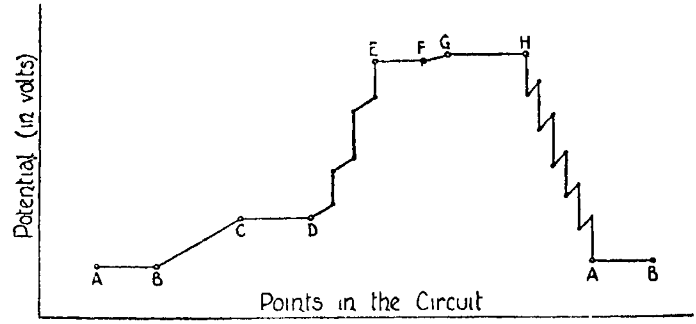


Figure 25-3.

Starting with A, the point at which the electrons have their highest concentration and which electricians speak of as a *low potential*, we proceed along the copper wire, AB, to the point B, which has almost as low a potential since the copper has so small a resistance; thus, in figures 25-3 and 25-4, the line AB is practically horizontal. BC in figure 25-2 is a resistance. There is a drop in potential from C to B equal to the product of the current I, by the resistance R, that is, an IR drop, or a rise from B to C as shown in figures 25-3 and 25-4. Again the wire CD of figure 25-2 has so little resistance that CD is nearly horizontal in figures 25-3 and 25-4. D is the negative end of the three-celled storage battery that is being charged and E is its positive terminal. In this battery, we

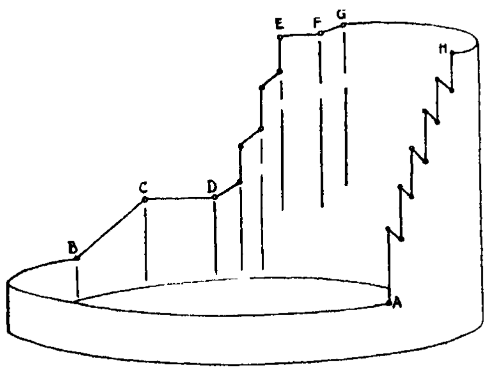


Figure 25-4.

meet three electromotive forces and three resistances. We may think of the seats of the electromotive forces as the surfaces separating the electrodes from the solutions; here there is an abrupt rise in potential of about two volts in going the slight distance necessary to get from one side of this surface to the other. Therefore the electromotive forces are represented on figures 25-3 and 25-4 as vertical lines, while the IR drops are again slant lines, and the potential at E is higher than at D for these two reasons. It is necessary to buck the back electromotive force of the three cells as well as to push the electrons through the three resistances. The lines EF and GH are again horizontal, and the line FG not far from horizontal, since the ammeter M has a very low resistance. H is the point in the circuit of highest potential; it is where the electrons are scarcest. In going from H to A we find six electromotive forces and six resistances. This time the effect of the electromotive forces is to concentrate the electrons more and more from H to A, so whereas the IR drops are still in the same direction the electromotive forces are in the other direction. We refer to them as *direct electromotive forces.* The voltage or potential drop between H and A is the sum of the electromotive forces of the six cells minus the IR drops of the cells. We are now back at A, where we started. It will readily be seen that, counting the back electromotive forces as negative, the sum of the electromotive forces in the entire circuit equals the sum of the IR drops. Or, since it is the same current everywhere in the circuit

$$I = \frac{\text{algebraic sum of e.m.f.}}{\text{sum of the resistances}}$$

The student will find it convenient to think of electromotive forces as vertical lines on a diagram like figure 25-4, and voltages or potential drops as slant lines on such a diagram.

25-6. Illustrative Problem. Assume the following numerical values in the preceding section: resistance of BC, 2.9 ohms; resistance of each cell, 0.01 ohm; resistance of the wires, zero; resistance of the ammeter, 0.01 ohm; electromotive force of each cell, 2 volts. Find the current I, also the potential drops AB, BC, CD, DE, EF, FG, GH, and HA.

Solving the equation at the end of the preceding section we have

$$I = \frac{6(2) - 3(2)}{2.9 + 3(0.01) + 0.01 + 6(0.01)}$$

or $I = 2$ amperes. We know at the outset that the potential drops across AB, CD, EF, and GH, which we shall represent respectively as V_{AB}, V_{CD}, V_{EF}, and V_{GH}, are all zero because these resistances are all taken as zero. V_{CB} is purely an IR drop, therefore $V_{\ B} = (2)\ (2.9)$ or 5.8 volts, and V_{BC} is $-$ 5.8 volts. That is, V_{BC} is a potential rise and therefore a negative potential drop. V_{DE} is another potential rise made up of 3(2^v) or 6 volts of e.m.f. together with 3(2) (0.01) or 0.06 volt of potential rises through the three resistances. Thus $V_{DE} = -6 - 0.06$ or -6.06 volts. $V_{FG} = -2(0.01)$ or -0.02 volt. V_{HA} is made up of six e.m.f.'s of 2 volts each, or 12 volts together with 6 potential rises due to resistance. $6(2)(0.01) = 0.12$ volt. $V_{HA} = 12 - 0.12 = 11.88$ volts. As a check, it will be seen that the drop from H to A, 11.88 volts, is equal to the sum of the rises from B to C, 5.8 volts; from D to E, 6.06 volts; and from F to G, 0.02 volt.

25-7. Series and Parallel Circuits. All the circuits thus far discussed have been series circuits. That is, the electrons have found it necessary to flow through a series of conductors, one after the other. In such a circuit, the current is the same everywhere; the total resistance is the sum of the separate resistances and the voltages add algebraically. Figure 25-5 is also an example of a simple series circuit, where again the total resistance is the sum of R' and R''. If, on the other hand, the arrangement is such that the current can divide, part flowing through one resistance and part through another, and then come together again (see figure 25-6), the resistances are said to be in parallel. Here the combination resistance is no longer the sum of the two; in fact it is less than either one alone. It is now the total conductance that is the sum of the individual conductances. Conductance may be defined as the *reciprocal of the resistance*. When the resistance is large, the conductance is small, and vice versa. It is quite customary to assign the letter G to conductance and to measure it in *mhos*. (Mho is *ohm* spelled backward.) Then

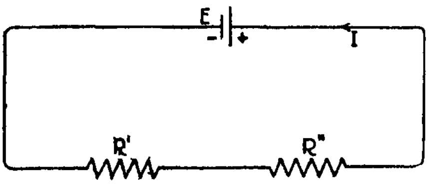


Figure 25-5.

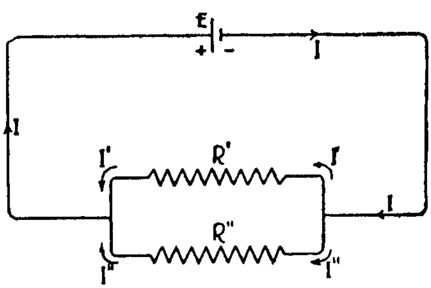


Figure 25-6.

$$G = \frac{1}{R}$$

and the relation $G = G' + G''$ becomes

$$\frac{1}{R} = \frac{1}{R'} + \frac{1}{R''}$$

for resistances in parallel. Furthermore

$$I = I' + I''$$

but the potential drop from A to B is the same whether we think of the upper resistance R', the lower one R'', or both together.

25-8. Cells in Parallel and in Series. If several cells are placed in parallel, the electromotive force of the combination is the same as the electromotive force of a single cell. This is because the electromotive force of a cell does not depend upon its size, but simply upon the chemical substances of which it is composed. Putting several cells together in parallel is equivalent to manufacturing one large cell from the same materials. One would never put two different *kinds* of cells in parallel because the one with the greater electromotive force would force a current backward through the weaker cell which would result in "charging" the weaker at the expense of the stronger. But, as we have seen, when several cells are arranged in series with each other, the total electromotive force is the sum of the individual electromotive forces.

25-9. Illustrative Problem. Given the circuit shown in figure 25-7 in which the battery consists of eight cells arranged with two rows in parallel

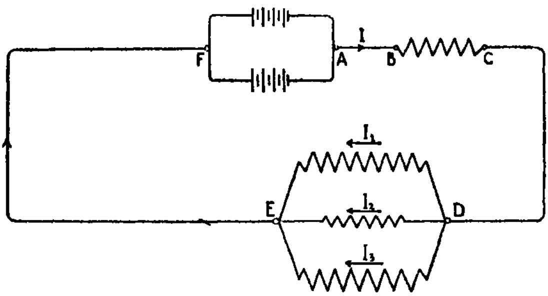


Figure 25-7.

and four cells in series in each row, each cell with an e.m.f. of 1.5 volts and negligible internal resistance. The rest of the circuit consists of a resistance BC of 10 ohms, in series with a group of three resistances which are in parallel with each other; $R_1 = 12$ ohms, $R_2 = 6$ ohms, and $R_3 = 4$ ohms. The problem is to find the current I, in the main circuit, also the currents in each branch of the circuit.

The current I may be found by dividing the electromotive force of the battery, which is the electromotive force of 4 cells (not 8) or 6 volts, by the total resistance of the circuit. In order to find this resistance we must solve the equation

$$\frac{1}{R} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4}$$

for R. This gives $R = 2$ ohms. The total resistance of the circuit then is $10 + 2$ or 12 ohms and the total current is

$$I = \frac{6}{12}$$

or 0.5 ampere. The current through each branch of the battery is half of this, or 0.25 ampere. The IR drop, V_{ED}, across the parallel circuit is (0.5) (2) or 1 volt. Since the same voltage holds for all three resistances between E and D, we have

$$I_1 = \frac{1}{12} \qquad I_2 = \frac{1}{6} \qquad I_3 = \frac{1}{4}$$

by using the relation $I = V/R$. This gives $I_1 = 0.0833$ amp., $I_2 = 0.1667$ amp., and $I_3 = 0.250$ amp.

SUMMARY OF CHAPTER 25

Technical Terms Defined

Polarization. The tendency to reverse the electromotive force of a cell by the plating out of hydrogen on the carbon terminal within the cell during the action of the cell.

Electrode. A solid plate or rod which conducts a current into or out of a solution.

Electrolysis. The formation at the electrodes of an electrolytic cell of substances derived from the solution.

Anode. The positive electrode. It collects negative ions during electrolysis.

Cathode. The negative electrode. It collects positive ions during electrolysis. The electrons may be said to enter a solution by the cathode and leave it by the anode.

Legal Coulomb. That quantity of electricity which when passed through the cathode into a silver solution results in plating out 0.00111800 grams of silver.

Legal Ampere. That steady current which when passed through the cathode into a silver solution plates out 0.00111800 grams of silver per second.

Series Circuit. A circuit where all the electrons have to pass through all the various elements, one after the other.

Parallel Circuit. A circuit in which the current divides, and part flows through each branch.

Conductance. The reciprocal of resistance. It is measured in mhos.

PROBLEMS

25-1. One coulomb will plate out 0.000329 gram of copper from a solution of copper sulphate. With a current of 20 amperes, how long a time is needed to purify by electrolysis a pound of copper?

25-2. Silver was plated on a platinum electrode from a silver nitrate solution. The current was controlled so that an ammeter in the circuit read precisely 0.500 ampere for a time of 90.0 minutes. By carefully weighing the platinum electrode before and after this period, it was found that 3.025 grams of silver had been deposited. By how much was the ammeter in error?

25-3. Draw a hill diagram for a circiut consisting of a battery of 5 cells each of which has an e.m.f. of 1.4 volts and an internal resistance of 0.05 ohm, all in series with a 3-ohm resistance. Find the current in this circuit. How would the hill diagram be changed if this circuit were opened somewhere?

25-4. Draw a hill diagram for that part of a circuit consisting of a 110-volt source, a series resistance of 20 ohms, and a 12-volt storage battery of negligible resistance being charged. What is the charging current?

25-5. Two resistors of 20 and 30 ohms respectively are connected (1) in series and (2) in parallel. Compute the resistance in each combination.

25-6. How many resistors, each of 20 ohms, will be needed to carry 23 amperes on a 115-volt line? Will they be in series or in parallel?

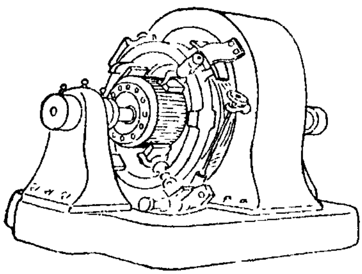
25-7. Three fixed resistors of 20, 30, and 40 ohms respectively have a combined resistance of 43.3 ohms. How are the resistances arranged?

25-8. A 32-volt lamp has a resistance of 10 ohms and is to be connected with a 110-volt line. Compute the resistance that must be used in series with the lamp.

25-9. Three pieces of apparatus of 20, 24, and 36 ohms respectively are connected in series across a 115-volt line. Compute (1) the current in, and (2) the voltage across, each piece.

25-10. Two resistors, A and B, of 140 and 100 ohms respectively are connected in parallel and placed in series with a third resistor C, of 100 ohms. This combination is connected with a 110-volt line. Compute (1) the current through each resistor, (2) the resistance of the combination, and (3) the voltage across each resistor.

CHAPTER 26



Magnetism and the Electric Current

26-1. Some of the Effects of an Electric Current Are Not Inside the Wire. Although the electrons do their moving within the wire, some of the most important effects of the electric current exist in the region outside of the wire. In this respect, the analogy between water flowing in a pipe and the electron flow in a wire breaks down. The effect just alluded to is magnetic and may readily be described in terms of the same type of lines of force as those mentioned in section 22-4. Furthermore, these magnetic fields are properties of space and exist in a vacuum as readily as in air or other material mediums. At one time, the "electromagnetic ether" was invented as a medium filling all space, endowed with properties necessary to explain electric and magnetic forces. But this medium became more and more complicated and contradictory as additional properties had to be given it, until it finally was discarded. It is very possible that the contradictory features of the ether are an indication of its polydimensional nature; if so our only hope of handling it would be through pure mathematics, since we humans are unable to visualize more than three dimensions. We shall see later that the same thing has happened to light waves; indeed there is an intimate relation between electromagnetic fields and light.

26-2. Magnetic Fields Around a Current in a Wire. A single straight wire carrying a current is surrounded by a magnetic field represented by lines in the shape of circles, the centers of which lie in the wire. All magnetic lines are closed curves; they act as if they repelled neighboring lines of force and they all tend to shorten. Therefore if the electrons cease their motion, first the inner lines shrink to zero, after which the lines farther out are free to shrink.

In order to remember the direction of the magnetic lines of force surrounding a current, one may think of grasping the wire with the left hand with the thumb pointing in the direction of flow of the electrons, in which case the fingers would curl about the wire in the direction of the lines of force.

26-3. The Electromagnet. If insulated wire is wound about a piece of soft iron, as in figure 26-1, and a current allowed to flow through the wire, one end of the iron bar will become a north pole and the other end a south pole. The left hand rule described in the preceding section may be used to determine which end is north. Or the rule may be reversed with the fingers of the left hand representing the direction of flow of the electrons and the thumb the north pole of the electromagnet. If the piece of iron is removed, the coil will still behave like a magnet while the current flows, but the effect will be much weaker. If hardened steel is used for the "core," the current may be shut off and the steel will retain most of its magnetism; but if soft iron is used for the core, the magnetic effect will be present only while the current is flowing—a very useful fact, since it makes possible the operation of lifting magnets, electric bells, the telegraph, telephone, and so on.

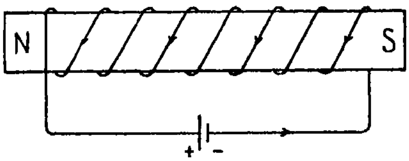


Figure 26-1.

26-4. The Electric Bell. Figure 26-2 is a diagrammatic representation of an electric bell. When the push button closes the circuit, the electron current flows in the direction of the arrows, producing a magnetic polarity as indicated. This pulls the iron armature, A, to the right and causes the hammer to strike the bell. But it also breaks the circuit at B, and this results in the release of the armature by the electromagnet. A spring causes the armature to fly back and complete the circuit again, when the whole action is repeated. The effect is therefore to move the hammer rapidly back and forth and ring the bell.

26-5. Comparison of Fields Produced by Currents and by Magnet Poles. The magnetic field produced by an arrangement like that in figure 26-1 but with an air core may be computed by the equation

$$H = \frac{4\pi In}{l}$$

in which H is the magnetic field strength at a point in the center of the core in millioersteds, which is the practical unit, I is the current in amperes, n is the number of turns of wire, and l the length of the coil in meters.

It will be remembered that magnetic fields also surround magnets (section 22-4). In this case the equation is

$$H = \frac{pk_m}{\mu d^2}$$

where H is again the magnetic field at a given point in millioersteds, p is the strength of the pole in practical pole units, $k_m = 10^7$ newton-meters^2 per pole unit squared, d is the distance between the pole and the given point in meters, and μ is a pure number equal to unity for a vacuum and 1.00026 for air. μ is called the *permeability* of the medium.

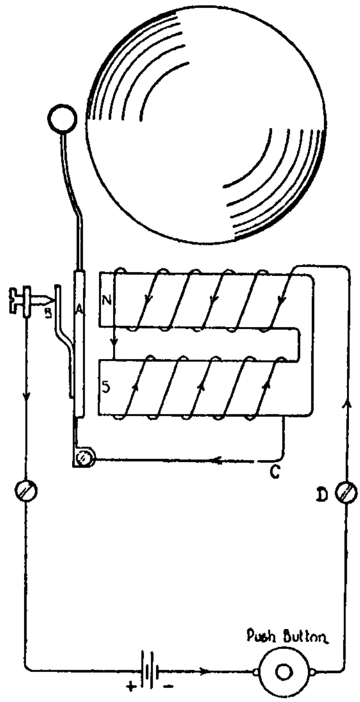


Figure 26-2.

Although these two methods of producing magnetic fields seem very different, they are actually very similar. The lines of force could be drawn in figure 26-1 to look just about as they did in figure 22-1. The field in figure 26-1 is caused by the circulation of electrons in the surrounding wire, whereas in figure 22-1, the field is due to "elementary magnets" which consist of certain electrons within the iron atoms, spinning on their axes in the direction of the fingers of the left hand when the thumb points toward the north pole. The student will also remember still another application of this principle in which a feeble magnetic field was produced by rotating rapidly a charged disk (section 23-3).

Illustrative Problems. (1) Find the magnetic field strength at the center of a helical coil of 100 turns of wire half a meter long through which a current of 3 amperes is flowing.

Solution: Such a coil is often called a ***solenoid***. The value of the field at the center is independent of the radius of the helix, also independent of the material inside the coil, that is, whether it is air, or iron, or a vacuum, since the permeability, μ, does not occur in the equation. To find the field strength it is necessary to substitute into the first equation of section 26-5 the values $I = 3$ amperes, $n = 100$, and $l = 0.5$ meter and obtain

$$H = \frac{4\pi(3)(100)}{0.5}$$

or H is 7,540 millioersteds, a rather feeble field.

(2) Find the numerical value of two opposite magnetic poles of like strength, which, when placed each 25 centimeters away from and on opposite sides of a given point, will produce a field strength, at the given point, of 7,540 milloersteds, in vacuo.

Solution: This time we must substitute into the second equation of section 26-5 the values $H = 3{,}770$ millioersteds, $k_m = 10^7$ newton-meters² per pole², $\mu = 1.000$, and $d = 0.25$ meter and solve for one of the poles, p. The other pole will produce a like effect and thus account for the entire field. Thus

$$3{,}770 = \frac{p\ 10^7}{(1)(0.25)^2}$$

and p is 23.6×10^{-6} pole units or 23.6 micropole units.

26-6. Flux Density. A technical term called *flux density* is obtained by dividing magnetic field strength by k_m and multiplying by the permeability. Flux density is represented by the letter B by electrical engineers. That is

$$B = \frac{\mu H}{k_m}$$

Thus, the two equations of section 26-5 become

$$B = \frac{4\pi I n \mu}{l k_m}$$

and

$$B = \frac{p}{d^2}$$

The flux density in a solenoid depends very much on the permeability of the material involved. For example, the permeability of various samples of steel and iron can easily run from a few hundred to several thousand. The unit of flux density in the practical system is the *weber* per square meter. This is ten thousand times as great as the corresponding unit in the c.g.s. electromagnetic system, the gauss, so that 1 weber/meter² = 10^4 gauss. Flux density and field strength in electromagnetism are related to each other somewhat as strain and stress are related in elasticity, or as effect and cause in logic. The field strength is the cause and the flux density is the effect.

26-7. Flux. As might be guessed from the expression, flux density, it is also thought of as the density of a quantity called *flux* and measured in webers. It is an unfortunate fact that flux is another entity visualized in terms of lines drawn in the same general direction as the field lines. In fact when we speak of magnetic lines we mean flux lines more often than we do field lines. Which is meant can generally be inferred from the context. From now on we shall always be specific. If Φ stands for flux in webers, B is flux density in webers/m^2, and A is area in square meters, the equation connecting them is

$$\Phi = BA$$

26-8. Dimensions. So many technical terms have now accumulated that some sort of classification is desirable. For this purpose we shall use a system which reduces them all to combinations of four fundamental entities: length, time, mass, and quantity of electricity. These we shall denote by the letters L, T, M, and Q respectively. For example, the dimensions of a velocity are those of length divided by time or L/T, usually written LT^{-1}. From this we could infer that the unit is the meter per second. A linear acceleration has the dimensions LT^{-2}. Since force equals mass times acceleration by Newton's second law, the dimensions of force are $LT^{-2}M$. Work or energy has the dimensions of force times distance or $L^2T^{-2}M$. Electrical potential is work per unit charge or $L^2T^{-2}MQ^{-1}$. Electric current is $T^{-1}Q$, measured fundamentally in coulombs per second. Electrical resistance is $L^2T^{-2}MQ^{-1}/T^{-1}Q$ or $L^2T^{-1}MQ^{-2}$. Magnetic field strength from the first equation of section 26-5 is current divided by length or $L^{-1}T^{-1}Q$. From the fact that H is also force per unit magnetic pole we can derive the dimensions of pole as F/H or $LT^{-2}M/L^{-1}T^{-1}Q$ or $L^2T^{-1}MQ^{-1}$. From the second equation of section 26-5 we obtain the dimensions of k_m, remembering that permeability, μ, is a pure number with no dimensions, as $L^{-1}T^{-1}QL^2/L^2T^{-1}MQ^{-1}$ or $L^{-1}M^{-1}Q^2$. From any of the equations of section 26-6 the dimensions of B are $T^{-1}MQ^{-1}$, while those of flux, Φ, are the same as those of pole, namely, $L^2T^{-1}MQ^{-1}$. Finally, the dimensions of k_e from Coulomb's law for electric charges are obtained from $F\epsilon d^2/q_1q_2$, remembering that ϵ like μ is a pure number. Thus the dimensions of k_e are $LT^{-2}ML^2/Q^2$ or $L^3T^{-2}MQ^{-2}$.

The product of the dimensions of k_e and k_m are $L^3T^{-2}MQ^{-2}L^{-1}M^{-1}Q^2$ or L^2T^{-2}, that is, a velocity squared. If we remember the numerical values we find that $k_ek_m = 9 \times 10^9 \times 10^7$ meters^2/sec.^{2} or 9×10^{16} meters^2/sec.^{2}. This is the square of 3×10^8 meters/second, which happens to be the speed with which radio, light, and other electromagnetic disturbances travel through free space. This is, of course, no accident. The dimensions of electric capacitance are obtained from $C = Q/V$ or $Q/L^2T^{-2}MQ^{-1}$ or $L^{-2}T^2M^{-1}Q^2$.

26-9. Effect of a Magnetic Field on a Current. Figure 26-3 represents a square electric circuit in a vertical plane in which flows an electron current, I, in a counterclockwise fashion as indicated by the arrows. If a vertical wire carrying a current, I', is placed near the square circuit, the direction of I' being downward, the result will be that current I' will tend to move toward the left. This is because electrons moving in the same direction lose some of their repulsion, while electrons moving in opposite directions repel each other more. A convenient way of remembering the direction of the force on I' is found to be as follows: observe first by means of the left hand rule of section 26-2 that the square circuit produces magnetic lines of force which inside the square are away from the observer; then using the left hand again as in figure 26-4, recite the physical fact, "*Current* (1) in a *field* (2) produces *motion* (3)," putting into position in turn the thumb, forefinger, and middle finger. That is, the thumb will represent the current, the forefinger the field, and the middle finger the motion of the wire, which is toward the left.

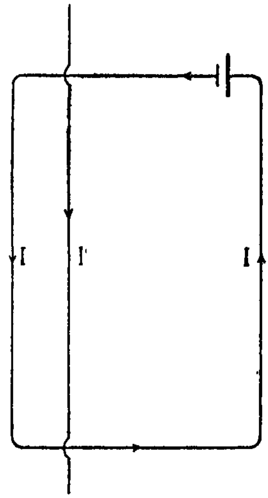


Figure 26-3.

26-10. Comparison of Forces Exerted by a Magnetic Field on Poles and Currents. A magnetic field, H, exerts a force, F, on a magnetic pole, p, which is given by the equation

$$F = Hp$$

where F is in newtons, H in millioersteds, and p in pole units. The direction of the force is the same as the direction of the field.

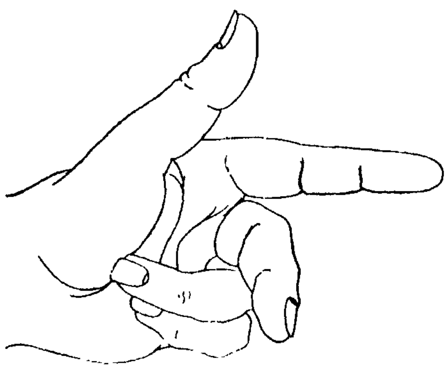


Figure 26-4.

A magnetic field, H, exerts a force, F, on a wire of length, l, carrying a current, I, in a medium of permeability, μ, given by the equation

$$F = \frac{\mu H I l}{k_m}$$

Comparing this with the first equation of section 26-6 it will be seen that it is simpler to say

$$F = BIl$$

where F is in newtons, B in webers/m^2, I in amperes, and l in meters.

The force, the flux density, and the current are all at right angles to each other; the force in the direction of the middle finger of figure 26-4, the flux density the forefinger, and the current the thumb.

26-11. Illustrative Problems. (1) Check the second equation of section 26-10 dimensionally.

Solution: The usual notation for stating that the dimensions of F are $LT^{-2}M$ is to write the equation

$$[F] = LT^{-2}M$$

Similarly

$$\begin{aligned} [\mu] &= 1 \\ [H] &= L^{-1}T^{-1}Q \\ [I] &= T^{-1}Q \\ [l] &= L \\ [k_{\mathrm{m}}] &= L^{-1}M^{-1}Q^2 \end{aligned}$$

The relation, $[\mu] = 1$, has a totally different meaning from that of $\mu = 1$. The first means that μ is a pure number while the second means that not only is it a pure number, but its numerical value is 1.000. Another way of indicating that μ is a pure number is to say $[\mu] = L^0T^0M^0Q^0$. Now to check the relation $[F] = [\mu HIl/k_{\mathrm{m}}]$ we have

$$LT^{-2}M = \frac{L^{-1}T^{-1}QT^{-1}QL}{L^{-1}M^{-1}Q^2}$$

Since the right hand side does reduce to the left hand side, we can say that we have checked the equation dimensionally. It should be possible to check dimensionally any equation of physics.

(2) Find the side push on a wire 20 centimeters long which carries a current of 2 amperes and lies in the armature of a motor in a field of 10,000 oersteds.

Solution: Assuming this wire to lie in the air gap between the pole faces, $\mu = 1$. We also have $l = 0.2$ meter, $I = 2$ amperes, $H = 10{,}000{,}000$ millioersteds, and $k_m = 10^7$ newton-meters²/pole². Thus from the second equation of section 26-10

$$F = \frac{(1)(10{,}000{,}000)(2)(0.2)}{10{,}000{,}000}$$

This reduces to $F = 0.4$ newton. Since there are 4.45 newtons to the pound, this is about 0.09 pound or 1.44 ounces. But if there are, say, 200 of these conductors under the pole pieces of the motor at any one time, the total force would be about 18 pounds and would exert a satisfactory torque.

26-12. Motors and Meters. Figure 26-5 is to be thought of as the cross section of a motor, the armature of which is free to rotate between the pole pieces (labeled N and S). Let us adopt the convention that ⊙ represents the cross section of a wire in which the electrons are moving toward us and ⊕ a wire with electron current away from us. In the former case, we are seeing the tip of an arrow and in the latter case, the tail. We should first check to see if the flow of current in the field as indicated by the four arrows would actually make the left hand north and the other south. Grasping the left hand pole with the left hand, with the fingers pointing in the direction of the field current, does result in pointing the thumb upward in the direction of the north pole. Next apply the thumb

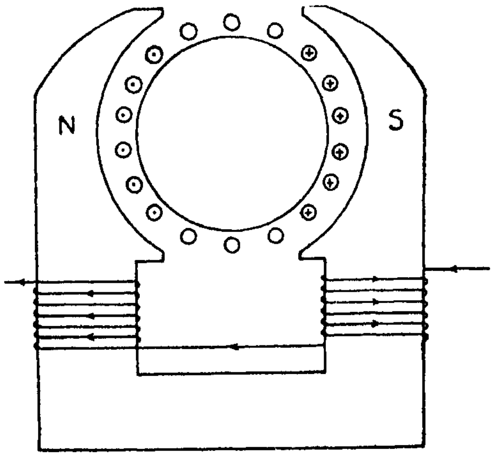


Figure 26-5.

and two finger rule to determine the direction of rotation of the armature, again using the left hand. Current (1) in a field (2) produces motion (3). Concentrating on the air gap between the north pole and the armature, we find wires with electrons coming toward us, so we point the thumb of the left hand toward us. The field goes out of the north pole through the armature into the south pole, so we point the forefinger in that direction. The middle finger by now is pointing down, which is therefore the direction of rotation of that side of the armature. For practice the student should apply the rule again to convince himself that the other side of the armature rotates upward.

The motor is arranged with brushes to feed in the current to the armature always in the same direction in spite of the rotation, so that the effect is continuous. If, on the other hand, an apparatus is constructed so that the tendency to rotate is counterbalanced by a spring, it is possible to move an indicator across a scale to an extent dependent on the strength of the current flowing. Such a "meter" is called a *galvanometer*. A galvanometer may be arranged to measure either current (ammeter) or voltage (voltmeter). Inside the box housing, the ammeter is a galvanometer in parallel with a heavy copper strap of low resistance. When the ammeter is put in series with the rest of the circuit, most of the current goes through the strap, but what little goes through the galvanometer is still proportional to the total current, and the scale may be graduated to give the reading of the total current. Inside the box housing of the voltmeter is a galvanometer in series with a considerable resistance. When the voltmeter is put in parallel with the apparatus whose voltage is desired, the small current through the voltmeter will be proportional to the potential drop across its terminals and again the scale of the instrument may be calibrated to give this reading in volts.

26-13. Induced Electromotive Force. Of the three common methods of producing an electromotive force, namely, the chemical method utilizing a voltaic or storage cell, the thermoelectric method, and the magnetic, the last is by far the most important commercially. We think of it as a process of "cutting lines of force" with a conductor, and the "induced electromotive force" in volts is numerically equal to the rate at which the lines of force are cut.

The reason that electrons tend to move through a wire while it is moving at right angles both to itself and to a field may be seen by considering a simple experiment. Imagine pushing a metallic wire, held parallel to the plane of the paper and perpendicular to the bar magnet, down into the paper just beyond the letter *N* in figure 26-1. The moving electrons, which constitute the electric current in the insulated wire of the electromagnet, must be thought of as coming up out of the paper at the top of the diagram and going down into the paper on the under side. The free electrons in the metallic wire that we are pushing down through the field will thus find themselves moving in the same direction as the electrons that are going into the paper. As we have seen, electrons moving parallel with each other lose some of their repulsion for each other. For this reason the free electrons in the moving wire will tend to move along the wire in a direction toward the bottom of the page; if the wire is part of a closed circuit there will be an electron current in this direction. At any rate there will be a redistribution of electrons constituting an electromotive force.

This e.m.f. exists only during the relative motion of the wire and the field, and it makes no difference which one of the two does the moving. If E represents the e.m.f. in volts, and Φ is the number of flux lines or webers cut by the wire in time, t seconds, then

$$E = \frac{\Phi}{t}$$

The student will notice that this equation checks dimensionally. Another equation closely resembling this is

$$E = Blv$$

where again E is e.m.f. in volts, B is flux density in webers/m^2, l is the length of the wire that is cutting the field in meters, and v is the velocity of this wire in meters per second.

26-14. Induction Coil; Transformer. We can easily arrange it so that a magnetic field moves or varies in the presence of a stationary wire. For example, in figure 26-1, assume an additional wire to be wound around the iron core with the two ends of the new wire electrically connected. If, now, the original circuit is broken, the accompanying magnetic field will shrink to zero in the presence of the new wire. The relative motion of the shrinking field and the wires will result in an electromotive force in both the old and the new wire in the same direction in which the original current was flowing. Since the circuit was broken in the original wire, no current flows in it, but since the new circuit is closed, a momentary current flows in it. When the original battery circuit is again closed, there will be a momentary current in the new wire in the opposite direction, due to the increasing magnetic field in the presence of the wire.

If the new circuit consists of several turns of wire, we may regard the e.m.f.'s in each turn as in series with each other. We shall now call the original windings the *primary*, and the new windings the *secondary*, circuit. An induction coil (see figure 26-6) is a device of this type. The primary circuit consists of a few turns of insulated wire on an iron core with connections that remind us of the principle of the electric bell. The hammer again flies back and forth, thus constantly interrupting current in the primary circuit. This causes the flux lines to shrink and grow regularly at a rapid rate. The secondary circuit consists of many

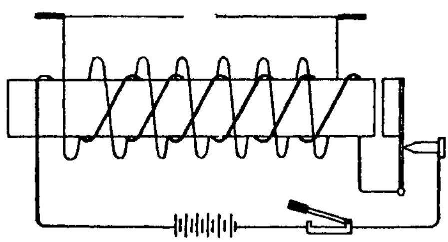


Figure 26-6.

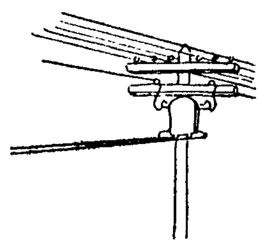
turns of fine wire also carefully insulated. Since these turns lie in the rapidly changing field, they will be the seat of an induced electromotive force and on account of the many turns, the total voltage will be very large, thousands of volts. A spark will thus jump between the terminals of the secondary circuit for a considerable distance.

Instead of interrupting the primary circuit mechanically, we may feed into it an alternating current (abbreviated to *a.c.*) in which the electrons reverse their direction of motion continuously. Thus the flux lines constantly shrink to zero and expand in the other direction. Again there will be a voltage induced in the secondary. Such an apparatus is known as a *transformer*. The relation between the *a.c.* voltage, E_p, applied to the primary of the transformer and that induced in the secondary, E_s, is given by the equation

$$\frac{E_s}{E_p} = \frac{n_s}{n_p}$$

where n_p is the number of turns of wire in the primary and n_s the number of turns in the secondary circuit. In the transformer it is much more efficient to make the iron core more in the shape of a torus (doughnut-shaped) so that the flux lines may exist in iron throughout their entire length. Transformers are used to "step up" and "step down" voltages. For example, electric companies find it more economical to transmit their power at high voltages and low currents (hence small I^2R heat losses) to the places where it is to be used, then step down the voltage to 110 volts by means of transformers located on poles at the point of delivery.

26-15. Inductance. One more technical term must be defined here for use in our coming discussion of alternating currents, namely *inductance*. Inductance plays just about the same part in electrical theory that mass does in mechanics. *Mass* may be defined as the ratio between a force and the resulting rate of change of velocity; similarly inductance may be defined as the ratio between an electromotive force and the resulting rate of change of current. Up to this time, we have considered our circuits after a steady state had



been established, in which case the only effect of the impressed voltage was to maintain the constant current through the given resistance. But when the current is first turned on, it must grow from zero to its final value, and part of the voltage is used for this purpose.

Electrons behave as if they possessed inertia; if they are at rest, it requires an electromotive force to start them moving, and once they are moving, they tend to continue in motion even if it means jumping the gap when the switch is opened. This electrical inertia is to be identified with inductance just as we identified mechanical inertia with mass. The equation connecting the quantities involved is

$$V = (L)\ (\text{rate of increase of current})$$

V represents a portion of the impressed voltage which is increasing the current and is measured in volts, and the rate of increase of current is measured in amperes per second. The unit of L is the *henry*, in memory of Joseph Henry (1797-1878), an American physicist. Dimensionally, inductance is L^2MQ^{-2}.

We are able to explain electrical inertia in terms of induced electromotive forces, whereas mechanical inertia must simply be assumed as a fact. At the instant when a voltage is first applied to a piece of apparatus such as the electromagnet in figure 26-1, no current is flowing and no magnetic flux exists. When the current starts to flow and the magnetic flux lines begin to come into existence, they cut across the wire in the electromagnet in such a direction as to induce an electromotive force opposing the impressed voltage. Thus at any instant previous to the establishment of the steady state, part of the applied voltage is utilized in opposing this induced electromotive force and the rest in maintaining the current that exists at the moment. This is why the V in the equation of the preceding paragraph of this section is merely a portion of the total impressed voltage. When the switch is opened and the current begins to shrink, the motion of the decreasing flux in cutting the wire is such as to induce a direct electromotive force which tends to keep the current flowing and makes an arc across the switch.

The kinetic energy of the electrons flowing in a circuit may be written

$$k.e. = \tfrac{1}{2}LI^2$$

just as mechanical translatory kinetic energy was $\frac{1}{2}mv^2$. In this equation, $k.e.$ is in joules, L in henries, and I in amperes. It is this energy which appears in the form of heat and light in the spark or arc when the switch is opened.

There are formulas for computing the inductances of various types of circuits. Only one of these will be given here, namely, that for an electromagnet like the one shown in figure 26-1. If l is the length of the bar of iron in meters, A its cross-sectional area in square meters, and μ its permeability, then

$$L = \frac{4\pi n^2 A\mu}{k_m l}$$

Again L is in henries, n is the number of turns of wire, and k_m is 10^7 newton-meters^2 per pole unit squared. By remembering the physical dimensions involved, it will be seen that k_m may also be expressed as 10^7 coulomb^2 per kilogram per meter or as 10^7 meters per henry.

26-16. Illustrative Problems. (1) Find the inductance of the primary of a transformer if the iron core is in the shape of a torus of mean circumference 0.7 meter, cross section 10 cm.^{2}, and permeability 2,000, wound with 200 turns of wire.

Solution: It is merely necessary to substitute into the last equation of the preceding section the values $n = 200$ turns, $A = 0.0010m^2$, $\mu = 2{,}000$, $k_m = 10^7$ meters/henry, and $l = 0.7$ meter. Thus we have

$$L = \frac{4\pi\ (200)\ (0.0010)\ (2{,}000)}{(10^7)\ (0.7)}$$

or $L = 0.718 \times 10^{-3}$ henries. It is more customary to express this as 0.718 millihenry.

(2) If the resistance of this electromagnet is half an ohm, find the rate at which the current is increasing a thousandth of a second after the switch has been closed in a 6-volt circuit; assume that the value of the current at this instant is 6.24 amperes.

Solution: When the current is 6.24 amperes, 3.12 volts is necessary to make it flow on a 0.5-ohm circuit. Thus of the 6-volt total, 2.88 volts is still available to make the current grow. Substituting then in the first equation of section 26-15 the values $V = 288$ volts and $L = 0.000718$ henry, we can solve and find that the rate of increase of current at this instant is 2.88/0.000718 or 4,010 amperes per second. Since the final current in this circuit is to be 6/0.5 or 12 amperes and it is already 6.24 amperes, at this rate it will take only 5.76/4,010 or 0.001436 second more to reach maximum value. As a matter of fact, the nearer the current gets to the 12-ampere mark the slower is the rate of increase, so that theoretically it would require an infinite time; for all practical purposes, however, we consider the current steady after a few thousandths of a second.

(3) How much energy will appear in the spark when this circuit, with a 12-ampere current flowing, is broken?

Solution: We need the second equation of section 26-15. L is 0.000718 henry, I is 12 amperes, therefore

$$k.e. = \tfrac{1}{2}\ (0.000718)\ (12^2)$$

Thus the energy of the circuit which was originally contained in the magnetic field but which now appears in the spark at the switch is 0.0517 joule. This is only about a hundred thousandth of a Calorie or about 0.00005 B.t.u.

26-17. Lenz's Law. There is always some motor action in a generator and some generator action in a motor. This means that while it is very easy to turn a generator before the circuit has been closed (that is, while no current is flowing), the moment a current commences to flow, a side push (motor action) develops *opposite* to the direction of rotation. If the side push were in the same direction instead of the opposite direction, the generator would run itself and we should have perpetual motion. As a motor operates, the armature wires cut lines of magnetic force and an electromotive force is produced (generator action) *opposite* to the direction in which the current is flowing (we call it a *back e.m.f.*). If this back electromotive force were a direct electromotive force, we could use it to run the motor and have another case of perpetual motion. But we have just seen that perpetual motion is a violation of the law of conservation of energy. Either of these statements, one for the generator and the other for the motor, may be considered a statement of Lenz's law. The statements may be reworded slightly so as to read (1) when a conductor moves in a magnetic field, a current tends to be produced, the side push on which is in the direction opposite to the motion of the conductor, and (2) when a conductor carries a current in a magnetic field, motion tends to be produced, the direction of which is such as to induce an electromotive force in *opposition* to the current already existing in the conductor.

SUMMARY OF CHAPTER 26

Technical Terms Defined

Permeability. The ratio between the force between two magnetic poles in vacuo and the force between the same poles the same distance apart in another medium is the permeability of this medium.

Flux Density. The product of the magnetic field strength by the permeability of the medium and divided by the constant k_m, which is 10^7 newton-meters^2 per pole unit squared, gives the flux density in practical units (webers per square meter).

Flux. The magnetic flux through a given area is the product of the flux density by the area. The practical unit of flux is the weber.

Dimensions. Dimensions of physical quantities are reductions of these quantities to the four fundamental quantities in physics, length, time, mass, and quantity of electricity. A few dimensions are as follows.

Physical term	*Symbol*	*Dimensions*
Length	l	L
Time	t	T
Mass	m	M
Quantity of electricity	q	Q
Velocity	v	LT^{-1}
Acceleration	a	LT^{-2}
Force	F	$LT^{-2}M$
Energy		$L^2T^{-2}M$
Potential	V	$L^2T^{-2}MQ^{-1}$
Current	I	$T^{-1}Q$
Resistance	R	$L^2T^{-1}MQ^{-2}$
Magnetic field strength	H	$L^{-1}T^{-1}Q$
Magnetic pole	p	$L^2T^{-1}MQ^{-1}$
Magnetic constant	k_m	$L^{-1}M^{-1}Q^2$
Flux density	B	$T^{-1}MQ^{-1}$
Flux	Φ	$L^2T^{-1}MQ^{-1}$
Electrostatic constant	k_e	$L^3T^{-2}MQ^{-2}$
Capacitance	C	$L^{-2}T^2M^{-1}Q^2$

Induced E.M.F. An induced electromotive force is produced by cutting magnetic flux by a conductor. The value of the induced electromotive force in volts may be found by dividing the flux in webers by the time in seconds consumed in cutting it.

Inductance. The inductance of a circuit is the ratio of that portion of the voltage employed in making the current increase by the rate of increase of the current. Its unit is the henry. Inductance in electricity corresponds to inertia in mechanics.

Laws, Rules, and Principles

Left Hand Thumb Rule. If the fist of the left hand be used with only the thumb extended, the thumb shows the direction of the magnetic flux if the closed fingers indicate the direction of flow of the electrons around an electromagnet. Or the thumb may indicate the direction of flow of the electrons along a wire and the fingers the magnetic flux around the wire.

Left Hand Thumb and Two Finger Rule. If the thumb and the first two fingers of the left hand are extended in the most natural manner so that all three are at right angles to each other, they may be used to illustrate

(1) For Side Push. the relations involved in the statement (1) "A current (a) in a field (b) produces motion (c)" in which the thumb is (a) and the next two fingers are (b) and (c) respectively, or

(2) For Induced Electromotive Force. (2) "Motion (a) in a field (b) produces e.m.f. (c) in which case (a), (b), and (c) still refer to the thumb and two fingers respectively.

Two Methods of Producing Magnetic Fields. Method (1), by magnet poles. Equation

$$H = \frac{pk_m}{\mu r^2}$$

Method (2), by an electric current. Equation

$$H = \frac{4\pi In}{l}$$

Two Effects of a Magnetic Field. Effect (1), on a magnetic pole. Equation

$$F = mH$$

Effect (2), side push on a current. Equation

$$F = \frac{\mu HIl}{k_m}$$

Lenz's Law. Both the induced electromotive force produced by cutting magnetic flux with a conductor and the side push exerted on a current-bearing conductor in a magnetic field are in such a direction as to avoid a violation of the law of conservation of energy.

PROBLEMS

26-1. Draw a diagram of an electromagnet shaped like a horseshoe with poles labeled north and south respectively, showing the necessary directions of the electron currents.

26-2. Find the current which must flow in a solenoid of 300 turns, one meter long, such that the magnetic field strength at its center may be 0.166 oersted. If the axis of this solenoid is placed in a horizontal position at right angles to the earth's magnetic field at a place where the latter is also 0.166 oersted, in what direction will a compass needle at its center point? Draw a diagram to illustrate.

26-3. A magnet, 25 centimeters between poles, has poles of strength 60 microunits. Find the strength of the magnetic field at a point 15 centimeters from one pole and 20 centimeters from the other.

26-4. What is the flux density through the center of the solenoid of problem 26-2, (1) with an air core, and (2) with an iron core of permeability 1,500?

26-5. Find the total flux in a toroidal iron ring of permeability 2,000, mean circumference 90 centimeters, and cross-sectional area 12 cm.^{2}, if it is wound with 300 turns of insulated wire which carries a current of 2 amperes.

26-6. Newton's law of gravitation is expressed

$$F = \frac{m_1 m_2 k_g}{d^2}$$

in which the numerical value of k_g is 6.66×10^{-11} when F is in newtons, m_1 and m_2 in kilograms, and d in meters. Find the dimensions of k_g and assign units to its numerical value.

26-7. *Magnetic moment* is defined as the product of the length of a magnet by the strength of one of its poles. Find the dimensions of magnetic moment. A current of I amperes flowing in a circular wire of one turn, such that the area of the circle is A square meters, is equivalent to a magnet of moment

$$\frac{AI\mu}{k_m}$$

Check this relationship dimensionally.

26-8. Assume the lines of force in the earth's magnetic field to run from south to north and to dip below the horizontal at an angle of 74 degrees. The total intensity of this field is 0.59 oersted. If a rod two meters long that is part of a closed circuit is held at right angles to this field in a vertical plane and moved toward the west at the rate of 30 feet per second, what electromotive force will be induced and in what direction?

26-9. If the rod of the preceding problem is stationary, but carries a current of 50 amperes, what force acts on it and in what direction?

26-10. If the horizontal intensity of the earth's magnetic field is 0.166 oersted, what force does this field exert on one pole of a compass needle the pole strength of which is 0.001 of a micropole unit?

26-11. A certain motor has 300 conductors under the pole pieces, each 0.3 meter long and carrying a current of 5 amperes. If the field through the air gap in which these conductors lie is 30,000 oersteds and the radius of the armature is 0.25 meter, find the torque acting on the armature.

26-12. As the armature of a motor rotates in the magnetic field supplied by the pole pieces, a back electromotive force of 1,105 volts is induced in the armature. This motor runs on 110 volts and the armature has a resistance of 0.2 ohm. What is the armature current? What power is supplied to the armature? How much of this power goes into heat? Find the electrical efficiency of the armature.

26-13. A galvanometer has a resistance of 25 ohms and requires a current of one milliampere to move its needle along its scale one division. What resistance must be put in series with it inside the instrument case so that there will be one volt drop across the terminals of the instrument when the needle moves one division? Into what instrument has the galvanometer now been transformed? Draw a diagram showing how it could be used.

26-14. A galvanometer has a resistance of 25 ohms and requires a current of one milliampere to move its needle along its scale one division. What resistance must be placed in parallel with it (shunted across it) inside the instrument case so that a combined current of one ampere will go through both galvanometer and shunt when the needle moves one division? Into what instrument has the galvanometer now been transformed? Draw a diagram showing how it could be used.

26-15. If, instead of feeding the motor described in problem 26-11 a current, it is driven at the rate of 2,330 revolutions per minute, it becomes a generator. What electromotive force will then be induced in each of its conductors?

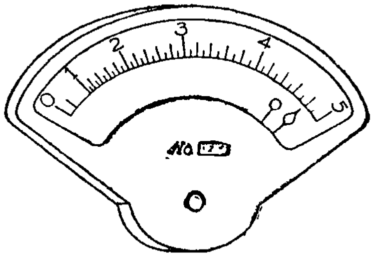
26-16. A step down transformer is desired, the high side of which may be attached to a 110-volt 60-cycle line and the low side to furnish 4 volts with which to ring a bell. If there are 1,000 turns of wire in the primary, how many turns must there be in the secondary?

26-17. Find the number of turns that must be wound on an iron core of permeability 1,600, length 80 centimeters, and cross-sectional area 20 square centimeters to make up an inductance of one henry.

26-18. Find the energy in joules residing in the magnetic field surrounding a current of 10 amperes as it flows through an inductance of 5 henries. What ultimately becomes of this energy?

26-19. A certain circuit has a resistance of one ohm and an inductance of 10 henries. One hundredth of a second after the switch is closed in this circuit, the current has risen to one thousandth of its full value. If 10 volts is impressed on this circuit, how fast is the current increasing at this instant? Ten seconds after closing the switch, the current has risen to 63.3 per cent of its full value. How fast is the current now increasing? One minute after closing the switch, the current has reached 99.75 per cent of its full value. How fast is the current now increasing?

CHAPTER 27



Alternating Currents

27-1. Qualitative Description of an Alternating Current. The simplest type of alternating current is that in which the electrons oscillate in the manner described in chapter 15, that is, in simple harmonic motion. Thus there is a certain instant in each cycle when they are approximately at rest, a quarter of a cycle later they are moving with maximum velocity in one direction, and half a cycle after this, they are moving with maximum velocity in the other direction. If the velocity of the electrons is graphed against time, we obtain a diagram like the curve ADF in figure 27-2, which is the well known sine wave. The accompanying magnetic field also goes through a similar cycle, increasing from zero to a maximum, shrinking to zero, building up to a negative maximum, and again becoming zero. The common length of a cycle is one sixtieth of a second, although other values are occasionally used, such as 25 cycles per second or 500 cycles per second. It is thus seen that in a discussion of alternating currents, several more variables are involved than in direct current theory. These will not only include such items as voltage, current, and resistance, but, since the current is continually varying, inductance will become important; also frequency and capacitance will enter into the calculations.

27-2. Mechanical Analogies. There is a close parallelism between the behavior of an electric condenser and the phenomenon of elasticity in mechanics. It is easy to take the first electron out of the conductor on one side of a condenser and put it in the other side. In order to take the second electron away from the now positive plate, we must oppose the attraction between the opposite charges, and to put the electron on the now negative plate, we must oppose the repulsion of like charges. And the more charge the condenser already has, the more voltage is necessary to produce any further charge, just as the more an elastic rod is bent, the more force is required to do any more bending, by Hooke's law. In the latter case, we have $F = kx$ while in the electrical case we have $V = \frac{1}{C}Q$ and we see that $1/C$ corresponds to "stiffness" in mechanics. Sometimes it is said that C in electrical theory corresponds to "compliance" in mechanics.

We have already seen that inductance in electricity corresponds to mass or inertia in mechanics.

It is furthermore true that electrical resistance corresponds closely to fluid friction, which is proportional to the velocity with which an object moves through the resisting fluid medium. Thus $E = RI$ corresponds to $F = R'v$ where F is the mechanical force necessary to make the object move through the fluid with velocity, v, and the proportionality factor, R', is the mechanical resistance.

The behavior of electrons forced to execute simple harmonic motion with frequency, n, in a circuit containing resistance, R, inductance, L, and capacitance, C, is then quite analogous to the behavior of a fairly heavy pendulum bob attached to a spring of medium stiffness and forced to move through a viscous liquid in accordance with the requirements of simple harmonic motion. It will pay us to consider these three effects separately.

27-3. Effect of Resistance Alone. In order to eliminate the effect of inductance (electrical inertia) we shall make the pendulum bob very light, say of balsa wood. By removing the spring, we are freed of capacitance effects. We may imagine the viscous fluid to be molasses. Due to Archimedes' principle, it will be necessary to exert a downward force on the balsa wood merely to hold it under the surface, but this force is constant and need not be considered further. Since we are interested in the relationship between voltage and current, the relations between force and velocity in the mechanical analogy are to be considered.

We are starting with the simplest case. It is merely necessary with viscous friction to exert a large force at the same time that we wish a large velocity; therefore we say that the force varies harmonically *in phase with* the harmonic variation of the velocity. This means that when the velocity is zero, the force is zero; when the velocity has reached its maximum value in one direction (at the center of the motion), the force is also a maximum in that same direction. Ohm's law therefore holds for this case with no amendments

$$E_R = IR$$

Here as usual E_R is in volts, I in amperes, and R in ohms.

27-4. Effect of Inductance Alone. In order to determine the effect of inertia alone, the pendulum bob must be made massive, say 20 pounds of lead; it must move in a frictionless medium, air, or better yet, in a vacuum; furthermore there must be no restoring force of a capacitance-like nature. This means that the supporting cord must be very long.

Suppose we start at A in figure 15-3 with zero velocity. Due to Newton's second law, if we wish an acceleration (which at this point is to be a maximum acceleration) we must apply a corresponding force. By the time our lead weight has reached the point O, this force must have diminished to zero, but the velocity will then be a maximum. The weight will "coast" through the central point with maximum velocity and zero acceleration (which implies zero force), but beyond O a small force must be applied in the negative direction. This negative force increases to a maximum while the velocity shrinks to zero and then picks up a negative value. If the cycle is divided into quarters and the quantities described respectively at A, O, B, O, and A, the forces at these points are respectively positive maximum; zero; negative maximum; zero; positive maximum, while the velocities are respectively zero; positive maximum; zero; negative maximum; and zero. It will be noticed that the velocities are a quarter cycle behind the forces. In the electrical analogy, we say that the current lags ninety degrees behind the voltage. The voltage and current in this case are related by the equation

$$E_L = 2\pi nLI$$

where E_L is in volts, n in cycles per second, L in henries, and I in amperes.

27-5. Effect of Capacitance Alone. The mechanical analogy in this case is obtained by replacing the lead pendulum bob with balsa wood again, retaining the nonviscous medium, but using a spring in such a way that the equilibrium position of the bob will be at O. Then the force will be a negative maximum value at A, where the velocity is again zero. This time the successive values of the forces at A, O, B, O, and A are negative maximum; zero; positive maximum; zero; negative maximum, while the velocities are as before zero; positive maximum; zero; negative maximum; and zero. This time it will be noticed that the velocities are a quarter cycle ahead of the forces and the current is also leading the voltage by 90 degrees. The equation is

$$E_C = \frac{I}{2\pi nC}$$

where E_C is in volts, I in amperes, n in cycles per second, and C in farads.

27-6. The Joint Effect of Resistance, Inductance, and Capacitance. Returning now to the heavy lead pendulum bob with spring attached and moving in molasses, the resultant force will be the vector sum of three forces two of which are 180 degrees out of phase with each other and the third 90 degrees out of phase with each of the others. Likewise in the electrical case, the total voltage E is related to the three component electromotive forces by the equation

$$E = \sqrt{E_R^2 + (E_L - E_C)^2}$$

This may be seen by a reference to figure 27-1.

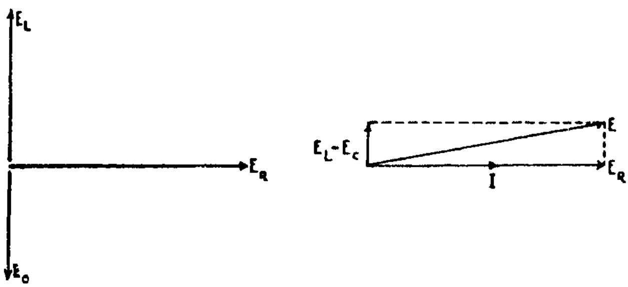


Figure 27-1.

Since the current is always in phase with E_R we have the same angle between I and E as between E_R and E. Thus if $O'A'$ of figure 27-2 represents E, then $O'B'$ will represent I.

27-7. The Rotating Vector Diagram. Figure 27-2 serves to show how a "sine wave" may be depicted by a vector rotating counterclockwise. The arrow $O'A'$ may be imagined to rotate uniformly and its projection at any instant on the axis of ordinates will give the points on the sinusoid. A is the projection of $O'A'$ at zero time, E_m after $O'A'$ has rotated counterclockwise through 90 degrees, D at 180 degrees and F at 360 degrees. The dashed sinusoid represents the current in a similar fashion; the current lags behind

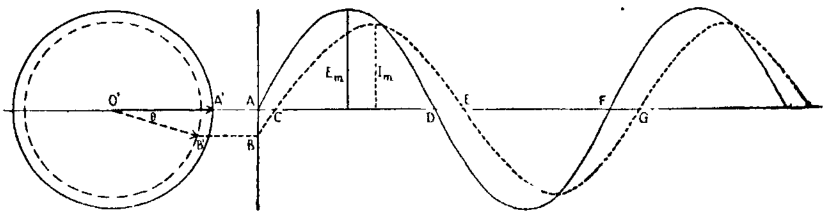


Figure 27-2.

the voltage in this diagram. It will be seen that the rotating vector diagram is the full equivalent of the data shown by the two sinusoids and is much more compact.

If E_L happens to be smaller than E_C, figure 27-1 will be replaced by figure 27-3.

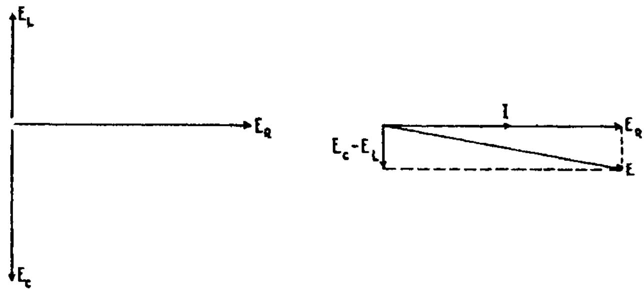


Figure 27-3.

In this case figure 27-2 is replaced by figure 27-4 and we say that the current leads the voltage.

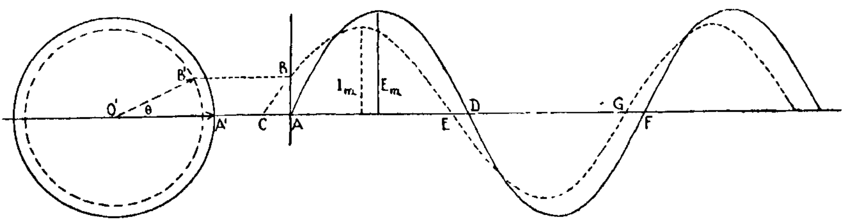


Figure 27-4.

27-8. The Alternating Current Equation. Putting together the equations of sections 27-3, 27-4, 27-5, and 27-6 gives

$$E = \sqrt{I^2R^2 + \left(2\pi nLI - \frac{I}{2\pi nC}\right)^2}$$

which when solved for I gives

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi nL - \frac{1}{2\pi nC}\right)^2}}$$

If we let X stand for $2\pi nL - \dfrac{1}{2\pi nC}$, this becomes

$$I = \frac{E}{\sqrt{R^2 + X^2}}$$

X is called reactance and is measured in ohms.

By definition

$$X_{\mathrm{L}} = 2\pi nL$$

$$X_{\mathrm{C}} = \frac{1}{2\pi nC}$$

X_{L} is called inductive reactance and X_{C} capacitive reactance.

If Z stands for

$$\sqrt{R^2 + X^2}$$

we have

$$I = \frac{E}{Z}$$

Z is called impedance and is likewise measured in ohms. Thus

$$Z = \sqrt{R^2 + \left(2\pi nL - \frac{1}{2\pi nC}\right)^2}$$

27-9. Illustrative Problem. A circuit consists of an alternating current generator of electromotive force 120 volts, frequency 60 cycles, and negligible impedance, in series with a resistance of 10 ohms, a coil of

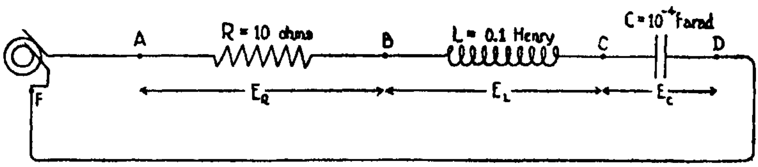


Figure 27-5.

negligible resistance but of 0.1 henry inductance, and a condenser of 100 microfarads capacitance. Find the current, the phase relation between current and voltage, and also the voltages across each part of the circuit.

Solution: The diagram representing such a circuit is shown in figure 27-5. A condenser represents a break in the circuit through which a direct current will not pass, but in the case of an alternating current, the electrons first pile up on one side of it, at the same time draining out of the other side, then drain out of the first side and pile up in the second side, repeating this at each cycle. The higher the frequency of an alternating current, the less is the reactance of a condenser, and the greater the reactance of an inductance; on the other hand, condensers offer more reactance to low frequency alternating currents and inductances very little reactance.

In our problem the inductance has a reactance, X_L, of $2\pi(60)(0.1) =$ 37.7 ohms and the capacitance a reactance, X_C, of

$$\frac{1}{2\pi(60)10^{-4}}, \text{ or } 26.5 \text{ ohms}$$

Then

$$X = X_L - X_C = 37.7 - 26.5 = 11.2 \text{ ohms}$$

The impedance, Z, of this circuit then is $\sqrt{10^2 + (11.2)^2} = 15.01$ ohms, and by the next to the last equation of section 27-8

$$I = \frac{E}{Z} = \frac{120}{15.01} = 7.99 \text{ amperes}$$

Since in this circuit everything is in series with everything else, the same current is flowing at each instant in all parts of the circuit, hence 7.99 amperes is the first result we seek.

By the equation of section 27-3, E_R, which is the voltage across AB, is $(7.99)(10) = 79.9$ volts. This is the horizontal vector of figure 27-1.

By the equation of section 27-4, E_L, which is the voltage across BC, is $(37.7)(7.99) = 301$ volts, which is greater than the impressed voltage (120 volts) on the entire circuit. It is thus more dangerous accidentally to get across the points BC with the fingers than across AF. This voltage is represented by the vector in figure 27-1 which points upward.

By the equation of section 27-5, E_C, which is the voltage across CD, is $(7.99)(26.5) = 212$ volts, which is again greater than the impressed voltage. This is shown in figure 27-1 by the downward vector.

In figure 27-1, the vector labeled $E_L - E_C$ is equal to $301 - 212$ or 89 volts. This is the voltage across BD of figure 27-5.

As a check, E should be $\sqrt{(79.9)^2 + (89.0)^2}$ or 119.6 volts, which rounds off to 120 volts.

The sine of the angle between E_R and E in figure 27-1 is 89/120 or 0.742. This corresponds to an angle of 47.9 degrees. See appendix 7. Therefore the phase relation between the current and the voltage in this circuit is that the current lags 47.9 degrees behind the impressed voltage. However, although the current in the circuit is everywhere the same at any given instant, the voltages are different, although they are such as to add vectorially to 120 volts. Thus in the resistance AB the current is in phase with the voltage. In the pure inductance, BC, the current lags 90 degrees behind the voltage, while in the pure capacitance, CD, the current leads the voltage by 90 degrees.

Actually there is no such thing as an inductance without resistance nor a resistance without inductance. When it is desired to wind a resistance "noninductively," the middle point of the wire is found; the wire is then bent back on itself and wound double, so that everywhere in the circuit, wherever the current is flowing in a given direction in one wire, there is a wire adjacent to it containing the same current in the opposite direction. Therefore the magnetic fluxes nearly cancel out.

27-10. Resonance. In the illustrative problem just solved, E_L and E_C were both larger than E, but added vectorially (also in this case algebraically) to something less than E. If E_L and E_C should be numerically alike and thus add to zero the condition of the circuit is described as that of "*series resonance.*" Thus we have resonance when the inductive reactance is numerically equal to the capacitive reactance. In this case the reactance, X, of the circuit reduces to zero and the impedance of the circuit is equal to the resistance.

Representing this situation by one equation we have

$$2\pi nL = \frac{1}{2\pi nC}$$

If we solve this equation for n, we obtain

$$n = \frac{1}{2\pi\sqrt{LC}}$$

It is therefore possible in any given circuit to find a frequency for which the circuit will be in resonance. We shall see that tuning a circuit to resonance becomes important in radio. This is because at the resonant frequency the current is very much larger for a given impressed voltage than at any other frequency.

27-11. Power. One of the power equations that we met in the discussion of direct current circuits still holds in alternating current theory; the other one does not.

$$\text{Power} = I^2R$$

still represents the rate of heating in watts if I is in amperes and R is in ohms. In fact, we could use this equation to give us a picture of the alternating current ampere. Put into words, we have the statement that an alternating current ampere is so chosen that it will generate heat in a given resistance at the same rate as one ampere of direct current.

In figure 27-6, we have three similar triangles, all containing the same acute angle, θ.

The second is obtained by multiplying each side of the first by the current, I, and the third by multiplying each side of the second by the current. Thus the hypotenuse of the second triangle is IZ or the impressed voltage, and in the third triangle, the horizontal side is I^2R or the power P. Since the cosine of an angle is the ratio of

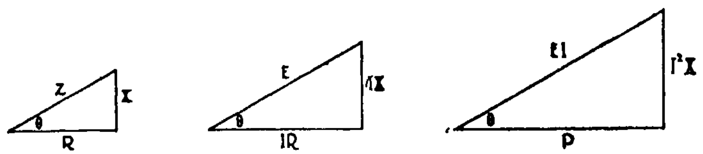


Figure 27-6.

the leg of a right triangle adjacent to the angle to the hypotenuse (see appendix 6) we have from the third triangle of figure 27-6

$$P = EI\cos\theta$$

which is more complicated than our similar direct current relation. $\cos\theta$ is called the *power factor* of the circuit. Its value varies between zero for a pure reactance, either inductive or capacitive, and unity for a pure resistance. That is, no power would be consumed in a pure inductance or a pure capacitance, if there were such things.

27-12. Alternating Current Meters. In the usual direct current ammeter, the side push on a wire that carries a current in a magnetic field is utilized. Since a "permanent magnet" is employed, the deflection is nearly proportional to the first power of the current. If an alternating current is put through this type of meter, the needle merely attempts to oscillate about the zero position with a frequency of n cycles per second. Therefore the permanent magnet is replaced by an electromagnet, and since this reverses at the same frequency with which the current reverses, a deflection of the needle is obtained which is now proportional to the square of the current. But it is possible to calibrate the scale directly in amperes by placing, for example, a 5 where 25 should be, a 4 where 16 should be, and so on. This results in compressing the low end of the scale in comparison with the upper end.

The deflection of the needle is actually proportional to the average (or mean) of the square of the current, and by the device of calibrating the scale as we do, we read directly the square root of the mean squared current. We abbreviate this to root-mean-square current. Similarly root-mean-square voltages are read from voltmeters. In the case of a sinusoid, such as those in figures 27-2 and 27-4, the root-mean-square values are 0.707 of the maximum values.

It is thus immaterial whether we use as vectors in our rotating vector diagrams the maximum values as in figures 27-2 and 27-4 or the root-mean-square values as in figures 27-1 and 27-3, since one is directly proportional to the other. In general the root-mean-square values are the more convenient.

27-13. Parallel Circuits. In this book we shall say very little about parallel alternating current circuits other than to point out that in this case the voltage is the same across the various elements that are in parallel, but the several currents now add vectorially to give the total current. The equations of sections 27-3, 27-4, and 27-5 still hold for the respective portions of the circuit.

If an inductance and a capacitance are in parallel and the frequency is such that $X_L = X_C$ we say we have a case of antiresonance. This is often referred to as a "tank circuit." The combination presents a high impedance at this particular frequency.

27-14. Illustrative Problem. Find the power consumed in the circuit of section 27-9.

Solution: We may do this in either of two ways. Since no power is consumed in either a pure inductance or a pure capacitance, it is only necessary to use the relation $P = I^2R$ for the resistance. Thus

$$P = (7.99)^2(10) = 638 \text{ watts}$$

It is also possible to use the relation $P = EI\cos\theta$. In this case $\cos\theta$ is cos 47.9 degrees which is 0.670. Thus

$$P = (120)(7.99)(0.670)$$

or 642 watts which checks 638 watts to the degree of precision to which we are working.

SUMMARY OF CHAPTER 27

Technical Terms Defined

Root-Mean-Square Value. The square root of the average of the squared instantaneous values of the current or voltage. For a sine wave, it is 0.707 of the maximum value. It is the current (or voltage) read by an alternating current ammeter (or voltmeter).

Inductive Reactance. The ratio of the root-mean-square voltage to the root-mean-square current in a pure inductance measured in ohms. It is equal to the product of 2π by the frequency by the inductance.

Capacitive Reactance. The ratio of the r.m.s. voltage to the r.m.s. current in a pure capacitance. Measured in ohms. It is equal to the reciprocal of the product of 2π by the frequency by the capacitance in farads.

Reactance. Inductive reactance minus capacitive reactance. Measured in ohms.

Impedance. The ratio between the r.m.s. voltage and the r.m.s. current in a circuit containing resistance, inductance, and capacitance. Measured in ohms. Is equal to the square root of resistance squared plus reactance squared.

Phase Angle. Angle between voltage and current on a rotating vector diagram.

Resonance. The condition of an alternating current circuit when the capacitive reactance is equal to the inductive reactance.

Power Factor. The ratio between the power and the product of r.m.s. voltage by r.m.s. current. It is the cosine of the phase angle.

PROBLEMS

27-1. What is the current through a resistance of 10 ohms, an inductance of 7 henries, and a capacitance of 1 microfarad when connected in series on a 115-volt line at 60 cycles? How much is the current out of phase with the voltage?

27-2. If one ampere, 60 cycles is flowing through the circuit of the preceding problem, find the *a.c.* voltage across each part of the circuit as well as the voltage across the entire circuit.

27-3. Solve problem 27-2 for 120 cycles instead of 60 cycles.

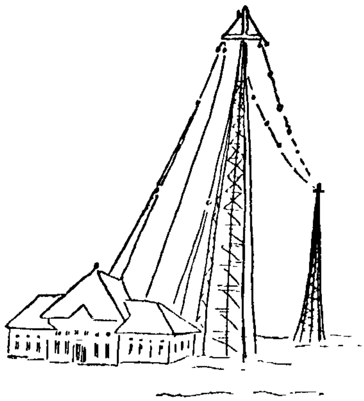
27-4. At what frequency will an inductance of 5 henries and a capacitance of 2 microfarads be in resonance?

27-5. A 110-volt *a.c.* line sends a current of 5.50 amperes through a series circuit the resistance of which is 17 ohms. Compute the impedance of the line, also the power factor.

27-6. A watt meter indicates that the input to a motor is 1,900 watts when connected to a 115-volt line. The ammeter shows that a current of 20 amperes is flowing. What is the power factor, the resistance, and the reactance?

27-7. When a coil is connected with a 120-volt *d.c.* line, 12 amperes flow through the coil. But when it is connected with a 60-cycle line of the same voltage, only two thirds of the original current flows. Calculate (1) the resistance of the coil, (2) its reactance, and (3) the capacitance needed to increase the current to its original value.

CHAPTER 28



Radio; Radar

28-1. Speed of Transmission of a Telephone Message Versus Speed of Sound. Sound travels in air at the speed of about 1,100 feet per second. But a man in Boston can carry on a telephone conversation with a friend in California and perceive no delay due to distance in the replies to his questions. If the telephone line were long enough to go around the world, there would still be a delay of less than a second in the transmission of a message. In the telephone, the sound at the transmitter makes a diaphragm vibrate; these vibrations modify the resistance of an electric circuit; the consequent variations in the direct electric current make an electromagnet vary in strength in the receiver at the other end of the line; this in turn sets up vibrations in a disk, and consequently in the adjacent air, closely similar to those in the transmitter. Thus, when one listens at the telephone, he ordinarily hears the words slightly sooner than someone in the same room with the speaker who is dependent on the speed of sound waves in air.

28-2. Electromagnetic Waves. When the current is turned on in an electromagnet, the magnetic field thus created theoretically extends to an infinite distance, but it is not all created at the same instant. It takes 1/60 of a second to establish the magnetic field

3,100 miles away, and one whole second for the effect to reach a point 186,000 miles from the electromagnet. If the current is continually reversed in the electromagnet, as happens when an alternating current is used, then the magnetic field in the surrounding space is also subject to reversals. In the case of a 60-cycle alternating current, there will be points in space 3,100 miles apart, where the magnetic fields are in the same direction at the same time; thus we can say that electromagnetic waves are created by the alternating electromagnet, with a "wave length" of 3,100 miles (or 5,000 kilometers). If the frequency were 60 kilocycles per second (60,000 cycles per second), the resulting "wave length" would be five kilometers (5,000 meters). The equation is (see section 16-4)

$$n\lambda = V$$

There is very close connection between these alternating electromagnetic fields and light waves, enough to warrant us in including in the same category of "electromagnetic radiation," radio waves, infrared, ordinary, and ultraviolet light, X rays, gamma rays, and secondary cosmic rays which result from charged particles entering our atmosphere at enormous speeds from outer space.

28-3. Four Reasons Why Radio at One Time Seemed Impossible. For a long time, it was supposed that the varying magnetic fields thus produced by oscillating currents would be too feeble to be detected at distances more than a few feet from the source. A second difficulty lay in the fact that, in order to radiate a reasonable fraction of the total power, more rapid oscillations were required than could be produced by mechanical means. The third problem was how to modify these waves, assuming they could be produced, so as to reproduce music and voices. Fourth, the details of a telephone receiver are such that the rapid alternations of the radio wave will produce in it an average effect of zero; the receiver, however, responds to a varying direct current, so that some device is necessary to rectify the alternating impulses. It is a remarkable fact that the solution of all four of these difficulties came with the invention of a single device, the radio tube. The four applications of it just suggested will be discussed separately under the headings *amplification*, *oscillation*, *modulation*, and *rectification.*

28-4. Amplification By Means of the Radio Tube. The radio tube (see schematic representation in figure 28-1) consists essentially of an evacuated glass tube with wires sealed in, connected to a *filament*, a *grid*, and a *plate* within the tube. The terminals of the filament are connected to a low voltage from a battery or transformer which causes a current called the *filament current* to heat the filament so that it glows. When a metallic wire is heated, free electrons are evaporated out of the wire and hover around in the space just outside of the wire. If now, a large positive voltage is applied to the plate, it will attract the negative electrons, resulting in a flow of electrons from the filament to the plate, called the *plate current.* The plate current may be controlled in three ways: (1) by changing the filament current, (2) by changing the positive voltage on the plate, and (3) by varying the voltage on the grid, which lies between the filament and the plate. If the grid is made negative, it repels the electrons which are trying to pass from the filament to the plate, and thus decreases the plate current. If the grid is made positive, it increases the flow. A slight change in the grid voltage has the same effect as a very large change in the plate voltage. If a fairly large resistance is inserted into the plate circuit, the IR drop across it is a reasonably large fraction of the plate voltage. If several tubes are used in such a way that the IR drop of each plate resistor is applied to the grid of the next tube, an extremely small variation in voltage in the first grid produces a large effect on the plate voltage of the last tube. We say in this case that we have employed several stages of amplification. Thus, an extremely small impulse at the microphone may be amplified to several hundred kilowatts at the antennae; also at the receiving end, a signal broadcast thousands of miles away may be picked up and amplified enough to be heard for several blocks.

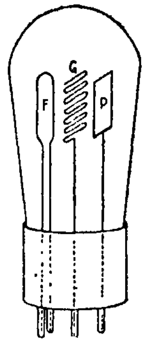


Figure 28-1.

28-5. Oscillation Produced By the Radio Tube. A high frequency oscillatory current may be created by means of batteries, a radio tube, a condenser, and two inductances. In figure 28-2, the A-battery serves to heat the filament. The filament, grid, and plate are all inclosed in one glass tube as in figure 28-1. The B-battery through L_P charges the plate positively. As the plate current builds up (in spite of the negative charges on the grid furnished by the C-battery) the increasing magnetic field in L_P induces a voltage in L_G which makes the grid increasingly negative. As stated in the preceding section, this has the effect of decreasing the plate current. As the plate current shrinks, the decreasing magnetic field in L_P induces a voltage in L_G which this time makes the grid less negative, which in turn increases the plate current. The cycle now repeats itself again and again, and the oscillations continue as long as the circuit is kept closed and the batteries hold out. The frequency, n, of the oscillations can easily be controlled since it depends on the capacitance, C, of the condenser and the inductance L_P. If n is in cycles per second, L_P in henries, and C in farads, the equation (see section 27-10) is

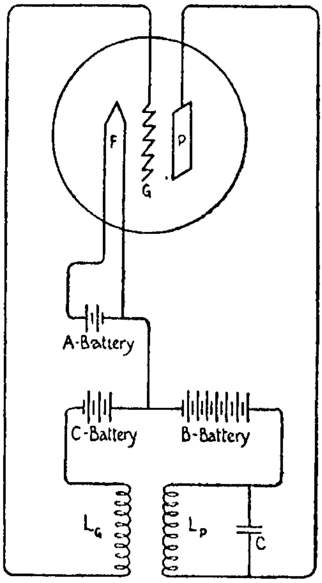


Figure 28-2.

$$n = \frac{1}{2\pi\sqrt{L_P C}}$$

Frequencies from one per second to 60,000,000 per second may be obtained in this way. Much higher frequencies may be obtained efficiently by means of the modern "cavity magnetron."

28-6. Illustrative Problems. (1) Find the frequency at which a condenser consisting of a glass plate 0.3 cm. thick with sheets of aluminum foil on each side of the plate, of area 16 square centimeters, and an inductance made up of a coil of 1,000 turns of wire on a core of permeability 2,000, cross-sectional area of 5 cm.^{2}, and length 20 cm., will oscillate when connected into a circuit with a suitable means of excitation.

Solution: Using the equation of section 23-6, and assuming the dielectric constant, ϵ, of glass to be 8, we have $A = 0.0016m^2$, $k_e = 9 \times 10^9$, $d = 0.003$ meters, and

$$C = \frac{(8)(0.0016)}{4(3.14)(9)(10^9)(0.003)} = 3.77 \times 10^{-11} \text{ farads}$$

Using the last equation of section 26-15, we have $n = 1{,}000$, $A = 0.0005m^2$ $= 2{,}000$, $k_m = 10^7$, $l = 0.20$ meters, and

$$L = \frac{4(3.14)(1{,}000)^2(0.0005)(2{,}000)}{(10^7)(0.20)} = 6.28 \text{ henries}$$

Now using the equation of section 27-10, with L and C as just computed,

$$n = \frac{1}{2\pi\sqrt{(6.28)(3.77)(10^{-11})}} = 10{,}350 \text{ oscillations per second}$$

This would ordinarily be expressed as 10.35 kilocycles.

(2) If a high rate of oscillation is desired, both the inductance and the capacitance must be as small as possible. Assuming that the inductance, L_P, in figure 28-2 is two microhenries, find the value of the capacitance, C, so that the frequency will be 60 megacycles.

Solution: In the equation of section 28-5, n = 60,000,000 and L_P = 0.000002 henries. Thus we have

$$60{,}000{,}000 = \frac{1}{2\pi\sqrt{(0.000002)C}}$$

Solving, we obtain $C = 3.52 \times 10^{-12}$ farads. This would commonly be expressed as 3.52 micromicrofarads.

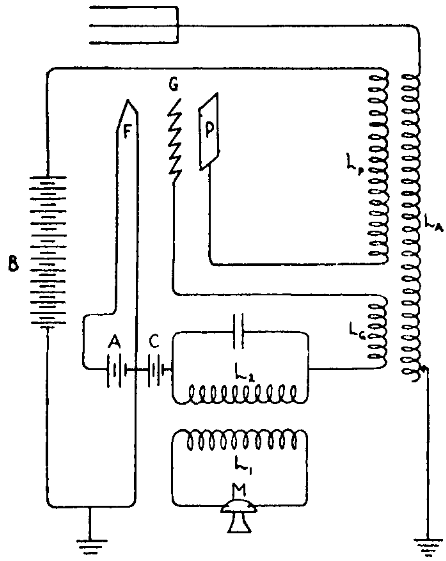


Figure 28-3.

28-7. Modulation Produced by the Radio Tube.

Figure 28-3 illustrates one method of arranging a circuit so that the sound vibrations at the microphone, M, can be made to modify the amplitude of the radio wave. First consider the original radio wave shown in figure 28-4, assuming silence at M. After a steady state is established, the situation is as follows: between the antenna and the ground there is a capacitance, C_A, (not represented in figure 28-3) which, together with the antenna inductance, L_A, determines the frequency of the tube oscillations and consequently the frequency of the emitted "carrier wave."

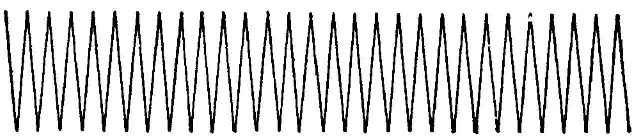


Figure 28-4.

An induced voltage in L_A is caused by the transformer effect from L_P and in turn induces a voltage in L_G, which by means of the grid controls the plate current in the tube and thus maintains the oscillations in L_P as depicted in figure 28-4, all at the expense of the B-battery. If, now, a sound wave of the form shown in figure 28-5 is created at the microphone, fluctuations will be introduced in the resistance of the microphone circuit, and the microphone current will vary. The transformer action from L_1 to L_2 will

Figure 28-5.

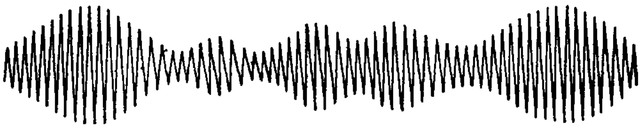


Figure 28-6.

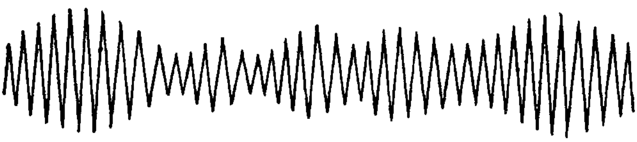


Figure 28-7.

superimpose on the grid current the form of the sound wave, and the result of the modification, or "modulation" as it is called, will be the wave shown in figure 28-6, which represents the final shape of the radio wave broadcast at the antenna. If the same sound wave were sent out by another station with a greater wave length, it would appear as in figure 28-7.

28-8. Rectification Produced by the Radio Tube. We have described the method of producing oscillations of sufficiently high frequency to broadcast efficiently; the method of controlling (modulating) the amplitude of these oscillations so as to represent music and speech; and the process of amplifying the signals, both at the sending and at the receiving end. We know that electromagnetic radiation represents a varying magnetic field and will therefore produce an alternating voltage in any conductor it encounters, such as the receiving antenna.

It now remains to show how the alternating currents due to these alternating voltages may be rectified so as to be detected by a telephone receiver, an instrument which responds not to alternating but to a fluctuating direct current. Figure 28-8 shows a simple receiving circuit, with the tube used now as a detector, or *rectifier*. It is to be understood that the plate, grid, and filament are all contained in the tube as in the previous figures. We have seen that heating the filament will evaporate out electrons; the plate, however, is not heated and hence cannot be made to serve as a source of electrons. Hence, if an effort is made to reverse the direction of the current, after sweeping all the electrons in the tube back into the filament, the action ceases for lack of electrons, that is, the current becomes zero. We thus have in the radio tube a device which will permit a current to pass in one direction but not in the other. Such an apparatus is said to "rectify" the alternating current. If, therefore, the telephone receiver is placed in the plate circuit, we shall obtain a reproduction of the original sound waves originating at the microphone of the sending apparatus. It will be noticed that in figure 28-8 there is an A-battery to heat the filament, a B-battery to make the plate positive, but no C-battery to give the "negative bias" to the grid necessary to control the plate current as described in section 28-4. The operation of the circuit in figure 28-8 may be described as follows: an alternating induced electromotive force is set up in the antenna with the frequency of the broadcasting station. By varying the capacitance of the condenser, represented with the arrow, the circuit may be "tuned" to correspond to the frequency of the broadcasting station, since the relations involved are those of the equation in section 28-5. Until this is done, the circuit will not oscillate at the required frequency.

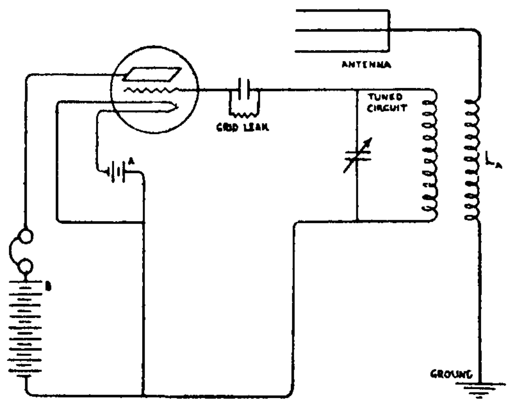
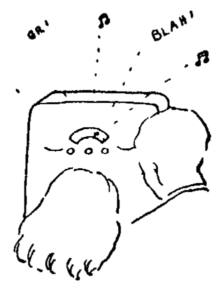


Figure 28-8.

If the connection described in figure 28-8 as a "grid leak" were omitted, the grid would be connected with the rest of the apparatus only through the grid condenser, and would be described as "floating." Under these conditions, electrons that happen to land on the grid on their way from the filament to the plate would have no way of escaping. Thus the grid would accumulate enough of a negative bias to cut off entirely the flow of electrons through the tube. The grid leak represents a resistance just large enough, say 3,000,000 ohms, so that sufficient of these electrons have a chance to escape through it from the grid to maintain the correct bias for proper operation of the circuit. The fluctuations in the voltage of the grid due to the incoming radiation control the fluctuations in the plate current which are detected in the telephone receiver, or which are amplified so as to operate a loud speaker.

28-9. Alternating Current Radio Sets. It has been simpler throughout this discussion to speak of *A*-, *B*-, and *C*-batteries, which indeed are used in sets where no electrical power is available. However, it is possible now to use 110-volt electric power as the source of all the voltage required. If this is direct current, suitable resistances will make the correct voltages available at the proper points. Radio sets are even designed to utilize alternating current power. Alternating current will heat the filament as well as direct current. In the portions of the circuit where we desire direct current, it may be obtained from alternating current, since radio tubes may be used as rectifiers. Thus transformers almost completely replace batteries in the modern electric radio.



28-10. Electronics. During recent years, there has grown up an enormous industry—now comparable in size with the automobile industry—which is based on multifarious uses of radio tubes. This new science is called *electronics;* there is at present no apparent limit to its future growth and expansion. Effects which have been known for many years, but which seemed too feeble for practical use, may now be utilized freely through the magic of the amplifying properties of radio tubes. Thus, light-sensitive photoelectric cells may be set to work in talking motion pictures, automatic door openers, burglar alarms, controls for automatic machinery, television, and thousands of other appliances. "Geiger counters" may be made so sensitive and equipped with so many stages of amplification that when a single electron enters an ionization chamber, a click of any degree of loudness may be produced.

28-11. Radar. Thousands of man-years during World War II went into the development of a system for *ra*dio *d*irection finding *a*nd *r*anging now expressed by the coined word *radar.* The principle dates back to the bat, which while flying emits a series of squeaks both high pitched and supersonic (above the audible range) and becomes aware of his surroundings by the way these sounds are reflected back to him. Figure 28-9 is an example of the block-diagram in electronics and serves to show the fundamental principles of radar. Each rectangle represents a complicated set of electrical connections. We begin with the modulator, the function of which

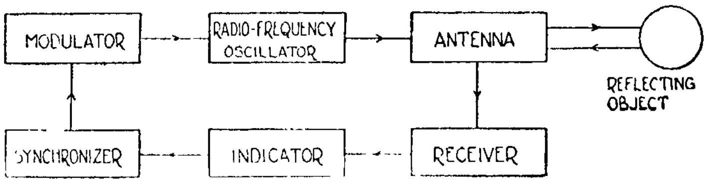


Figure 28-9.

is to turn on the oscillator for about a microsecond (a millionth of a second), turn it off abruptly, and wait a millisecond (a thousandth of a second) or so until time to turn it on again. The job of the radio-frequency oscillator is to deliver at the rate of millions of watts electromagnetic radiation of about 3,000 megacycles (wave length 0.1 meter) to the antenna. Prior to the development of the so-called "cavity magnetron," obtaining this amount of power at this frequency was a sheer impossibility. The antenna must serve for both sending and receiving purposes. It must be highly directional as well as capable of more or less rotation in order to "scan" the reflecting object. Its physical dimensions must be large in relation to the wave lengths utilized.

After the radiation has been reflected back from the object of interest, the task is to receive the impulse and, by the superheterodyne principle of generating a local oscillation which beats with the incoming signal, prepare an intermediate frequency (15 megacycles or so) signal for the indicator. It is important that the delicate receiver be turned off while the violent bursts of energy are being emitted by the oscillator, then turned on instantly afterward to receive the echo from an object sometimes only a few yards away. The indicator is usually a cathode-ray oscilloscope which contains a fluorescent screen. A line, which is straight up to the point where the echo appears, is thrown on this screen. The position of the kink in this straight line is a measure of the time consumed in the to-and-fro trip of the radio signal. Another device called the *plan position indicator* literally draws a map on a fluorescent screen of the region being scanned.

28-12. Radar in War. Since radar impulses penetrate darkness, fog, smoke, and rain, it may easily be seen how useful it was in both defense and offense during World War II. Combined with a device for distinguishing friend from foe, it enabled England to survive the German air attacks; it won the war against the submarines in the Atlantic; it enabled the Pacific fleet to sink Japanese warships miles away at night; it vastly improved the performance of antiaircraft guns; and it was an invaluable adjunct to the bomber. The "combat information center," an enclosed room on the ship in which the radar indicators were located, was a much more likely place to find the commodore of the destroyer force than up on the bridge with the captain.

28-13. Radar in Peace. Radar will likewise be useful in both air and sea navigation and piloting in times of peace. Just as a plane may be spotted from ground by radar, so ground or mountain peak or skyscraper may be detected by radar from a plane. Indirectly, too, radar will have a great influence on peacetime electronic industry by reason of the vast amount of research that it has already stimulated, and the large number of technicians trained in the field of electronics during the war. Even in the field of purely theoretical research, uses for radar will be found. When it was announced, early in 1946, that the moon had been picked up by radar, it immediately became obvious that the Michelson-Morley experiment of 1887, which ultimately resulted in the theory of relativity, could now be repeated on a colossal scale, to say nothing of the possibility of checking the dimensions of one of our convenient astronomical yardsticks, the distance from earth to moon. It was even of interest to know that a radar beam could penetrate those layers of ions in the earth's upper atmosphere, called collectively the *ionosphere*, from which low frequency radio waves are reflected back to earth. This meant that from this point on, it was physically possible to direct a V-2 rocket to the moon by radio control. Possibly, some day we shall have a photograph of that farther side of the moon which man has never yet seen!

SUMMARY OF CHAPTER 28

Technical Terms Defined

Electromagnetic Radiation. A term including radio, infrared, ordinary, and ultraviolet light, X rays, gamma rays, and secondary cosmic rays, all of which travel through space with the speed of 300,000,000 meters per second.

Electronic Tube. A tube made of glass or metal, either exhausted or filled with a gas at low pressure containing two or more electrodes. Diodes, triodes, tetrodes, pentodes, and so on.

Amplification. A function of electronic tubes with three or more electrodes which has the effect of permitting a very feeble voltage to direct the flow of a comparatively large amount of energy.

Oscillation. A function of circuit containing one or more electronic tubes, condensers, and inductances which results in the production of a wide range of electric oscillations without involving any moving parts.

Modulation. The superposition of electrical oscillations of audio frequencies upon carrier waves of radio frequencies.

Rectification. Conversion of an alternating electrical voltage or current into one that is pulsating but unidirectional. This is accomplished by an electrical valve which permits the flow of electrons in one direction but not the other.

Electronics. The study of the behavior of electrons under various conditions either in a vacuum or in a gas at low pressure.

Radar. A device for finding the direction and the distance of an object by reflecting high frequency radiation from it.

PROBLEMS

28-1. What is the wave length corresponding to a frequency of 550 kilocycles?

28-2. What is the frequency of a short-wave radiation with a wave length (1) of two meters; (2) of one centimeter?

28-3. Two flat metal discs, 20 centimeters in diameter and 0.5 centimeter apart, have a capacitance of 0.0000555 microfarad. Find the dielectric constant of the intervening medium.

28-4. Find the capacitance of an air condenser, the plates of which have an area of one square centimeter each and are separated by a distance of one centimeter.

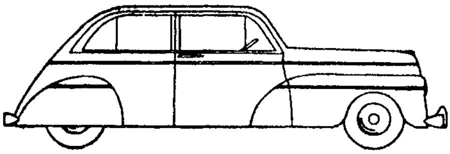
28-5. It is desired to design a step down transformer which will take the place of the *A*-battery in an *a.c.* radio set. Find two integers that will represent the number of windings in the primary and secondary of a transformer which will step the voltage down from 110 volts to 4.89 volts.

28-6. If, in the previous problem, the inductance of the primary is two henries, find the inductance of the secondary.

28-7. In order to tune to a frequency of 550 kilocycles, assuming the inductance to be fixed at 0.0485 millihenry, at what capacitance must the dial be set?

28-8. If the oscillator of a radar delivers a million watts for a microsecond, then rests for a millisecond, what is its average output?

CHAPTER 29



Photometry; Reflection and Refraction of Light

29-1. Brief History of the Theory of Light. Christian Huygens (1629-1695) and Isaac Newton (1642-1727) were contemporaries and both were familiar with the same experimental data, yet Huygens argued strenuously for a wave theory of light and Newton for a projectile theory. Newton won at the time, and so great was his prestige that for a hundred years after his death, physicists still held to his projectile theory of light. Largely through the efforts of Fresnel (1788-1827), a brilliant young French physicist, the wave theory was again enthroned and held undisputed sway until 1887. In the meantime, Maxwell (1831-1879) had shown that electromagnetic waves were of the same nature as light, but of longer wave length. In 1887, the photoelectric effect was discovered by Hertz (1857-1894), a physical fact which completely contradicted the wave theory. In accordance with the photoelectric effect, feeble light from a distant star will knock out what we now know to be electrons from a sheet of zinc. With the same speed and wave length, the most powerful electric arc available will expel more electrons, but each electron will have the same kinetic energy as those produced by the starlight. A discovery of Planck's, in 1900, was also incompatible with the wave theory, yet for a period, physicists used first one theory and then the other, whichever was convenient for the purpose at hand.

29-2. The "Wave Mechanics" Theory of the Nature of Light. Light is one more form of energy. It does not represent continuous energy, but is made up of discrete portions called photons. This was Planck's contribution. Each photon is created as such by being flung off from an electron or other atomic constituent while the latter is revising its status in the atom. We describe the process by saying that the electron has dropped from a certain energy level in the atom to a lower energy level, and that the photon represents the energy lost by the electron. The process is also reversible. But progress in explaining light and atomic structure seemed to be blocked until a theory was adopted in 1925 to the effect that the behavior of both photons and electrons is indeterminate and subject only to probability laws. This indeterminacy therefore appears to be a basic assumption and is justified because the conclusions derived from the theory check marvelously with experimental results. The probability functions which describe the behavior of ultimate particles are factored into so-called *psi-functions* (ψ-functions), and due to the fact that there is a strong resemblance between these mathematical functions and the equations that represent physical waves, the theory is called *wave mechanics*, and we shall call the functions *psi-waves.*

The only phase of the theory which interests us in this chapter is the point of view that is now being adopted in regard to radiation, and more particularly to that special type of radiation known as *visible light.* It is convenient to speak of psi-waves as exerting a certain type of control over photons. For example, if we find that the probability that a photon is at a certain point is zero, it amounts to saying that we shall never find a photon at that point. In field-free space, if such a place could exist anywhere in the universe, a psi-function would be shaped like a transverse wave, and would move through space with what we are used to calling "the speed of light"; in fact, it will do very nearly this in space that is not quite free of fields. By fields we mean not only magnetic fields, but electric and gravitational as well. Psi-waves are found to obey a certain definite group of mathematical laws. For example, they may be added together, so that it is proper to speak of the "resultant" psi-wave. If the medium is not a perfect vacuum, the velocities for different wave lengths are different, so that the resultant psi-waves will travel at an apparent rate which is called the *group velocity.* The most likely thing for a photon, once under the arch of the resultant psi-wave, is to stay there and be carried along with the group velocity of the waves. Thus the velocity of the photon may be very different from the velocity of any of the component psi-waves.

Hence, from one point of view, we might as well continue to think of light as a wave motion, since the position of the photon comes so near to being described by these wave functions. But the waves themselves are mathematical and not physical; they carry no energy. For example, probability is a mathematical and not a physical quantity. As an alternative, in some quarters, the possibility is toyed with that psi-waves exist in a multidimensional space, such that our three-dimensional space is a cross section of the whole. But even in this case we should be obliged to classify the waves as mathematical and not physical. The wave patterns may exist without any photons, but in this case we have no light, no radiation. The modern theory of light is therefore a curious combination of projectile and wave theory.

29-3. Meaning of "Frequency" and "Wave Length" in Photon Theory. Although physical light has now become an assemblage of photons, it is still possible to find an almost physical meaning for the terms "frequency" and "wave length." The "frequency" is now regarded as the energy of the photon divided by a number, known as Planck's constant, represented by the letter h, and equal numerically to 6.60×10^{-34} joule-seconds. That is

$$\text{the energy of a photon} = hn$$

Having determined the "frequency" by this method, we may also think of it as the actual frequency of the psi-wave. Moreover, the photon is thought of as having momentum; when the photon strikes an object, it exerts a pressure on it. The "wave length," on the basis of the new theory, is the quotient of h divided by the momentum of the photon. This may also be thought of as the actual wave length of the psi-wave. The product of the "frequency" by the "wave length" is therefore equal to the energy of the photon divided by its momentum, and also equal to the velocity of the psi-wave. We shall continue therefore to use these two terms in the remainder of the book, but to remind the student of their somewhat artificial meaning in modern theory we shall inclose them in quotation marks.

29-4. Speed of Light. The speed of light in a vacuum, although great (186,000 miles per second or 300,000,000 meters per second, usually written 3×10^8 m./sec.), has been measured by several distinctly different methods with results that check very closely. Photons of all energies, as well as their psi-waves, travel with the same speed in a vacuum, but in other transparent substances, as has just been mentioned, the speed of the psi-waves is less than the figure given above, and not only different in different substances but, in a given transparent substance, different for different "wave lengths." The statements we are now making about the psi-waves are the same as were formerly made about light itself.

29-5. Electromagnetic Radiation. Not all light is visible. For this reason, the larger term, *electromagnetic radiation*, will be used to include both visible and invisible photons together with their psi-waves. The longest electromagnetic "waves" are those used in radio broadcasting; their "wave lengths" are such as to be expressed conveniently in meters. Their speed is that mentioned in the previous paragraph. The shortest "waves" which can be detected by electrical methods have a "wave length" of about 0.001

meter (one millimeter). At this "wave length," the same photons can also be detected by sensitive thermometric devices such as radiometers, radiomicrometers, or thermopiles which will detect a rise in temperature of as little as a millionth of a degree Fahrenheit. In fact, the tremendous bursts of short-wave radiation used in radar may be felt distinctly as heat by intercepting the rays with the hand. From a "wave length" of a millimeter down to that of 0.00079 millimeter (0.79 micron) the radiation is called *infrared light.* In this region, photography becomes possible. At 0.79 micron, the photons just begin to become visible to us as red light. As the "wave lengths" decrease still more, the colors change to orange, yellow, green, blue, and violet; the extreme edge of the violet represents about 0.39 micron (3,900 Ångströms) in "wave length." Throughout the visible region, the radiation will still slightly raise the temperature of whatever object it falls upon (conversion of light energy into heat energy) so that the thermopile may be used; moreover, photography is still an available experimental tool.

Below 3,900 Ångströms, light again becomes invisible, but can be photographed more readily than ever, and can still be detected by the thermopile. In this region, it is called *ultraviolet light.* A third method of detecting the existence of ultraviolet light is to make use of the fact that when it passes through air, the air becomes ionized, that is, the air becomes temporarily a conductor of electricity. All "wave lengths" shorter than those we have just described can be detected by both the photographic and the ionization method. When the "wave lengths" become as short as about 50 Ångströms, the same radiation is referred to as short *ultraviolet* "waves" or long X rays; it is also customary to refer to long X rays as "soft" X-radiation, and as the "wave lengths" become shorter, we speak of them as becoming "harder." The shortest X rays (or Roentgen rays) have a "wave length" of about 0.05 Ångströms (50 X-units); below that "wave length" we have gamma rays (50 X-units to 5 X-units), which are given off by radioactive materials. Still shorter "wave lengths" or higher "frequencies" are to be found in the secondary cosmic radiation which is present in the earth's upper atmosphere and to a somewhat lesser extent at sea level.

There are no regions missing in this entire range from the radio wave to the secondary cosmic ray; all are alike in nature and all travel with the same speed in empty space.

Since $$\text{“wave length”} = \frac{3 \times 10^8 \text{ m./sec.}}{\text{“frequency”}}$$

and since $$\text{“frequency”} = \frac{\text{energy of photon}}{6.60 \times 10^{-34} \text{ joule-sec.}}$$

it follows that $$\text{“wave length”} = \frac{(3 \times 10^8)(6.60 \times 10^{-34})}{\text{energy of photon}}$$

$$\text{“wave length”} = \frac{1.98 \times 10^{-25} \text{ meter-joule}}{\text{energy of photon}}$$

It is therefore possible to translate either "frequency" or "wave length" directly into the energy of the accompanying photon. Since we can make this transition at any time, we shall continue to use the language of the wave theory and therefore avail ourselves of all of its advantages. The only limitation is that we cannot compute the energy of light as if it were a wave; the energy is expressible only in terms of photons.

29-6. Units of Length. It may be of interest to list in one place the enormous range of units used by the physicist.

1,000 X-units	= 1 Ångström
10,000 Ångströms	= 1 micron
1,000 microns	= 1 millimeter
1,000 millimeters	= 1 meter
1,000 meters	= 1 kilometer
300,000 kilometers	= 1 light-second
31,560,000 light-seconds	= 1 light-year
3.258 light-years	= 1 parsec
1,000,000 parsecs	= 1 megaparsec
3.08×10^{35} X-units	= 1 megaparsec.

The number of light-seconds in a light-year is obtained by multiplying the number of days in a year by the number of seconds in a day. The light-year is the distance light will travel in a year. The present estimate of the radius of the universe is about 400 megaparsecs.

29-7. Photometry. As in the case of sound, it is also true of light that the intensity is inversely proportional to the square of the distance from the source. See section 17-5. This fact furnishes the basis of *photometry*, which is the determination of the relative strengths of light sources. The amount of light falling on a surface of area A square feet, coming from a source of C candlepower, d feet from A, the dimensions of both C and A being small compared with d, and the direction of d making an angle of i with the normal to A (see figure 29-1), is given by the following equation

$$L = \frac{CA}{d^2} \cos i$$

See appendix 6 for a discussion of the *cosine*. The amount of light, L, in this equation comes out in *lumens*, which may be considered as defined by the equation. The angle i is called the *angle of incidence*; it will be noticed that when the light falls directly upon the surface so that this angle is zero, $\cos i$ will be unity. A could have been expressed in square meters, in which case d would be in meters, the other units remaining the same.

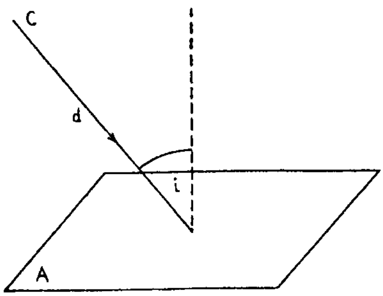


Figure 29-1.

The definition of the *lumen* just referred to, when put into words, is as follows: the lumen is the amount of light which, coming from a one-candlepower source of small dimensions, will fall upon unit area, unit distance from the source, with zero angle of incidence. If the area is curved so that i is zero at every point, the area need not be small.

Illumination is the amount of light falling upon unit area, and is measured in lumens per square foot, or lumens per square meter. If we let I stand for illumination, then $I = L/A$; therefore

$$I = \frac{C}{d^2} \cos i$$

and we shall find that in many cases the conditions are so arranged that the light falls squarely upon the illuminated surface, so that i is zero and $\cos i$ is one. Unfortunately, it is also customary to use the illogical units *foot-candle* and *meter-candle* as units of illumination. The foot-candle is the same as the lumen per square foot, and the meter-candle the same as lumens per square meter. Throughout physics, we have been able to predict a new unit by noting the algebraic processes that led to it. Thus, feet divided by seconds give feet/second and feet multiplied by pounds give foot-pounds. But here candles divided by feet squared give unexpectedly *foot-candles!* But fortunately there are practically no other cases of this type.

In photometry, it is often the candlepower of the source which it is desired to measure. Suppose, for example, that we have two electric light bulbs, the candlepower of one known, and of the other unknown. The procedure would be to mount the two bulbs at opposite ends of a three-meter bench and to place a screen at such a point between them as to receive equal illumination from each bulb. Various devices are employed for determining when the two illuminations are equal, the simplest being the so-called *grease-spot photometer*, the essential feature of which is a sheet of paper containing a translucent grease-spot. If the illumination on the rear of the screen is greater than that on the front, then the grease-spot appears brighter than its surroundings, and if less, vice versa. Two mirrors so placed as to enable the observer to see both sides of the screen at once help in judging the equality of the illumination. Since i is zero in each case, one illumination, I, is C/d^2 and the other, I', is C'/d'^2, and when $I = I'$

$$\frac{C}{d^2} = \frac{C'}{d'^2}$$

29-8. Illustrative Problems. (1) A sheet of paper, 8.5 by 11 inches, lies on a table which is illuminated by a 100-candlepower lamp on the ceiling, six feet above the table top and eight feet to one side. How many lumens does the sheet receive, and what is its illumination in foot-candles? From figure 29-2 we see that $\cos i$ is 0.6. We have therefore $C = 100$,

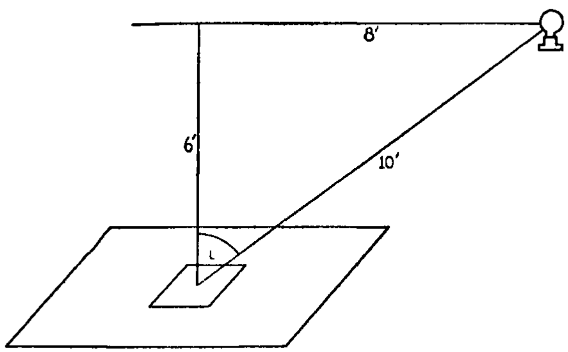


Figure 29-2.

$A = 0.65$ square foot, and $d = 10$ feet. Thus $L = (100)(0.65)(0.6)/100$ which gives us $L = 0.390$ lumen. The illumination is 0.390 lumen/0.65 foot² or 0.6 lumen per square foot which is the same as 0.6 foot-candle. The illumination may also be obtained directly from the original data from the formula $C \cos i/d^2$; that is, $I = (100)(0.6)/100$; or 0.6 foot-candle. This would be considered very poor illumination; about five foot-candles is considered a minimum for ordinary reading.

(2) A certain lamp has been certified by the Bureau of Standards as giving 25 candles. When this lamp is placed at one end of a three-meter optical bench and an electric lamp of unknown candlepower at the other end, it is found that the photometer must be placed one meter from the standard lamp to make the illumination equal. Find the unknown candlepower. One illumination is $25/1^2$ and the other is $C/2^2$. Setting these equal, we have $25 = C/4$, or $C = 100$ candles.

29-9. Reflection of Light. If light strikes an optically smooth surface, the reflected light follows the law enunciated in section 16-5 (see also figure 16-6). In figure 29-3, the angle of incidence is i and the angle of reflection is r; the law just referred to simply equates i to r. On a surface not optically smooth, such as a sheet of paper, the light would be reflected in all directions; this we call diffuse reflection. It is this property of diffuse reflection that enables us to see the objects about us. If we could make a perfect mirror that would reflect 100 per cent of the light incident upon it, we should be able to see the objects reflected in it, but we should be unable to see the mirror itself. A chemically deposited silver surface will reflect about 92 per cent of the incident light.

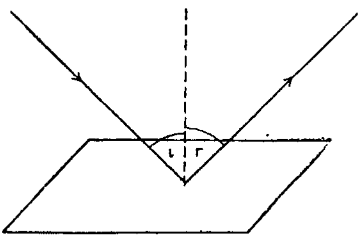


Figure 29-3.

29-10. Images. The eye is accustomed to assume that the light entering it has followed a straight line from its starting point. For example, in figure 29-4, the eye assumes that the light entering it comes from the point I rather than the point O. It sees an image just as far back of the mirror as the object, O, is in front of the mirror. As a matter of fact, however, there is nothing behind the mirror which the naked eye can see from this point, so that we refer to this type of image as a *virtual image* rather than a real image. On the other hand, if we should focus the rays of the sun by means of a concave mirror, such as the parabolic mirror in figure 29-5, upon the head of a match, and thus start it blazing, we should be entitled to say that the concave mirror

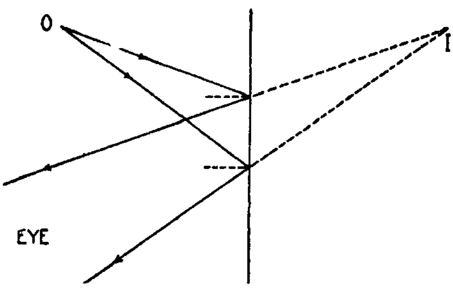


Figure 29-4.

had formed a *real image* at the head of the match (point F in figure 29-5). We can make the general statement that virtual images are always behind a mirror, and real images always in front. We shall see that the reverse is true of lenses.

29-11. Curved Mirrors. The only type of curved mirror of practical importance, not counting freak mirrors at amusement parks, is parabolic. A parabolic mirror (figure 29-5) has the property of reflecting a group of parallel rays through a single point called the *focus.* The largest telescopes are constructed on this principle. Also, if a source of light is placed at the point F, the rays, after striking the mirror, will be reflected along parallel lines. This is the principle of the searchlight.

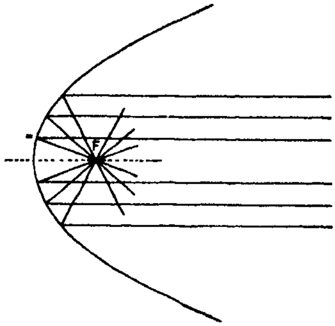


Figure 29-5.

29-12. Refraction of Light. Refraction, while not very important in sound, has many applications in the case of light. In section 16-6, the index of refraction is defined as the ratio of the wave velocities in two mediums. Referring to figure 29-6, when i is the angle of incidence and r is the angle of refraction, the index of refraction being denoted by μ, we have

$$\mu = V/V'' = \sin i/\sin r$$

This assumes that V is the velocity of light in the medium where the angle is i and V'' in the other medium. With reference to light, the index of refraction of a substance is the ratio of the velocity of light in a vacuum to the velocity of light in that substance. Since measurements of the velocity of light are somewhat inconvenient, it is very fortunate that we also have the relationship between the sines of the angles as well. In figure 29-6, the ray CE may be thought of as traveling in a vacuum, and the ray EH in some other medium, such as glass. The reflected ray is omitted in figure 29-6 for the sake of simplicity. The velocity of light in air is nearly (99.7 per cent) as great as in a vacuum, so that for our purposes

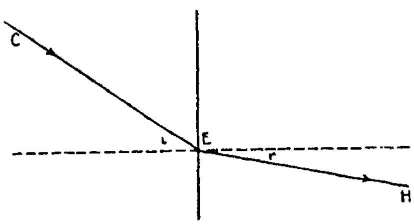


Figure 29-6.

(slide-rule accuracy) we need not consider our substances surrounded by a vacuum. If a ray of light passes through a plate of glass with parallel surfaces (figure 29-7), it will come out parallel to

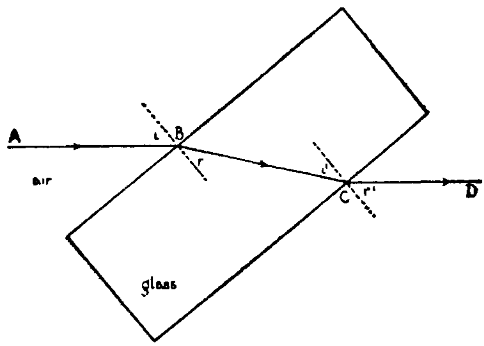


Figure 29-7.

the original ray, but displaced laterally. If, however, as in figure 29-8, the light is made to go through a glass prism, it will not come out parallel to the incident direction. If the eye is placed at the point H in figure 29-8, and looks in the direction of G, it will see objects at the points F and E, but F and E will appear to be along an extension of the line HG. This fact is the basis for the construction of lenses, one of the important applications of the study of light.

29-13. Illustrative Problem. Consider the angles of the triangular prism in figure 29-8 all to be 60 degrees, and the angle i 45 degrees. If the index of refraction of the glass is 1.414, find angles r, r', and i'.

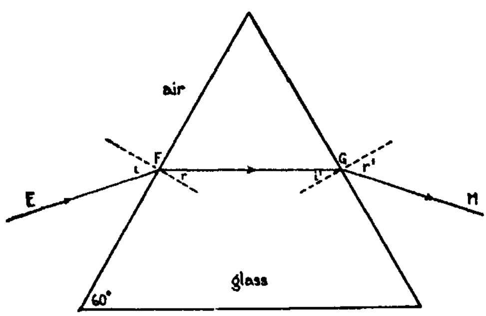


Figure 29-8.

Using the equation $\mu = \sin i/\sin r$, where $\mu = 1.414$, and $i = 45$ degrees, we have $1.414 = 0.707/\sin r$. Solving, we have $\sin r = 0.500$ and $r = 30$ degrees by appendix 7. If $r = 30$ degrees, then the ray makes an angle of 60 degrees with the side of the prism and is therefore parallel with the base of the prism. It immediately follows that i' is also 30 degrees. To find r', we substitute into the equation $\mu = \sin r'/\sin i'$. This gives us $1.414 = \sin r'/0.500$ from which we obtain $\sin r' = 0.707$, and therefore $r' = 45$ degrees.

SUMMARY OF CHAPTER 29

Technical Terms Defined

Light "Waves." Light "waves" obey all the properties of waves in general with the exception of those having to do with energy. For example, the energy of a sound wave or a water wave is proportional to the square of the amplitude, whereas the energy of light is instead proportional to the first power of the frequency. Removing the energy from a light wave virtually removes it from physics and puts it into the field of mathematics.

Photons. Light as a physical entity is now considered to consist of discrete particles called photons. The energy of a photon of a given color is proportional to the "frequency" of the accompanying "wave." In free space photons travel with a speed of 300,000,000 meters per second.

Electromagnetic Radiation. Electromagnetic radiation is a term rather more general than light. It includes radio, infrared, visible, and ultraviolet light, as well as X rays, gamma rays, and secondary cosmic radiation. The photons of these types of electromagnetic radiation increase progressively in energy from radio to secondary cosmic radiation.

Photometry. The measurement of intensity of light sources.

Lumen. The amount of light which, coming from a one-candlepower source of small dimensions, will fall upon a unit area curved so that the angle of incidence is everywhere zero, unit distance from the source. For example, if a one-candlepower source were placed at the center of a hollow sphere, 4π lumens would be delivered to the inner surface of the sphere.

Illumination. The number of lumens falling on unit area of a surface.

Foot-Candle. One foot-candle is the same as an illumination of one lumen per square foot.

Meter-Candle. One meter-candle is the same as an illumination of one lumen per square meter.

Virtual Image. A point from which light appears to diverge after suffering reflection or refraction.

Real Image. A point toward which light converges after suffering reflection or refraction.

Index of Refraction of Light. The ratio between the velocity of light in vacuo and that in some other medium.

Law of Refraction. The index of refraction is the ratio between the sines of the angles of incidence and refraction.

PROBLEMS

29-1. How long does it take light to make the trip from the sun to the earth, 92,000,000 miles? How long does it take for a radar beam to travel from the earth to the moon, 240,000 miles? How long does it take a radio wave to go around the earth once?

29-2. If the dimensions in figure 29-2 are changed from 6, 8, and 10 feet to 12, 5, and 13 feet respectively, find how strong a lamp will be needed to produce 5 foot-candles at the given point on the table.

29-3. Two electric lamps give 25 and 40 candlepower respectively. If they are placed at opposite ends of a 3-meter optical bench, where must a screen be placed between them to receive equal illumination from each?

29-4. Using the last equation of section 29-5 together with the table of length units found in section 29-6, find the energies of photons corresponding to "wave lengths" of (1) 0.001 meter, (2) 0.79 micron, (3) 3,900 Ångströms, (4) 5 X-units.

29-5. Prove that if a candle could give off light uniformly in all directions, it would emit 4π lumens.

29-6. How tall a mirror will be needed so that when placed vertically it will show a six-foot man his full length?

29-7. From data in section 29-12, find the index of refraction of air.

29-8. In figure 29-9, assume that the index of refraction is 1.150, and find the values of angles KCH and MDG. See appendix 7.

29-9. In the preceding problem, assume instead that the index of refraction is 1.414 and recompute the values of the same two angles. See appendix 7.

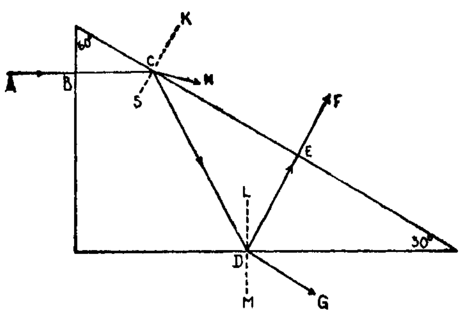
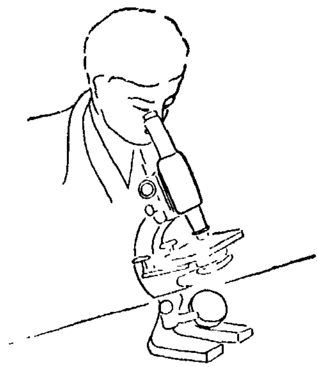


Figure 29-9.

CHAPTER 30



Lenses; Miscellaneous Properties of Light

30-1. Lenses. We have seen that light rays, emerging from a prism, are not parallel to those entering the prism. If we join together parts of several prisms, as in figure 30-1, we obtain a lens-shaped figure. We should therefore also expect light to come out of a lens in a direction different from that of the incident ray. Lenses are classified as *converging* and *diverging*. As in figure 29-8, the rays are always bent toward the thicker part of the prism or lens, so that when the lens is thicker at the center than at the rim, the rays are all bent toward the axis of the lens, that is, they converge, whereas the rays are bent away from the axis (diverge) when the center is thinner than the rim. There are thus three types of converging lenses (figure 30-2). Converging lenses are often called *positive* and diverging lenses *negative* lenses.

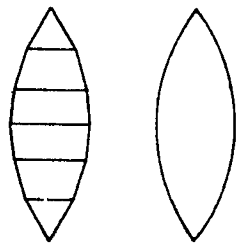
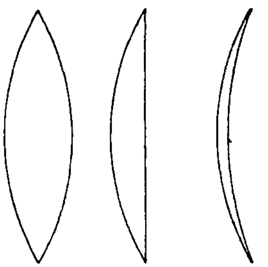


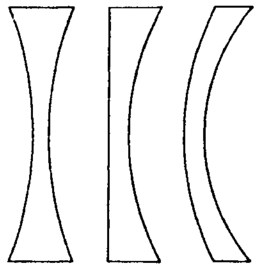
Figure 30-1.

30-2. Formation of a Real Image by a Converging Lens. In figure 30-4 we have a converging lens, and regardless of whether the curvature of one side of the lens is the same as that of the other side, there will be two points (F) called *foci* (the singular is *focus*) at equal distances from the center of the lens. If light enters the lens in a direction parallel to the axis of the lens, it will, after passing through the lens, go through a focus. It is also true that if a ray of light passes through a focus before entering the lens, it will emerge



Converging or Positive Lenses

Figure 30-2.



Diverging or Negative Lenses

Figure 30-3.

in a direction parallel to the axis of the lens. But at the center of the lens, the opposite sides of the lens are so nearly parallel to each other that a ray passing through the center of the lens will come out in a direction parallel to the incident ray, but displaced laterally a little, as in figure 29-7. Let $O'O$ in figure 30-4 be called the *object*,

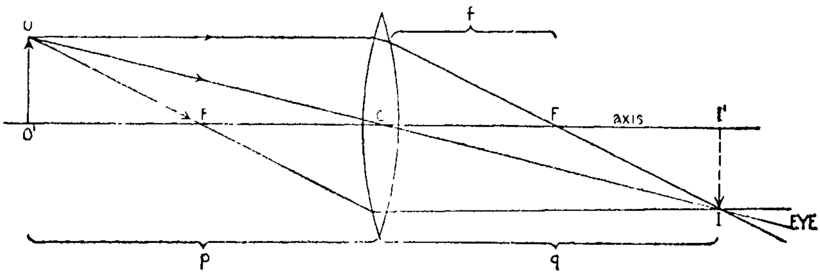


Figure 30-4.

and consider a group of rays going out in all directions from the point O toward the lens. Three of these rays are shown in the figure, the same three which have just been described. It will be seen that these three rays meet again at the point I. In fact, any ray that enters the lens from the point O will, after leaving the lens, pass through the point I. Similarly, any ray that enters the lens from the point O' will also pass through the point I'. We therefore speak of $I'\ I$ as being the *real image* of the object $O'O$. It will be noticed that $I'I$ is inverted; real images are always inverted with respect to the object. There are two ways in which the eye can see this image. The normal eye is supposed to see things best when they are about ten inches from the eye; this is called the "distance of distinct vision." Therefore, in figure 30-4, the eye could be placed about ten inches beyond I to see the image. But this is not the usual method. It is more customary to place a screen at I and view the image by reflected light. Thus we are looking at a series of real images on the moving picture screen; and the film must be run through the machine upside down.

30-3. Algebraic Relationships. It is clear from figure 30-4 that the triangle $O'OC$ is similar to the triangle $I'IC$, therefore the size of the image is to the size of the object as q is to p. p is called the *object distance* ($O'C$ in the diagram) and q the *image distance* (CI' in the figure). That is

$$\frac{I'I}{O'O} = \frac{q}{p}$$

Another relation that is approximately true for lenses is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

where f is the focal length of the lens (CF in the figure). All three of these quantities are positive in figure 30-4. f is positive for any lens that is thicker in the center than at the edges, that is, for converging lenses; f is likewise negative for diverging lenses. When q is in its natural position (on the opposite side of the lens from the object), it is positive; when it is on the same side as the object, q is negative.

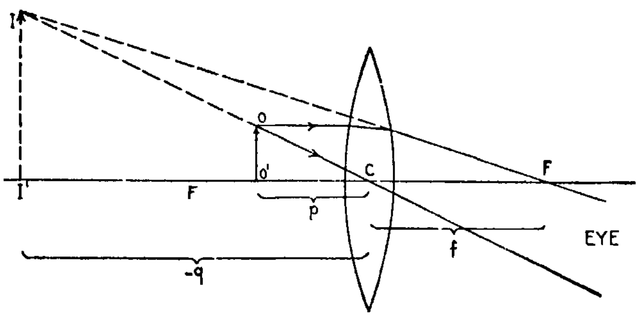


Figure 30-5.

30-4. Formation of Virtual Images. Figures 30-5 and 30-6 illustrate the formation of negative or *virtual images* with both converging and diverging lenses. It will be noticed that in figure 30-6 the lens has no real focus, but that when the ray is parallel to the axis on one side of the lens, it diverges from the axis on the other as if it had come from the virtual focus, F', on the first side. If, in figure 30-5, the distance p were ten inches and the distance q were

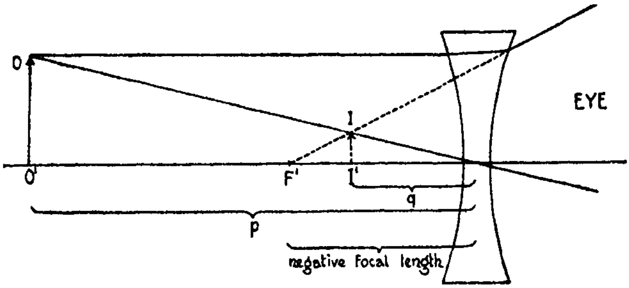
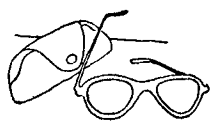


Figure 30-6.

minus thirty inches, the lens would be ideal for the so-called far-sighted person whose distance of distinct vision is thirty inches. He would be able to hold his book ten inches away while what he saw would appear to be thirty inches away. On the other hand, the lens in figure 30-6 would be convenient for a near-sighted person, whose distance of distinct vision is, say, four inches. In this case p is ten inches and q is minus four inches. The equations of section 30-3 are sometimes called *thin lens* formulas because they ignore the fact that the lens actually has a finite thickness instead of being infinitely thin. Likewise in this illustration of the eye-glasses, the distance between the glasses and the eye is ignored.



30-5. Illustrative Problem. The focal length of a converging lens is 30 inches. If an object 10 inches high is placed 50 inches from the lens, find the size and position of the image.

In this case, $f = 30$ inches, $p = 50$ inches, and $O'O = 10$ inches. Subsituting in the equation $1/f = 1/p + 1/q$, we have

$$\frac{1}{30} = \frac{1}{50} + \frac{1}{q}$$

Multiply both sides of the equation by $150q$ and get

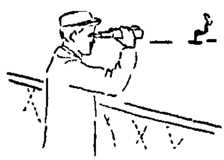
$$5q = 3q + 150$$

Solving, $q = 75$ inches, the image distance.
Substituting now in the equation, $I'I/O'O = q/p$, gives us

$$\frac{I'I}{10} = \frac{75}{50}$$

Therefore, $I'I = 15$ inches, the height of the image.

30-6. Dispersion by Refraction. In section 29-4 the statement was made that although in a vacuum all "wave lengths" (still having in mind the psi-waves) travel with the same speed, they not only travel with less speed in a medium like glass, but each "wave length," that is, each color travels with a speed of its own. We must therefore think of figure 29-8 as representing a ray of some one color. A *ray of light* is the path of a succession of photons as more or less determined by the psi-waves. The psi-functions have about the same degree of control over a photon as the banks of a brook have over the course of a drop of water in the brook; they determine its general path but not every last detail of its motion. If, on the other hand, the original ray consisted of white light, which is a mixture of all colors, each color would come out of the prism in a different direction. It will be noticed that since a given color represents a given "wave length," it therefore also designates the energy of each photon in a ray of that color. (See end of section 29-5.) The direction of two of these rays can be seen in figure 30-7. We refer to this splitting of white light into colors as *dispersion.*



We should expect some color effect in a lens, but as a matter of fact the color effect is not very great, especially in a thin lens. But in a thick lens, or in a situation where precision is demanded, the color effect, or *chromatic aberration* as it is called, is troublesome enough so that it is customary to use two or more lenses together, of different shapes and of different kinds of glass, so that the colors produced by one lens will neutralize those produced by the other. This is called an *achromatic combination* of lenses; it is not unusual for a good microscope to contain over a dozen lenses. It is simpler, however, to think of both telescopes and microscopes as consisting of two *sets* of lens; the first of these sets, called the *objective,* forms a real image (inverted) of the object, and the second set, or *eyepiece,* forms a virtual image of this real image.

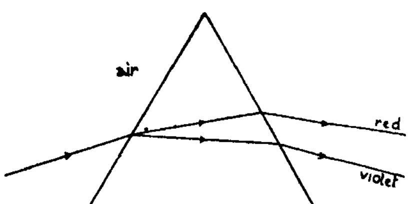


Figure 30-7.

30-7. Diffraction and Interference. Figure 30-8 contains an illustration of both *diffraction* and *interference* of light. *S* is a point source of light of some one color, say yellow. If S lies at the focus of lens L, the rays will be parallel with one another emerging from the lens. G represents a screen containing a series of parallel slits, A, B, C, and so on. Each slit is narrow enough so that on the other side we get complete *diffraction,* that is, the light goes in all

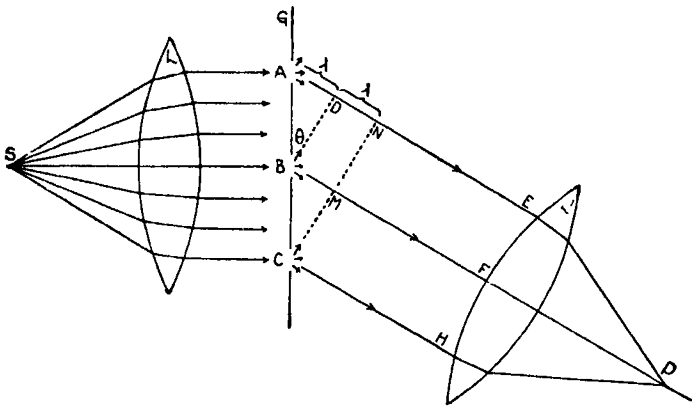


Figure 30-8.

directions from each of the points A, B, C, and so on. (See section 16-8). Now let us consider three rays, AE, BF, and CH, which happen not only to be parallel with one another, but which make an angle with the direction CA such that when the perpendicular BD is dropped to the line AE, the distance AD comes out just one "wave length." Let the angle ABD be called θ. CA represents a "wave front" (see section 16-6) which has emerged from the lens L, so that the vibrations at any given instant at A and B are alike. If D is just one "wave length" from A, then at any given instant, D is also in phase with A and B, so that BD may also be considered a "wave-front", and likewise CMN. If the rays AE, BF, and CH now pass through the lens L', the photons constituting these rays will converge at the point P, which is the focus of L'. If L' happens to be the crystalline lens of the eye, P is some point on the retina of the eye and will receive photons, the energies of which are characteristic of yellow light. The manner in which these psi-waves reinforce one another serves as an excellent example of *constructive* interference. If, however, the angle θ were slightly larger or smaller than the one we have chosen, AD would be larger or smaller than a "wave length"; no "wave fronts" would be formed for that particular color, therefore no photons of that energy would be guided in the new direction. This would be an example of *destructive* interference.

30-8. Dispersion by Diffraction. If in figure 30-8 we had started with white light instead of light of some one color, then no matter what the angle ABD, there would always be some "wave length" present equal to AD, and that color would be the one brought to a focus at P. In other words, the eye, by looking in different directions, would see different colors, a situation similar to that of figure 30-7. The screen, G, of figure 30-8, containing a set of equidistant parallel slits, is called a *diffraction grating.* One way of making a diffraction grating is to rule scratches on a piece of glass with a diamond point; 15,000 scratches to the inch is common practice.

30-9. Measurement of "Wave Lengths." The apparatus which is diagrammed in figure 30-8 consists of a turntable upon which the grating is mounted, and two telescope tubes arranged to rotate about a vertical axis which is also the axis of the turntable. One telescope is called the *collimator* and contains the slit, S, and the collimating lens, L. The other telescope, the *observing telescope*, contains the lens, L'.

The simplest way of using the instrument for the measurement of "wave lengths" is to set the axis of the collimator at right angles to the grating, as in figure 30-8, and then measure the angle between the normal to the grating and the observing telescope when the latter is focused upon the desired spectrum line. We talk about "spectrum lines" because what we see in the observing telescope is a series of colored images of the slit, and the slit is a long narrow opening shaped like a "line." Scales are provided for the accurate measurement of the angular positions of the turntable and both telescopes. The angle between the normal to the grating and the observing telescope is equal to the angle θ. If we know how many lines have been ruled to the inch in our grating, then we also know AB. The "wave length," λ, which is AD, is therefore given by the equation, $\lambda = (AB) \sin \theta$. It may readily be verified that all through the discussion of figure 30-8 $N\lambda$ could have been substituted for λ, where N is any integer. N is called the *order* of the spectrum. The equation therefore becomes

$$N\lambda = (AB) \sin \theta$$

30-10. Illustrative Problem. A grating has 21,561 lines to the inch. Find the necessary angle between the normal to the grating and the observing telescope, assuming the collimator to be normal to the grating, to get the sodium D line, $\lambda = 5{,}890.2$ Å in the first order.

It will be convenient to find how many lines there are to the centimeter since the "wave length" has been expressed in Ångströms, the customary unit: 21,561/2.5400 = 8,488.6 lines per centimeter. The "wave length" expressed in centimeters is 0.000058902 cm. Since we are in the first order, N equals 1.00000. Substituting in the equation at the end of section 30-9 gives us

$$0.000058902 = (1/8488.6)(\sin\theta)$$

Solving, we obtain $\sin\theta = 0.49999$. Looking this up in a five-place table of sines, we find that the sine of 29°59′ is 0.49975, and the sine of 30°0′ is 0.50000. (The latter value may also be found in appendix 7). Our value is so much closer to an even 30 degrees than it is to 29°59′, that we shall submit 30°0′.0 as our result.

30-11. Spectra. We have now met two devices, different in principle, capable of dispersing white light into its various constituent colors: the prism and the grating. The array of colors produced is called a *spectrum.* A white-hot solid or white-hot liquid will each produce a spectrum containing all the colors of the rainbow, and one could not tell by examining the spectrum what substance constituted the source of light. On the other hand, if it is a gas emitting the light, a limited number of "wave lengths" will be present in the spectrum with empty spaces between the spectral lines. The arrangement of "wave lengths" in the spectrum is typical of the gas emitting the spectrum so that the gas may be identified by means of its spectrum. This process is called *spectroscopic analysis.*

30-12. Polarization of Light. The statement was made in section 16-2 that the psi-waves of light are *transverse* waves, and the reason for our thinking that the waves are transverse is that they can be *polarized* (see section 16-10). When these psi-waves pass through certain crystals, all vibrations are absorbed except those parallel to some given plane. Tourmaline, for example, behaves in this way. By rotating the piece of tourmaline, the plane of the emerging polarized waves is also rotated. It is possible, with two pieces of tourmaline, to arrange them so that what light gets through one of them also goes through the other. If, however, the second piece of tourmaline is now rotated 90 degrees, the light that passes through the first crystal will not be able to go through the second. Solutions of certain carbon compounds, such as sugar, as well as certain transparent solids under strain, such as celluloid, have the property of *rotating* the plane of polarization. This fact is made use of in the analysis of sugar solutions. The principle is also applied by making models of such things as dirigibles out of celluloid and observing the effect on polarized light when various stresses are applied. A commercial material called *polaroid* is now available; this has the same properties as tourmaline. One of the many uses for a material of this kind is in the headlights and windshields of automobiles to prevent the glare experienced in night driving.

SUMMARY OF CHAPTER 30

Technical Terms Defined

Dispersion. The separation of the frequencies of white light into those of the constituent colors. This is usually accomplished either by a prism or a grating.

Diffraction. The spreading experienced by light after passing through a small opening.

Interference. The combining of crests and troughs of one wave train with those of another, which has the effect of neutralizing the wave motion in some cases and intensifying it in others.

Grating. A piece of metal or glass ruled with many parallel lines, several thousand to the inch, and used either to reflect or transmit light. Its effect is to separate white light into its constituent colors.

Spectrum. An array of colored images of a slit, characteristic of the incandescent gas which is serving as source.

Polarization. The removal from a beam of transverse waves all except those vibrating in planes parallel to a given plane.

PROBLEMS

30-1. A converging lens is being used in such a way that the object and image are on the same side of the lens. (See figure 30-5). If the object is ten and the image thirty inches from the lens, find the focal length.

30-2. The object is ten, and the image four inches from a diverging lens. Find the focal length of the lens.

30-3. A picture on a lantern slide has dimensions of two by three inches. Find the focal length of a lens which will project an image of this picture two by three yards in size, on a screen thirty feet from the slide.

30-4. It is possible to observe the same object simultaneously with one eye unaided and the other eye provided with a small telescope, and by comparing the apparent sizes of the object, it is possible to determine the magnification of the telescope. Give reasons why the same procedure could or could not be employed successfully to obtain the magnification of a single converging lens.

30-5. If a pin an inch long is placed three inches from a converging lens with a focal length of four inches, find the position and size of the image. Will the image be real or virtual? Where will the eye have to be to see the image?

30-6. Given a converging lens with the object at infinity, that is, an infinite distance away, where is the image? Describe the successive positions of the image as the object is moved in toward the lens, finally arriving at the lens.

30-7. Repeat problem 30-6, except that the converging lens is replaced by a diverging lens.

30-8. The three principal methods of producing colored light from white light are (1) passing the light through an object which is transparent to certain colors and opaque to others, such as blue glass, (2) utilizing refraction, as in the case of the prism, and (3) utilizing interference, as in the case of the grating. Decide whether the following belong under one or another of these three cases or under other cases not listed: (a) the color of copper sulphate, which looks blue both by reflected and by transmitted light, (b) very thin gold leaf which looks yellowish by reflected light and greenish by transmitted light, (c) a rainbow, (d) the beveled edge of a mirror, (e) a soap bubble, and (f) a thin film of oil on a puddle of water.

30-9. It is desired to measure the "wave length" of one of the so-called sodium D lines by using the second order spectrum produced by a transmission grating ruled with 20,000 lines to the inch. The angle between the normal to the grating and the observing telescope is 68°12.4′, the sine of which is 0.92853. Find the "wave length." How many significant figures is it proper to keep in your answer?

APPENDIX I

Common Physical Constants and Conversion Factors

A physical constant consists of a number with or without (but usually with) a more or less complicated unit. Some of the more common physical constants used in this book are given here in both English and metric units.

	English Units	*Metric Units*
Normal height of barometer	30 inches	76 0 centimeters
Atmospheric pressure	14.7 lb /in.^{2}	1,033 grams/cm.^{2}
Density of water	62.4 lb./ft ^{3}	1 00 gram/cm.^{3}
Specific gravity of water	1 00	1 00
Acceleration of gravity	32 2 ft./sec.^{2}	9 80 m./sec.^{2}
π	3.14	3.14
Heat of fusion of water	144 B.t.u./lb.	80 Cal /kgm.
Heat of vaporization of water	972 B.t.u /lb	540 Cal./kgm.
Specific heat of water	1 00	1 00
Coefficients of linear expansion	See section 20-2	
Speed of sound	1,087 ft /sec	331 meters/sec.
Speed of light, radio waves, etc.	186,000 miles/sec.	3×10^8 m./sec.

A conversion factor always consists of a number together with a unit. Furthermore, the unit of a conversion factor always consists of a numerator and a denominator, both of which represent the same kind of physical quantity. All conversion factors may be equated to unity; it is thus possible to multiply or divide any physical quantity by a conversion factor without changing the value of the original quantity. Consider the equation, 3 feet = 1 yard. As it stands it is correct; without the units it is, of course, incorrect. Now divide both sides of the equation by the unit "yard." The result is 3 feet/yard = 1. Unity on the right-hand side of the equation is now a pure number, that is, it has no units, and 3 feet/yard is a conversion factor. Try the effect of multiplying 4 yards by 3 feet/yard. The result is 12 feet; the yards cancel each other; and 12 feet is the same as 4 yards. Similarly the student may obtain conversion factors from any of the following equations.

9.80 newtons	= 1 kilogram
4.45 newtons	= 1 pound
1 newton-meter	= 1 joule
1 erg	= 1 dyne-centimeter
980 ergs	= 1 gram-centimeter
42,800 gm.-cm.	= 1 Calorie
10,000,000 ergs	= 1 joule = 1 watt-second
4,190 joules	= 1 Calorie
0.252 Calories	= 1 British thermal unit (B.t.u.)
3,600 joules	= 1 watt-hour
1,000 watt-hours	= 1 kilowatt-hour (Kw.-hr.)
3,410 B.t.u.	= 1 Kw.-hr.
0.746 kilowatt-hours	= 1 horsepower-hour (hp.-hr.)
550 foot-pounds	= 1 horsepower-second
3,600 hp.-seconds	= 1 hp.-hr.
0.738 foot-pounds	= 1 joule
778 foot-pounds	= 1 B.t.u.
980 ergs/second	= 1 gram-centimeter/second
10,000,000 ergs/second	= 1 watt = 1 joule/second
1,000 watts	= 1 kilowatt
746 watts	= 1 horsepower
550 ft.-lb./second	= 1 horsepower
33,000 ft.-lb./minute	= 1 horsepower
96,500 coulombs	= 1 faraday
980 dynes	= 1 gram
454 grams	= 1 pound
2.20 pounds	= 1 kilogram
32.2 pounds	= 1 slug
60 seconds (″)	= 1 minute (′)
60 minutes	= 1 degree (°)
57.3 degrees	= 1 radian
90 degrees	= 1 quadrant
360 degrees	= 1 revolution
2π radians	= 1 revolution
1,000 X-units	= 1 Ångstrom
10,000 Angstroms	= 1 micron
1,000 microns	= 1 millimeter
10 millimeters	= 1 centimeter
100 centimeters	= 1 meter
1,000 meters	= 1 kilometer
300,000 kilometers	= 1 light-second
31,560,000 light-seconds	= 1 light-year
3.258 light-years	= 1 parsec
1,000,000 parsecs	= 1 megaparsec
2.54 centimeters	= 1 inch

APPENDIX 2

Significant Figures and Computation Rules

In general, a physical quantity involves both a number and a unit, and must be thought of in nearly every case as either the direct or the indirect result of a measurement. Thus, 3 meters is a physical quantity where 3 is the number and meters is the unit. Moreover, the physicist makes a distinction between 3 meters, 3.0 meters, and 3.00 meters. The first means that he has merely estimated the first figure, 3, and has no idea whether the tenths figure is a 0, 1, or even a 4. If he has any idea of what the next figure is, he puts it down, even though it be a zero. Thus the last significant figure in the statement of a physical quantity is understood to be the best estimate for that position; the figures that precede it are known to be exact. In the illustration just used, 3 meters is referred to as a number of one significant figure and represents only an estimate, whereas the 3.00 meters is said to have three significant figures, the last of which is an estimate. In counting the number of significant figures, the position of the decimal point is not considered. For instance, 30.0 millimeters and 3.00 centimeters each have three significant figures, and in fact represent the same measurement.

We can also have insignificant figures. When a figure can not possibly be the result of measurement, or is merely used to occupy space between the decimal point and the figures that are significant, we say that it is insignificant. For instance, in the statement that the population of the United States is 140,000,001, the last 1 is insignificant (and therefore absurd) for the first reason, and several of the zeros are insignificant for the second reason. Insignificant zeros put in for the second reason are of course excusable and in fact necessary. But zero is the only figure that can legitimately be insignificant, and then only for the purpose of occupying space to the decimal point. Thus, the 3.00 centimeters in the previous paragraph may be expressed as 0.0300 meters and still have but three significant figures, since the zero before the 3 merely occupies space between the decimal point and the first significant figure. The zero in front of the decimal point is purely optional.

There are certain rules for the proper number of significant figures to keep in a computation. If it is a matter of multiplication or division, keep no more significant figures in the product or quotient than in the numbers started with. And the numbers started with should both have the same number of significant figures in the first place. There is one exception to this rule. When a number begins with the digit 1, it is customary to keep one more significant figure than the above rule prescribes. This is because of a practice that grew out of the structure of the slide rule. Numbers like 999 and 1,001 each represent measurements to the same degree of precision, yet it requires one more digit in the second case than in the first to express the quantity. Unless otherwise specified, in solving problems in this book, we limit ourselves to three significant figures, unless the number happens to begin with a *one*, when we allow ourselves the luxury of four significant figures. The only occasions when the decimal point influences the number of significant figures to be used are in addition and subtraction. The rule then is to round off the numbers to be added or subtracted so that they have the same number of decimal places, and then keep that number of decimal places in the answer. Thus in multiplication and division, it is the total number of significant figures that interest us, regardless of the position of the decimal point, while in addition and subtraction, it is the number of decimal figures that are important, regardless of the total number of figures in the number.

As an illustration, suppose it is required to add the following distances: 5.01, 0.1429, 0.00737, 0.000927, all in centimeters, it being understood as usual that the last digit in each number is only an estimate.

Incorrect	*Correct*
5.01	5.01
0.1429	0.14
0.00737	0.01
0.000927	0.00
5.161197	5.16

On account of the uncertainty of the hundredths places, it is understood that it is equally reasonable that the sum should be 5.18 or 5.15 centimeters, and the string of digits, 1197, are absolutely meaningless and out of place in the result.

In rounding off a number, we increase the last digit retained by one unit if the next digit (the first rejected digit) is more than a 5. This accounts for the replacement of 0.00737 by 0.01 in the previous paragraph. On the other hand, 0.1429 is replaced by 0.14. If we drop a 5, the origin of which we know to be either slightly more or slightly less than a 5, we know whether or not to increase the last digit retained. If the figure rejected is *exactly* 5, we might as well toss up a penny to decide whether to raise the previous digit by one or to leave it as it is. Trained computers follow some rule in this instance which will insure half of this type of 5's being treated in each way in the long run. One such rule, for instance, is to do whatever is necessary to leave the number even.

As another illustration of the computation rules, suppose that it is desired to determine the area of a rectangle 12.343 meters long and 3.47 meters wide. In the length we have an estimate to the fifth figure, but in the width to the third only. A strict adherence to the rules would require rounding off the first number to 12.3 meters; but, remembering the exception to the rule, since it begins with a 1, we are allowed to keep four figures, namely, 12.34 meters. The multiplication could be carried out by either of the following two methods of "long multiplication":

$$
\begin{array}{r@{}l}
3.4 & 7 \\
12.3 & 4 \\
\hline
1 & |388 \\
10 & |41 \\
69 & |4 \\
347 & | \\
\hline
42.8 & |198
\end{array}
\qquad\qquad
\begin{array}{l@{}l}
3.47 & \\
\quad\;\; 12.34 & \\
\hline
347 & | \\
\;\;69 & |4 \\
\;\;10 & |41 \\
\quad\; 1 & |388 \\
\hline
42.8 & |198
\end{array}
$$

The second is preferable because it gives us the most important part of the product first, but both are open to a criticism which will be obviated in the third section of the appendix. In the original statement of the width, we are sure of the 3 and the 4, but the 7 is only an estimate. Another estimate of the third digit might have made it an 8. If we had multiplied 3.48 by 12.34 we should have obtained 42.9432. Obviously, in the product we are sure only of the 42 and the best estimate of the next figure is an 8 or possibly a 9. But for all we know the area may be 42.6 or even 43.0 square meters. If we follow the rules for significant figures to be retained in a multiplication, we shall round off our answer to three figures and have 42.8 square meters for our result, thus expressing the value with two significant figures, the correctness of which we are sure, and a third significant figure, which represents the best estimate for that position. This means that the figures to the right of the vertical line are all superfluous; at best they influence merely the uncertain 8.

APPENDIX 3

Abbreviated Multiplication and Division

A series of multiplications and divisions that could be performed in sixty seconds with the slide rule would require about three minutes with logarithms (see appendix 8), five minutes by the abbreviated methods described in this section, and seven minutes by ordinary long multiplication and division. One does not always have a logarithm table or slide rule conveniently at hand, whereas he can always throw out the figures that play no part in the final result. Consider these two modifications of the second multiplication in appendix 2.

$$\begin{array}{r} 3.47 \\ \underline{12.34} \\ 347 \\ 70 \\ \underline{9} \\ 42.6 \end{array} \qquad\qquad \begin{array}{r} 3.47 \\ \underline{12.34} \\ 347 \\ 694 \\ 105 \\ \underline{12} \\ 42.81 \end{array}$$

In the first modification, we multiply the 347 by 1 and put down 347. We are now through with the 1 of the multiplier, also with the 7 of the multiplicand, since the latter simply gives us digits from now on to the right of the vertical line above. So cross out the 1 of the multiplier, also the 7 of the multiplicand, but since the latter is more than 5, increase the 4 by 1. Next we multiply the 35 by 2, which gives us 70. We are now through with the 2 in the multiplier and with the 5 in the multiplicand, therefore we cross them both out. This time we do not increase the 3 by unity, since the 47 which has been crossed out was less than 50. We now multiply 3 by 3 which gives 9. We are not disturbed that the sum is 42.6 instead of 42.8 because the third figure is merely an estimate anyway. The second method is the same as the first except that we do not begin crossing out digits of the multiplicand quite so soon, thus the third figure is more reliable. The result should, of course, be rounded off to 42.8 square meters.

Suppose now that it is desired to divide 42.8 by 12.34. Here are two methods, the long division and the abbreviated division.

```
             3.468                     3.472
      12.34|42.80 000           12.34|42.80
            37 02|                    37 02
             5 78|0                    5 78
             4 93|6                    4 92
               84|40                     86
               74|04                     84
               10|360                     2
                9|872
                 |488
```

In either case the result should be rounded off to 3.47. Again the figures to the right of the vertical lines are superfluous. In the right hand method, after the 3,702 has been subtracted from the 4,280, instead of bringing down a fictitious zero, we cross out the 4 of the divisor. Since 4 is less than 5 we do not increase the 3 by unity. 123 goes into 578 four times. Four times 123 is 492. And so forth. Each time, instead of bringing down a zero from the dividend, we cross out a digit of the divisor. By so doing (1) it is no longer necessary to handle so many insignificant figures, (2) the process becomes more and more simple as it progresses, and (3) the third advantage of the method is that when we get the proper number of figures in our quotient we have to stop! There is no more divisor left with which to continue. If we use a slide rule for multiplication and division, we shall also automatically obtain the correct number of significant figures in our results, without recourse to a set of regulations.

APPENDIX 4

Summary of Essentials of Algebra

This appendix naturally will not take the place of a course in algebra, but will merely serve to emphasize the portion most important for our purpose, namely, the handling of equations. The fundamental fact is that if we start with an equation, which is a statement that two things are equal to each other, then whatever we do to one side of this equation we must also do to the other side of the equation. The operations that we carry out most often on the two sides of the equation are addition, subtraction (which is merely the addition of a negative quantity), multiplication, division, squaring, and extracting the square root.

Addition and subtraction are represented by plus and minus signs as in arithmetic. Thus

$$5 + 10 - 2 - 7 = 6$$

If we add 7 to both sides of the equation we have $5 + 10 - 2 - 7 + 7 = 6 + 7$, or

$$5 + 10 - 2 = 6 + 7$$

The result is simply to shift the 7 to the other side of the equation and give it the opposite sign. This is called *transposing* the 7. With letters instead of figures we have

$$x + a = b$$

Transposing the a we have

$$x = b - a$$

Multiplication is represented by merely placing the quantities next to each other, preferably surrounded by parentheses. Example: $(6)\ (7) = 42$; $ab = c$, or $(a)\ (b) = c$; $(a - b)\ c = d$. The last is read: the product of $a - b$ and c is equal to d. Division is represented by making use of the fact that in a fraction the numerator is divided by the denominator. Thus 42 divided by 21 would be written

$$42/21 = 2 \text{ or } \tfrac{42}{21} = 2$$

$\frac{6}{2}3$ is the same as $\dfrac{(6)\ (3)}{2}$, and both are equal to 9. If we start with

$$xa = b$$

and divide both sides by a, we shall have $\frac{xa}{a} = \frac{b}{a}$, which is the same as

$$x = b/a$$

Let us start with the equation $\frac{x + a}{c} = b$, and let the task be to perform enough of the above operations on both sides of this equation to result in leaving x alone on the left hand side. First multiply both sides of the equation by c, giving $\frac{x + a}{c}c = bc$, or

$$x + a = bc$$

Now transpose the a and get

$$x = bc - a$$

which is the required result.

The problem solved in the previous paragraph is known as *solving an equation for x*. In this book, the equations are furnished by the facts of physics. A large number of the problems amount simply to furnishing numerical values for all but one of the quantities in an equation and asking the student to solve for the unknown quantity.

A proportionality constant is a number used to change a proportion into an equation. Suppose A to be proportional to B. For example, suppose that when A is 1, B is 3; when A is 2, B is 6; when A is 5, B is 15; and so on. Instead of saying that A is proportional to B, the situation could have been equally well expressed by saying that A is equal to B times a constant. $A = kB$, the constant, k, in this case being 1/3. This kind of a constant is called a *proportionality constant.*

If C has the same ratio to D that A has to B, we say "A is to B as C is to D" and write it $A:B = C:D$, or better, $A/B = C/D$. This is nothing more than the equating of the two fractions, A/B and C/D. A and D are called the *extremes* of this proportion and B and C the *means*. The product of the means of a proportion is equal to the product of the extremes; that is, $AD = BC$.

The only case of factoring involved in this text is based on the fact that

$$(x + y)(x - y) = x^2 - y^2$$

This is read, the product of the sum of x and y by the difference of

x and y is equal to the difference of the squares of x and y. This may be checked with numerical values as follows

$$(7 + 5)(7 - 5) = 7^2 - 5^2 \text{ or } (12)(2) = 49 - 25 = 24$$

Another way of checking this is as follows

$$(7 + 5)(7) - (7 + 5)(5) = 7^2 - 5^2$$

or

$$(7)(7) + (5)(7) - (7)(5) - (5)(5) = 7^2 - 5^2$$

If x is small compared with unity, then x^2, x^3, and so on, may be neglected and we have the following *approximate* equation

$$(1 \pm x)^n = 1 \pm nx$$

where n may be either positive or negative, integral or fractional.

The formula for the solution of the quadratic equation $ax^2 + bx + c = O$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

APPENDIX 5

Geometrical Propositions Essential to This Book

1. If one straight line intersects another, the opposite angles (vertical angles) at the vertex are equal.

2. Given two angles; if the sides of one angle are parallel respectively to the sides of the other angle, the two angles are equal or supplementary.

3. Given two angles; if the sides of one angle are perpendicular respectively to the sides of the other angle, the two angles are equal or supplementary.

4. The sum of the three angles of a triangle equals 180 degrees.

5. The sum of the four angles of a quadrilateral equals 360 degrees.

6. Two triangles are equal if (1) the three sides of one are equal to the three sides of the other, (2) if two sides and the included angle of one are equal to two sides and the included angle of the other, and (3) if two angles and the included side of one are equal to two angles and the included side of the other.

7. If the angles of one triangle equal the angles of another triangle, the corresponding sides are proportional.

8. In a parallelogram, the opposite sides are equal.

9. The sum of the squares of the two legs of a right triangle is equal to the square of the hypotenuse.

10. An angle inscribed in a semicircle is a right angle.

11. The area of a triangle is half the base times the altitude.

12. The area of a parallelogram is the base times the altitude.

13. The area of a trapezoid is the average of the two bases times the altitude.

14. The area of a circle is πr^2 or $\pi D^2/4$.

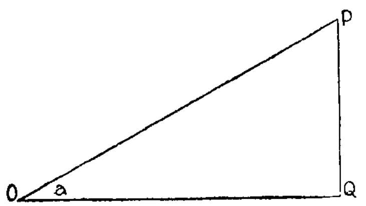
15. The volume of a sphere is $4\pi r^3/3$ or $\pi D^3/6$.

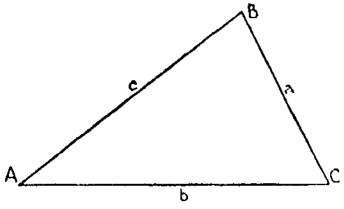
APPENDIX 6

Definition of Sine and Cosine; Sine Law, Cosine Law

The usual mathematical method of measuring an angle is to construct an arc with center at the vertex; the angle (in radians) is then equal to the ratio of the portion of the arc intercepted between the sides of the angle to the radius of the arc. It is often more convenient to obtain a measure of the angle in terms of ratios between straight lines as follows.

From any point P in one side of an angle drop a perpendicular PQ to the other side OQ. This forms a right triangle two sides of which are adjacent to the given angle. The shorter of these sides, OQ, divided by the longer, OP, is called the *cosine* of the angle a. The value of the cosine determines the angle and the value of the angle determines the cosine. Since we always divide the shorter by the longer, the cosine of an angle is always less than one. If the angle is obtuse, the perpendicular will land on a side of the angle produced and the cosine is then considered negative. The ratio between the side PQ, opposite to the angle a, to the hypotenuse OP, is called the *sine* of the angle a. The sine also is always less than one. The sine is considered positive whether the angle is acute or obtuse. We may summarize these two statements by the equations





$$\cos a = \frac{OQ}{OP} \qquad\qquad \sin a = \frac{PQ}{OP}$$

Numerical values of these ratios will be given in the next appendix.

In any triangle, ABC, where side a is opposite angle A, side b opposite angle B, and side c opposite angle C, the following two relations hold:

Sine law: $(\sin A)/a = (\sin B)/b = (\sin C)/c$

Cosine law:
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = c^2 + a^2 - 2ca \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

APPENDIX 7

Table of Sines and Cosines

$\sin 0 = \sin 180 = \cos 90 = -\cos 90 = 0.0000$
$\sin 5 = \sin 175 = \cos 85 = -\cos 95 = 0.0872$
$\sin 10 = \sin 170 = \cos 80 = -\cos 100 = 0.1736$
$\sin 15 = \sin 165 = \cos 75 = -\cos 105 = 0.259$
$\sin 20 = \sin 160 = \cos 70 = -\cos 110 = 0.342$
$\sin 25 = \sin 155 = \cos 65 = -\cos 115 = 0.423$
$\sin 30 = \sin 150 = \cos 60 = -\cos 120 = 0.500$
$\sin 35 = \sin 145 = \cos 55 = -\cos 125 = 0.574$
$\sin 40 = \sin 140 = \cos 50 = -\cos 130 = 0.643$
$\sin 45 = \sin 135 = \cos 45 = -\cos 135 = 0.707$
$\sin 50 = \sin 130 = \cos 40 = -\cos 140 = 0.766$
$\sin 55 = \sin 125 = \cos 35 = -\cos 145 = 0.819$
$\sin 60 = \sin 120 = \cos 30 = -\cos 150 = 0.866$
$\sin 65 = \sin 115 = \cos 25 = -\cos 155 = 0.906$
$\sin 70 = \sin 110 = \cos 20 = -\cos 160 = 0.940$
$\sin 75 = \sin 105 = \cos 15 = -\cos 165 = 0.966$
$\sin 80 = \sin 100 = \cos 10 = -\cos 170 = 0.985$
$\sin 85 = \sin 95 = \cos 5 = -\cos 175 = 0.996$
$\sin 90 = \sin 90 = \cos 0 = -\cos 180 = 1.000$

APPENDIX 8

Three-Place Logarithm Table

Logarithms are exponents of 10. For example, $100 = 10^2$, therefore 2 is the logarithm of 100, written $2.000 = \log 100$. Also $3.000 = \log 1{,}000$ and $4.000 = \log 10{,}000$. Numbers between integral powers of 10 may be expressed as fractional exponents of 10. For example, $10^{2.301} = 200$, $10^{2.477} = 300$, $10^{1.301} = 20$, and $10^{0.301} = 2$. These relations may also be written $\log 200 = 2.301$, $\log 300 = 2.477$, $\log 20 = 1.301$, and $\log 2 = 0.301$. It will be observed that in stating the value of the logarithm, the integral portion of the logarithm (called the *characteristic* of the logarithm) is always one less than the number of digits to the left of the decimal point in the number itself. Thus it is not necessary to include the characteristic in the logarithm tables, but only the *mantissas* or decimal portion of the logarithm. Let us look in the table on page 309 in the row numbered "8" under the column numbered "6" and find the mantissa 934. We could now write $\log 8{,}600 = 3.934$, $\log 86 = 1.934$, $\log 8.6 = 0.934$, $\log 0.86 = 9.934 - 10$, $\log 0.086 = 8.934 - 10$, and so on. Similarly we may look up the logarithm of 8,700 and obtain $\log 8{,}700 = 3.940$. The logarithm of 8,650 is 3.937, just halfway between 3.934 and 3.940. It is always possible to estimate the values of logarithms between the ones in the table in this manner; the process is called *interpolation.* The use of the logarithms follows from the fact that $(10^x)(10^y) = 10^{x+y}$. That is, when we multiply the numbers, we merely add the exponents. Therefore $\log 2 + \log 3 = \log 6$. Checking from the table, we find $0.301 + 0.477 = 0.778$. Division is similarly converted into subtraction.

Let the student solve the following problem from the table above and then check it with the work below.

$$\frac{(0.234)\,(1.478)\,(92.7)\,(0.0439)}{(0.567)\,(0.0872)\,(3.14)\,(15.79)} = ?$$

THREE-PLACE LOGARITHM TABLE

	0	d*	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d	
2	301	21	322	20	342	20	362	18	380	18	398	17	415	16	431	16	447	15	462	15	2
3	477	14	491	14	505	14	519	12	531	13	544	12	556	12	568	12	580	11	591	11	3
4	602	11	613	10	623	10	633	10	643	10	653	10	663	9	672	9	681	9	690	9	4
5	699	9	708	8	716	8	724	8	732	8	740	8	748	8	756	7	763	8	771	7	5
6	778	7	785	7	792	7	799	7	806	7	813	7	820	6	826	7	833	6	839	6	6
7	845	6	851	6	857	6	863	6	869	6	875	6	881	5	886	6	892	6	898	5	7
8	903	5	908	6	914	5	919	5	924	5	929	5	934	6	940	4	944	5	949	5	8
9	954	5	959	5	964	4	968	5	973	5	978	4	982	5	987	4	991	5	996	4	9
10	000	4	004	5	009	4	013	4	017	4	021	4	025	4	029	4	033	4	037	4	10
11	041	4	045	4	049	4	053	4	057	4	061	3	064	4	068	4	072	4	076	3	11
12	079	4	083	3	086	4	090	3	093	4	097	3	100	4	104	3	107	4	111	3	12
13	114	3	117	4	121	3	124	3	127	3	130	4	134	3	137	3	140	3	143	3	13
14	146	3	149	3	152	3	155	3	158	3	161	3	164	3	167	3	170	3	173	3	14
15	176	3	179	3	182	3	185	3	188	2	190	3	193	3	196	3	199	2	201	3	15
16	204	3	207	3	210	2	212	3	215	2	217	3	220	3	223	2	225	3	228	2	16
17	230	3	233	3	236	2	238	3	241	2	243	3	246	2	248	2	250	3	253	2	17
18	255	3	258	2	260	2	262	3	265	2	267	3	270	2	272	2	274	2	276	3	18
19	279	2	281	2	283	3	286	2	288	2	290	2	292	2	294	3	297	2	299	2	19
	0		1		2		3		4		5		6		7		8		9		

*d is called the "tabular difference." Each number in a column headed d is the difference between the two numbers immediately adjacent to it.

Solution:

$$\begin{aligned}
\log 0.234 &= 9.369 - 10 \\
\log 1.478 &= 0.169 \\
\log 92.7 &= 1.967 \\
\log 0.0439 &= \underline{8.642 - 10} \\
&\ 20.147 - 20
\end{aligned}
\qquad
\begin{aligned}
\log 0.567 &= 9.754 - 10 \\
\log 0.0872 &= 8.941 - 10 \\
\log 3.14 &= 0.497 \\
\log 15.79 &= \underline{1.199} \\
&\ 20.391 - 20 \\
\text{or}\ &\ 10.391 - 10
\end{aligned}$$

Subtracting

$$\begin{array}{r}
20.147 - 20 \\
\underline{10.391 - 10} \\
9.756 - 10
\end{array}$$

9.756 − 10, from the table, is the logarithm of 0.570. The slide rule gives for this problem the result, 0.574. The third place is necessarily uncertain when a three-place table is used. The value 20.391 − 20 was changed to 10.391 − 10 so that after subtracting, the remainder would be represented by a positive number minus some multiple of 10. This is convenient because the mantissas in the table are all positive.

APPENDIX 9

The Two Fundamental Theories of Physics

The same element in our make-up that is responsible for our enjoyment of detective stories, puzzles, and mysteries, results in an interest in physical theories. As examples of some of these enigmas we could ask, for example, in what respects is the space surrounding a magnet different from that surrounding an unmagnetized bar of iron, or the space surrounding an electric current unlike the same space with the current turned off. And why does light have all the properties of a wave in a rigid solid along with several other properties that no wave could possibly have? Why does the planet Mercury have many tons more mass when traveling rapidly through the part of its orbit nearest the sun than while moving leisurely through the more distant parts? Why do the same physical laws, which work so well for the engineer and the astronomer, fail utterly to give correct results within the atom? How is it that physical phenomena on a very small scale obey nothing but probability laws, while on a larger scale, everything seems quite determinate and predictable? The answers to these and similar questions lie in the region of physical theory. Most of the present-day physical theory is embodied in two well-known theories, one propounded by Einstein in 1916, called "general relativity theory" and the other developed by several men practically simultaneously in 1925 and 1926 and called "quantum mechanics."

General relativity theory is a new type of geometry not confined to three dimensions, and based on a set of axioms and postulates a little different from those of Euclid. The quantities in this geometry have an exact parallelism with the quantities of physics. The result has all the advantages of being a closely knit, highly deductive branch of mathematics, yet is at the same time a description of large scale physics. Several rather startling predictions made by this theory have been verified experimentally. Some of these were variation of mass with speed and the possibility of annihilating matter with the attending creation of vast quantities of heat, as in the atomic bomb.

Quantum mechanics may be considered to be based logically on Heisenberg's principle of indeterminacy which makes an entity called "action" (dimensions $L^2T^{-1}M$) fundamental both in physics and in atomic physics. Two physical quantities whose dimensions will multiply together and give the dimensions of action (such as energy and time, or momentum and distance) possess a small scale indeterminacy of such a nature that it is useless to try to measure quantities of each so small that their product has an order of magnitude less than Planck's "quantum of action." This quantum is represented by h and is numerically equal to 6.6×10^{-34} joule-second. The result of this theory is that in dealing with such things as the electrons within the atom and other entities like neutrons, positrons, protons, and so on, it is necessary to use a type of probability theory which has grown up into a beautifully consistent, logical system. The net result is that while we have to recognize the fact that Newton's laws of motion, also classical electromagnetic theory, do not hold for such small scale events as those within the quantum of action, yet on a larger scale, the probabilities involved become so extremely near to unity that we may confidently regard them as certainties. Thus there is no contradiction between quantum mechanics and engineering physics; the expression is often used that the former "extrapolates" into the latter.

APPENDIX 10

List of Symbols Used in This Book

A area
a acceleration
B bulk modulus, blackness ($0 < B < 1$), flux density
C candle power, capacitance, conductivity, heat capacity per unit mass
D density, diameter
d distance
E electromotive force, voltage, effort
e linear expansion, elongation
F force
f focal length
G conductance
g acceleration of gravity
H heat
h height, Planck's constant
I current, moment of inertia, illumination
i angle of incidence
K coefficient of cubical expansion
k coefficient of linear expansion, any constant
L latent heat, torque, lumens, inductance
l length
m mass
N order of spectrum
n frequency, number of turns of wire
P power, pressure
p object distance, pitch of screw, magnet pole
q image distance, electrical quantity
R resistance
r radius, amplitude
S shear modulus
s distance
T absolute temperature, period
t temperature, time
u initial velocity, velocity of observer
V volume, velocity of wave
v change in volume, final velocity, velocity of source of wave
W weight
w width of slit
X reactance
x abscissa
Y Young's modulus
y ordinate
Z impedance
Σ summation
Φ magnetic flux
α angular acceleration
θ angle
λ wave length
μ coefficient of friction, index of refraction, permeability
ω_0 initial angular velocity
ω final angular velocity

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